Experimental and Theoretical Investigations on Performance of Macro-Fiber Composites (MFC).

Sreenivasa Prasath. S

Guide: **Dr. Arockiarajan. A**

Solid Mechanics Group

Department of Applied Mechanics

Indian Institute of Technology-Madras, India

June 15, 2015

Outline

1. Introduction
2. Motivation
3. Theoretical models
   - Equivalent layered approach
   - Unit-cell model
4. Experiments
5. Preisach model
6. Summary

Macro-Fiber Composites

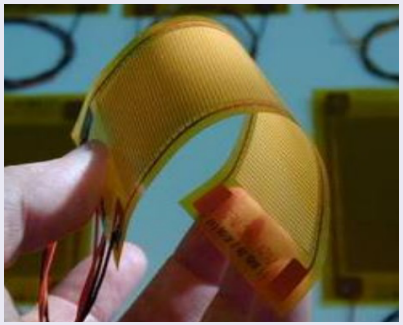
- The Macro-Fiber Composite is a seven layered piezoelectric composite.
  - one active layer (PZT fibers embedded in epoxy matrix)
  - two electrode, acrylic and kapton layers.

Advantages

- Flexibility, directional actuation, environmentally sealed package etc.

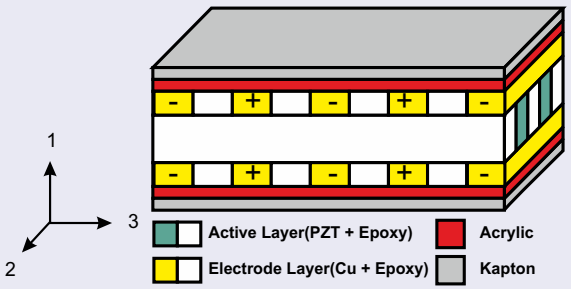
Applications

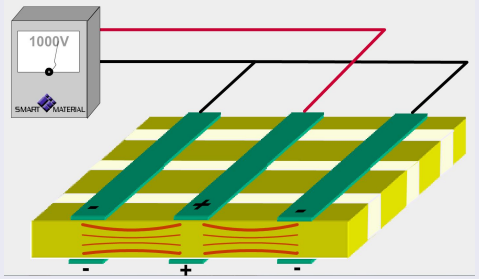
- Actuators, sensors, energy harvesters, structural health monitoring etc.



http://www.smart-material.com

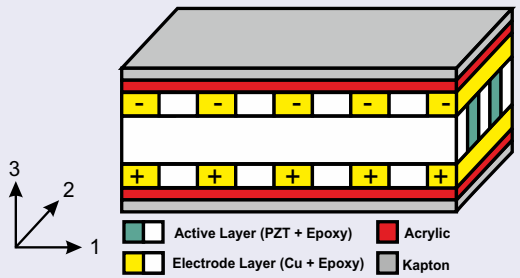
d_{33} MFC Elongator

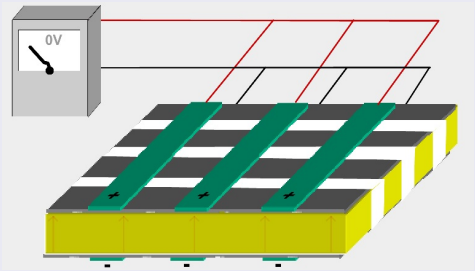




http://www.smart-material.com

d_{31} MFC Contractor





http://www.smart-material.com

Motivation and Objective

Motivation

- In various applications, the MFCs are subjected to wide range of temperature due to which the understanding of thermo-electro-elastic behavior of MFCs becomes vital.
- The data available for modeling the MFC is limited.
- Dearth of data available in literature.

