# Designing climate stabilization scenarios via optimal control theory

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## Abstract

Solar radiation management (SRM) by injection of aerosols into the stratosphere is one of the most effective solutions to decelerate the global warming and stabilize the climate [1, 2]. Here, an optimal control theory-based approach is introduced for designing SRM scenarios. The use of optimal control theory allows one to find a control law (the stratospheric aerosol emission rate as a function of time) and the corresponding climate system trajectory such that a desired goal of climate engineering expressed in the form of a certain performance measure is achieved. To illustrate the applicability of this method, a minimum-energy optimal control problem is considered is which a two-box energy-balance model is taken as the model of control object.

## Introduction

Climate models play a key role in exploring climate engineering techniques, and estimating and predicting their effects on the Earth's climate; but due to the uncertainties always existed in climate models that have not been noticeably reduced over the last decade, the resulting range of possible outcomes of hypothetical geoengineering efforts remains quite large. To handle the climate response uncertainties, some relatively recent studies (e.g. [3, 4]) have suggested considering the climate as a control system with feedbacks, which allows for designing the scenarios for geoengineering using relatively simple climate models.

Here, we introduce a new technique for planning climate engineering scenarios and, therefore, designing a climate engineering strategy. By considering the Earth's global climate as controlled dynamical system, we can approach the climate engineering from the perspective of optimal control theory (e.g. [5, 6]). Within an optimal control framework, the goal of geoengineering can be formulated in terms of extremal problem, which entails finding control parameters (or control functions) as well as the corresponding state vector of climate system that minimize or maximize a certain objective functional (also referred to as performance measure or index) subject to a number of constraints.

## Methods

The object of control is the earth's climate system. To simulate the dynamics of controlled object under the influence of external radiative forcing we apply the two-box energy-balance model

$$C(dT/dt) = -\lambda T - \gamma(T - T_D) + \Delta R_{CO_2} + (1-\alpha)\Delta R_A$$
$$C_D(dT_D/dt) = \gamma(T - T_D)$$

where T and $T_D$ are temperature anomalies of atmosphere-mixing ocean and deep ocean layers; $C$ and $C_D$ are the corresponding effective heat capacities; $\lambda$ is the feedback parameter; $\gamma$ is a coupling parameter; $\Delta R_{CO_2}$ is the anthropogenic radiative forcing; $\alpha$ is a planetary albedo; $\Delta R_A$ is the negative radiative forcing generated by aerosols at the top of the atmosphere, which is determined by $\Delta R_A = -\alpha_A Q_0$. Here $\alpha_A$ is the instant albedo of the artificial aerosol layer and $Q_0$ is the insolation. The albedo $\alpha_A$ is assumed to be the control parameter.

The following relationship between the $\alpha_A$ and the mass of aerosol particles $M_A$ can be obtained:

$$\alpha_A = M_A(\beta_A k_A / Q_0 S_e)$$

where $\beta_A = 24\,W/m^2$; $k_A$=7.6 $m^2/g$ is the mass extinction coefficient; $S_e$ is the Earth's area. If the optimal $\alpha_A$ is found then the optimal aerosol emission rate $E_A$ can be obtained via the mass balance equation

$$dM_A/dt = E_A - M_A/\tau_A$$

where $\tau_A \approx 2\ yr$ is the residence time of stratospheric aerosol particles.

The optimal control problem is: Find the admissible control $\alpha_A(t)$ and the corresponding surface temperature anomaly $T$ such that the cost function

$$J = \frac{1}{2}\int_0^{t_f} \alpha_A^2(t)dt$$

is minimized under dynamic constraints (the model equations) and given initial $T(0)$=0 and $T_D(0)$=0, as well as terminal $T(t_f)$ conditions.

Pontryagin's maximum principle is used to solve the optimal control problem. We can rewrite the model equations as

$$dT/dt = -aT + bT_D + ct - q\alpha_A$$
$$dT_D/dt = pT - pT_D$$

where $a = (\alpha+\gamma)/C$; $b = \gamma/C$; $p = \gamma/C_D$; $c = \eta t$; $q = Q_0(1-\alpha)/C$.