Play/Pause

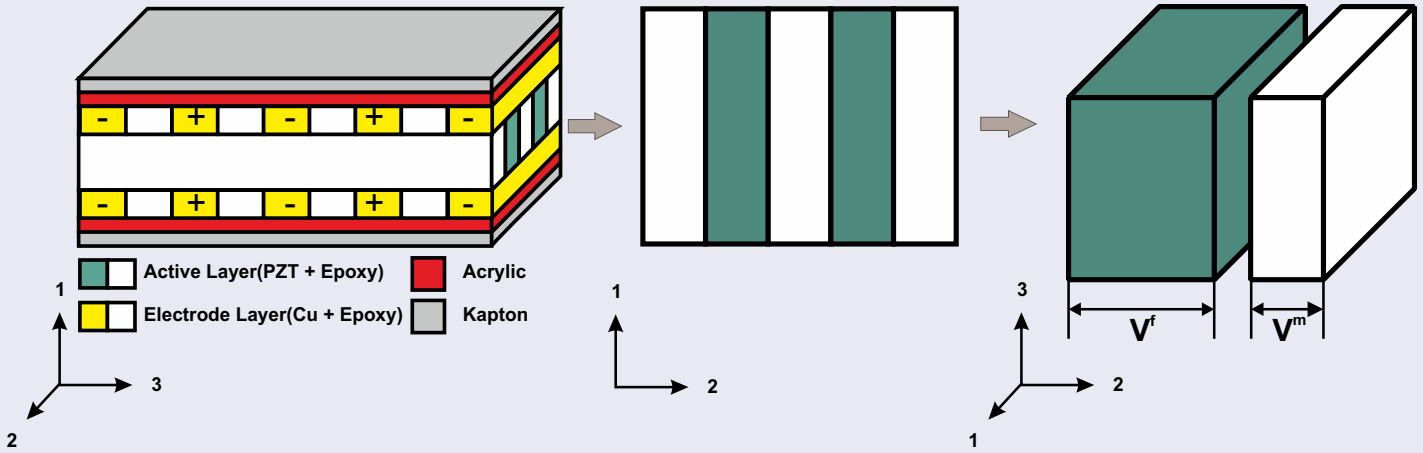
Objective

- To develop a simplified micro-mechanical analytical model to study the effective properties of Macro-Fiber Composites (MFC) under thermal environment.
- To develop a numerical model based on finite element calculations.
- To perform the experiments on available MFCs to validate the proposed models.
- To characterize the non-linear hysteretic behavior of Macro-Fiber Composites.

Analytical model

Equivalent layered approach

- The second phase layer thickness variation is directly proportional to the volume fraction of individual phases [7].
- The assumptions are based on the concept of "rule of mixtures" and "series & parallel capacitance theory".



Assumptions involved

Sequence of formulation	Iso-field assumptions $x^c = x^f = x^m$	Weighted average assumptions $x^c = V^f\, x^f + (1 - V^f)\, x^m$
	where x can represent (S, T, E or D)	
Active layer (V^{act})	S_1, S_3, S_5 T_2, T_4, T_6 $E_3, \Delta\theta$	S_2, S_4, S_6 T_1, T_3, T_5 D_3

Analytical model formulation

- The strain fields of piezoceramics can be expressed in the equation form as,

$$S_k^x = s_{kl}^x T_l^x + d_{km}^x E_m^x + CTE_k^x \theta^x \tag{1}$$

- The variables which do not change with the phases S_i ($i = 1, 3\&5$), T_j ($j = 2, 4\&6$), $\Delta\theta$ and E_3 - independent variables. The variables S_j, T_i, and D_3 which change - dependent variables.
- The formulation starts with replacing the matrix dependent variables (Eq. (1); $x = m$) in-terms of fiber and composite field variables as shown below:

$$S_i^m = s_{ij}^m((T_i^c - T_i^f v^f)/(1 - v^f)) + s_{ij}^m(T_j^c) + d_{3i}^m E_3^c + CTE_i^m \Delta\theta^c \tag{2}$$

- From the assumptions it is evident that S_i^f (Eq. (1); $x = f$)= S_i^m (Eq. (2)), which leads to deriving a relation of fiber dependent field variables expressed in-terms of independent field variables as follows:

$$(s_{ij}^m v^f - s_{ij}^f(1 - v^f))T_j^f = s_{ij}^m T_j^c + (s_{ij}^m(1 - v^f) - s_{ij}^f v^f)T_i^c + (d_{3i}^m(1 - v^f) - d_{3i}^f v^f)E_3^c \tag{3}$$

- Substituting the values for $i\&j$ and writing in a matrix form, the fiber dependent field variables can be represented in terms of independent variables as shown below:

$$\left[T_1^f T_3^f T_5^f\right]' = [A]^{-1}[B]\left[T_1^c T_2^c T_3^c T_5^c E_3^c \Delta\theta^c\right]' \tag{4}$$

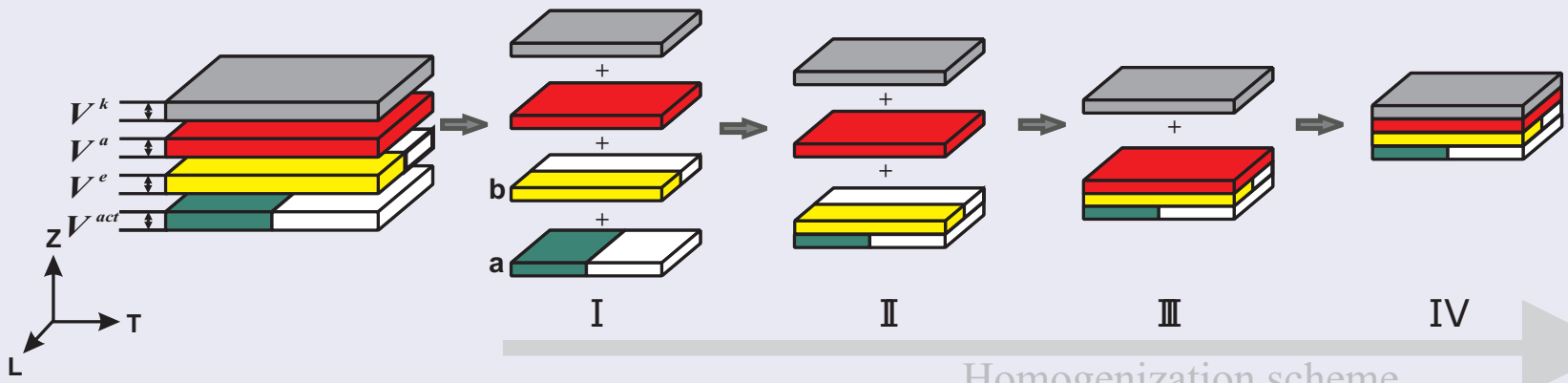
Analytical model formulation cont.

- Substituting the derived fiber dependent field variables in Eq. 1, an equation as a function of independent variables alone is obtained. Finally the electromechanical constants are obtained by equating the coefficients in derived fiber strain field (Eq. 1) and composite strain field (Eq. 4) as follows:

$$\begin{aligned} s^c_{33} &= Z_{13}s^f_{13} + Z_{23}s^f_{33} \\ d^c_{33} &= Z_{15}s^f_{13} + Z_{25}s^f_{33} + d^f_{33} \\ CTE^c_{33} &= Z_{16}s^f_{13} + Z_{26}s^f_{33} + CTE^f_{33} \end{aligned} \tag{5}$$

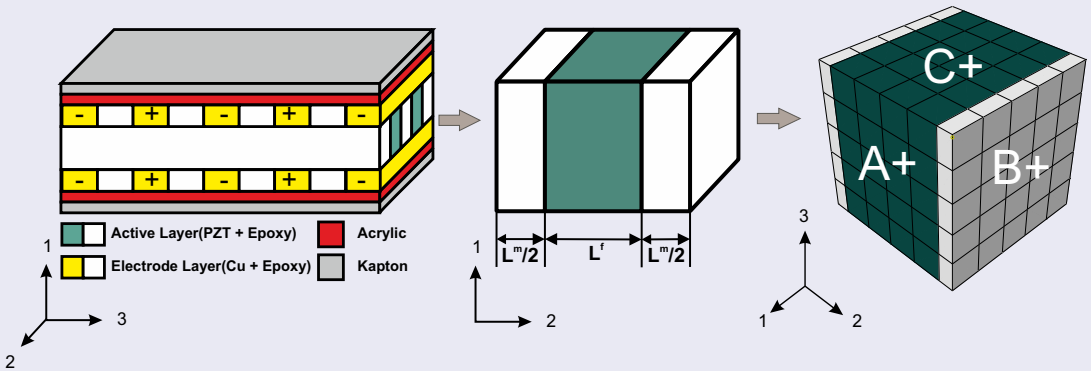
where, the required constants Z_{mn} are functions of volume fraction of constituents and their material properties.