The Hamiltonian function for this problem is defined as

$$H = -\frac{1}{2}\alpha_A^2 + \psi_1(-aT + bT_D + ct - q\alpha_A) + \psi_2(pT - pT_D)$$

where $\psi_1$ and $\psi_2$ are time-varying Lagrange multiplier.
Necessary conditions for optimality

$$d\psi_1/dt = -\partial H/\partial T = aT - p\psi_2; \quad d\psi_2/dt = -\partial H/\partial T_D = -b\psi_1 + p\psi_2.$$

The optimal control $\alpha_A^*$ is one that would maximize the Hamiltonian at each fixed time $t \in [0, t_f]$:

$$\alpha_A^* = \arg\max H(\alpha_A)$$

which requires

$$\partial H/\partial \alpha_A = -\alpha_A - q\psi_1$$

Optimal control and optimal trajectory $T(t)$ are given by

$$\alpha_A^*(t) = -q\psi_1 = -C_1 q v_{11} e^{\lambda_1 t} + C_1 q e^{(\lambda_1-\lambda_2)t_f} v_{21} e^{\lambda_2 t}$$
$$T^*(t) = C_3 e^{-\lambda_1 t} + C_4 q e^{-\lambda_2 t} + C_1\left(\alpha e^{-\lambda_1 t} - \beta e^{-\lambda_2 t}\right) + w_1 t + w_2$$

where $C_1$, $C_3$ and $C_4$ are constants of integration; $\lambda_1$ and $\lambda_2$ are eigenvalues of matrix of homogeneous system of model equations; $v_{11}$ and $v_{21}$ are the corresponding eigenvectors; $w_1$ and $w_2$ are known variables that depend on $a$, $b$, $c$ and $p$; $\alpha$ and $\beta$ are known variables that depend on $a$, $b$, $c$, $p$, $\lambda_1$, $\lambda_2$, $v_{11}$ and $v_{21}$.

## Results

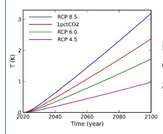



**Figure 1.** Changes in global mean surface temperature calculated for different climate change scenarios in the absence of climate engineering.

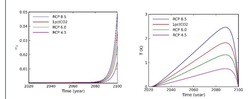


**Figure 2.** Optimal albedo of aerosol layer and corresponding global mean surface temperature (without constraints).

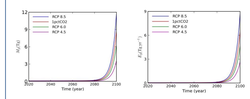


**Figure 3.** Optimal mass of aerosols and corresponding emission rates (without constraints).

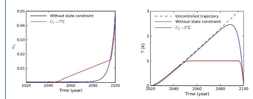


**Figure 4.** Optimal albedo of aerosol layer and corresponding global mean surface temperature with 1°C constraint for temperature increase.

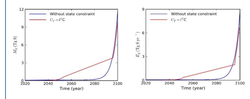


**Figure 5.** Optimal mass of aerosols and corresponding emission rates with 1°C constraint for temperature increase.

**Table 1.** Mass of aerosols loaded into the stratosphere during 2020-2100

| Scenario | RCP 8.5 | 1pctCO$_2$ | RCP 6.0 | RCP 4.5 |
|---|---|---|---|---|
| $M_{S,tot}$, Tg S | 61.97 | 45.87 | 33.32 | 18.83 |

**Table 2.** Mass of aerosols and emission rate in 2100

| Scenario | RCP 8.5 | 1pctCO$_2$ | RCP 6.0 | RCP 4.5 |
|---|---|---|---|---|
| $M_{S,2100}$, Tg S | 11.70 | 8.67 | 6.25 | 3.56 |
| $E_{S,2100}$, Tg S/yr | 8.71 | 6.45 | 4.68 | 2.65 |

By changing control and state constrains, we can evaluate the effectiveness of different strategies for achieving the desired outcome of geoengineering for a specific climate change scenario and estimate the amount of aerosols required.

## Conclusions

Considering climate as controlled system, we propose to approach the geoengineering from the standpoint of the optimal control theory, thereby formulating the goal of geoengineering projects in terms of extremal problem. We hope that the proposed method will provide additional useful insights for the development of optimal climate manipulation strategies to counter global warming.

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