Homogenization scheme



Numerical model - Unit-cell model

- A unit-cell is the smallest part of a periodic composite which contains sufficient information on the geometrical and material parameters at the microscopic level to deduce the effective properties of the composite.



Periodic Boundary Condition

- To ensure continuous deformation and to avoid overlap between unit-cells.
- In a cubic unit cell, the displacements of a pair of opposite boundary surfaces can be represented as [5]:

$$u_i^{S+} = S_{ij}^{avg} x_j^{S+} + u_i^p ; u_i^{S-} = S_{ij}^{avg} x_j^{S-} + u_i^p \tag{6}$$

- The periodic boundary condition for mechanical loading is represented as:

$$u_i^{S+} - u_i^{S-} = S_{ij}^{avg} (x_j^{S+} - x_j^{S-}) \tag{7}$$

- The applied and induced mechanical and electrical fields are evaluated using total volume average technique.

$$S_{ij}^{avg} = \frac{1}{vol} \int_{vol} S_{ij} dvol \quad ; T_{ij}^{avg} = \frac{1}{vol} \int_{vol} T_{ij} dvol \quad ; E_i^{avg} = \frac{1}{vol} \int_{vol} E_i dvol \quad ; D_i^{avg} = \frac{1}{vol} \int_{vol} D_i dvol \tag{8}$$

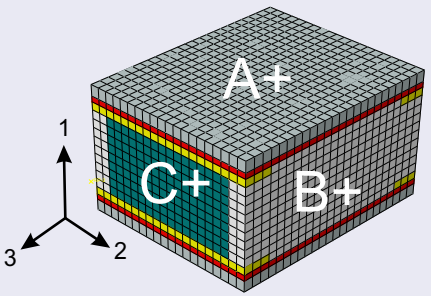
Numerical model cont.

Periodic Boundary Condition

Cases	Electrical PBC	Mechanical PBC		
1 (S_1)		$u_1^{A+} - u_1^{A-} = k$	$u_2^{B+} - u_2^{B-} = 0$	$u_3^{C+} - u_3^{C-} = 0$
2 (S_2)		$u_1^{A+} - u_1^{A-} = 0$	$u_2^{B+} - u_2^{B-} = k$	$u_3^{C+} - u_3^{C-} = 0$
3 (S_3)		$u_1^{A+} - u_1^{A-} = 0$	$u_2^{B+} - u_2^{B-} = 0$	$u_3^{C+} - u_3^{C-} = k$
4 (S_4)	$\phi^{A+} - \phi^{A-} = 0$	$u_1^{A+} - u_1^{A-} = 0$	$u_3^{B+} - u_3^{B-} = k$	$u_2^{C+} - u_2^{C-} = k$
5 (S_5)	$\phi^{B+} - \phi^{B-} = 0$	$u_3^{A+} - u_3^{A-} = k$	$u_2^{B+} - u_2^{B-} = 0$	$u_1^{C+} - u_1^{C-} = k$
6 (S_6)		$u_2^{A+} - u_2^{A-} = k$	$u_1^{B+} - u_1^{B-} = k$	$u_3^{C+} - u_3^{C-} = 0$
7 (E_3)		$u_1^{A+} - u_1^{A-} = 0$	$u_2^{B+} - u_2^{B-} = 0$	$u_3^{C+} - u_3^{C-} = 0$

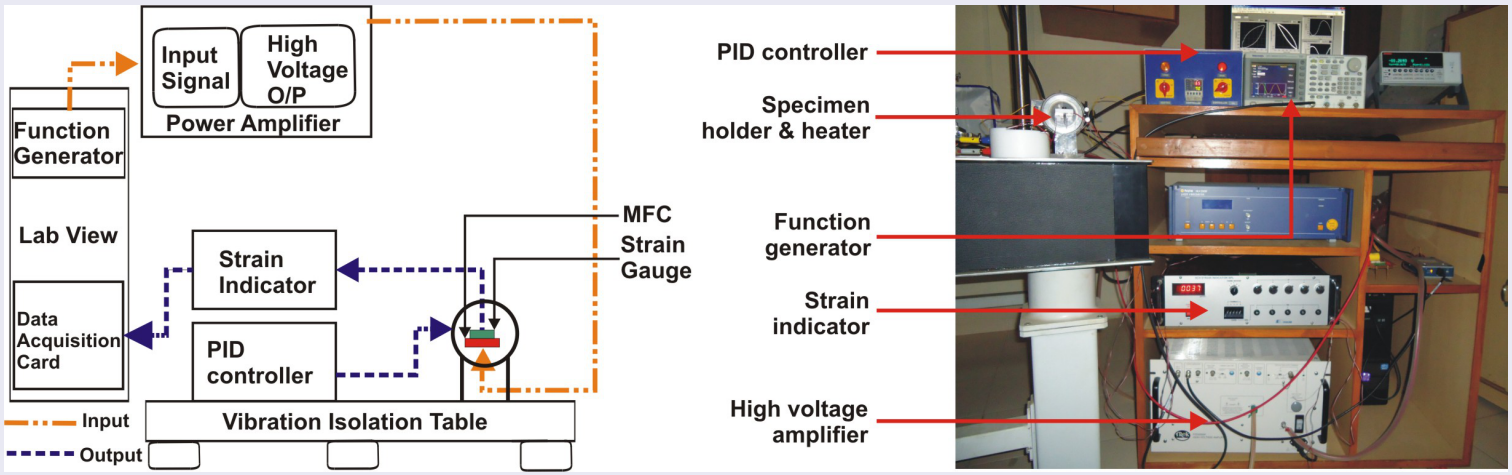
Thermal loading

- The nodes in the A-, B- and C- planes are constrained along x, y and z directions, respectively.
- To get uniform displacement, the nodes in the A+, B+ and C+ planes are coupled along respective directions.
- Reference temperature is set as 0^0C and 1^0C temperature is applied on all nodes.

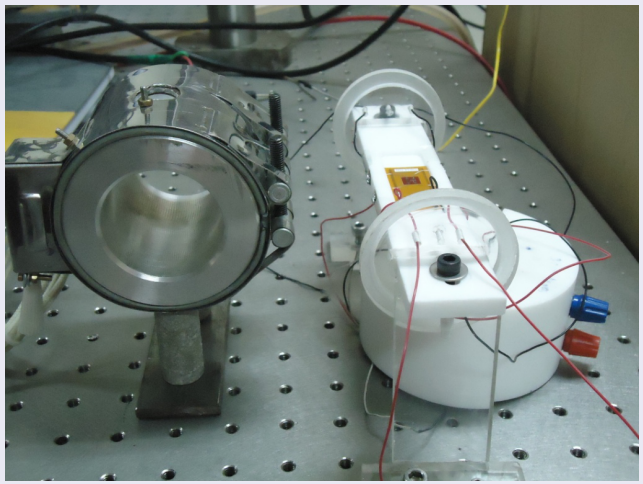


Experimental characterization cont.

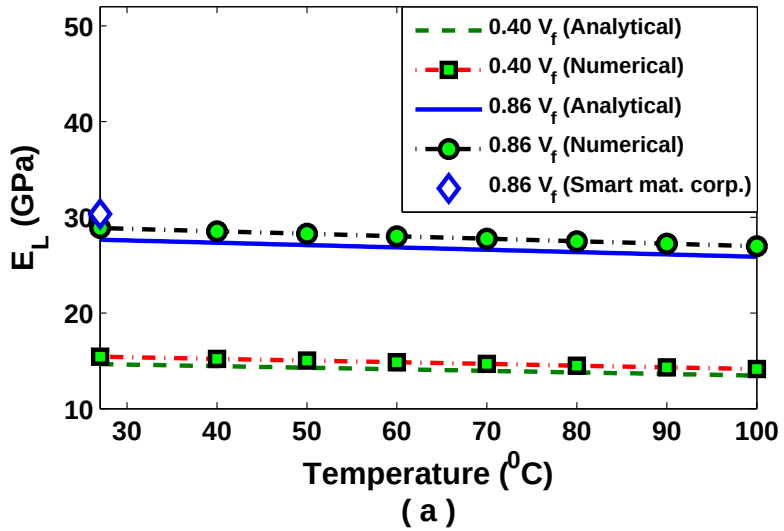
Photograph and schematic of experimental setup

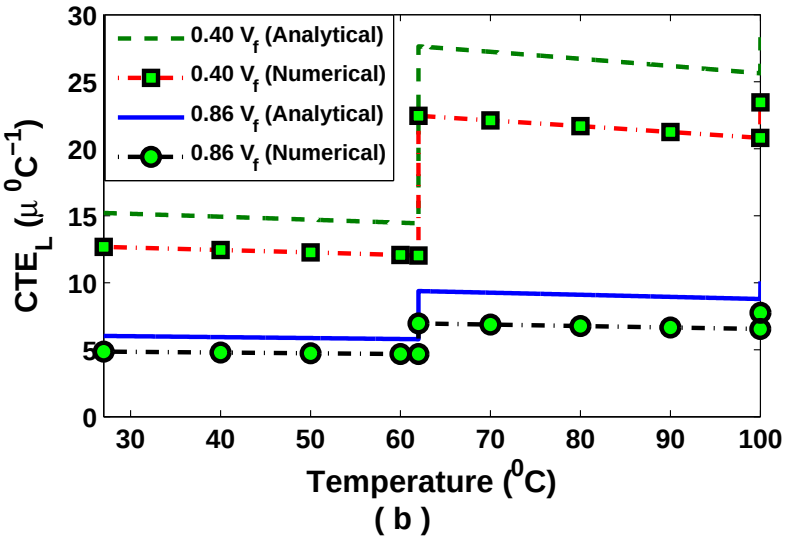


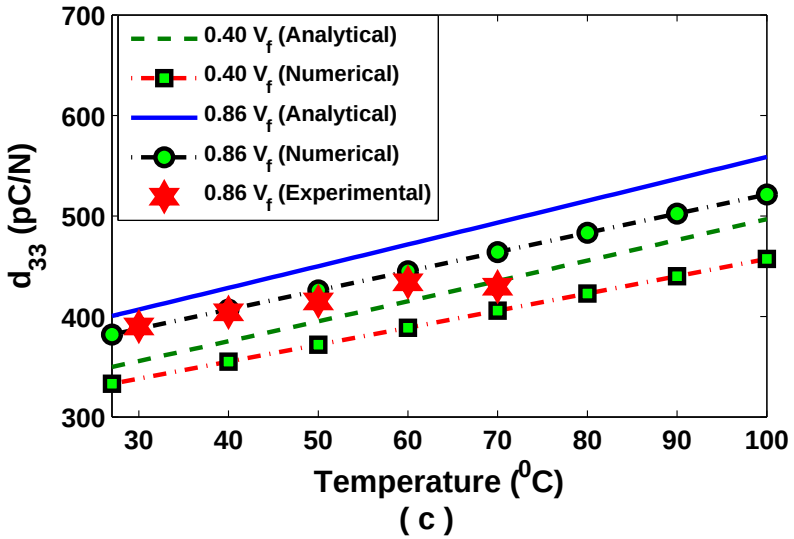
Heater with Specimen holder

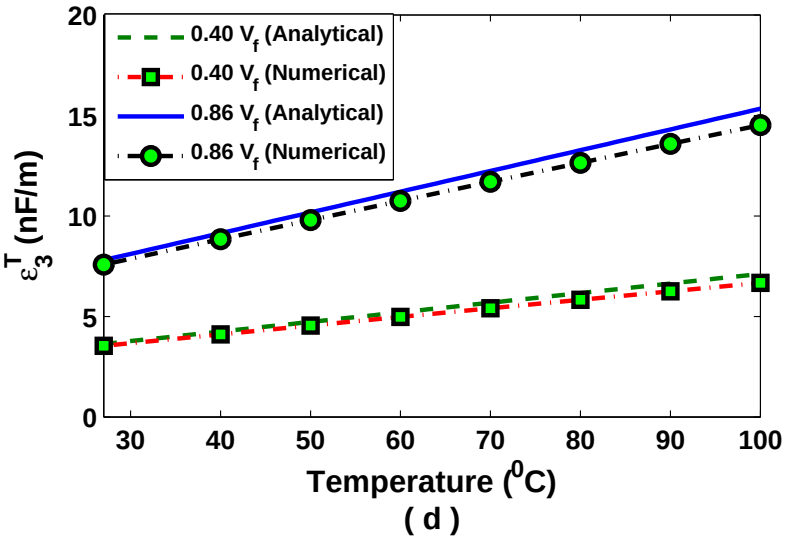


Results


0.40 V_f (Analytical)
0.40 V_f (Numerical)
0.86 V_f (Analytical)
0.86 V_f (Numerical)
0.86 V_f (Smart mat. corp.)
E_L (GPa)
Temperature (^{0}C)
(a)


0.40 V_f (Analytical)
0.40 V_f (Numerical)
0.86 V_f (Analytical)
0.86 V_f (Numerical)
CTE_L (μ $^0C^{-1}$)
Temperature (^{0}C)
(b)

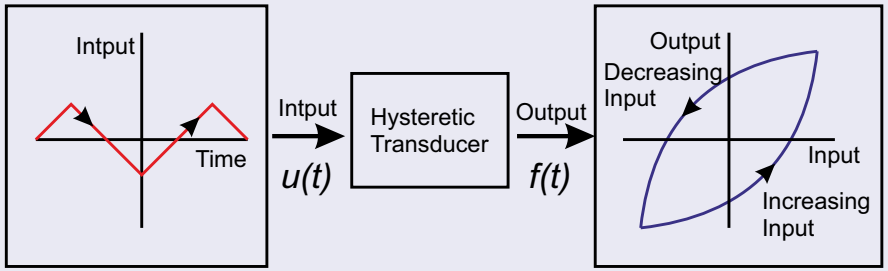

0.40 V_f (Analytical)
0.40 V_f (Numerical)
0.86 V_f (Analytical)
0.86 V_f (Numerical)
0.86 V_f (Experimental)
d_{33} (pC/N)
Temperature (^{0}C)
(c)


0.40 V_f (Analytical)
0.40 V_f (Numerical)
0.86 V_f (Analytical)
0.86 V_f (Numerical)
ε_3^T (nF/m)
Temperature (^{0}C)
(d)

Hysteretic behaviour

Static hysteresis

- The piezoelectric materials exhibits a hysteretic loop even when subjected to moderate cyclic electric field.

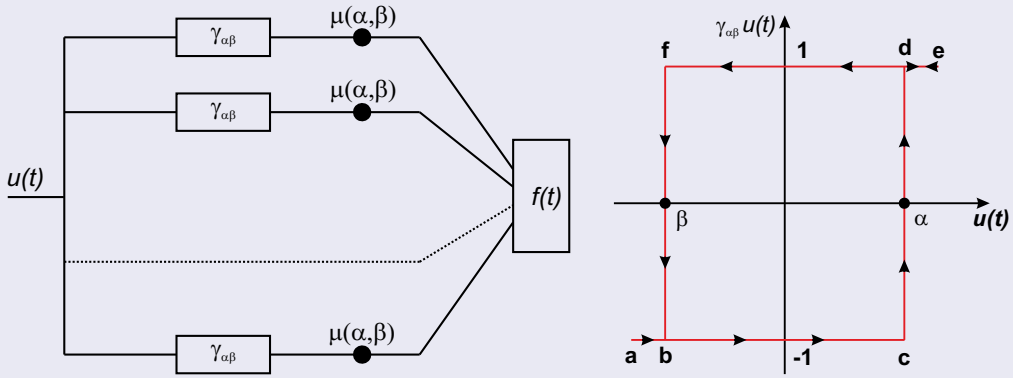


Preisach Model

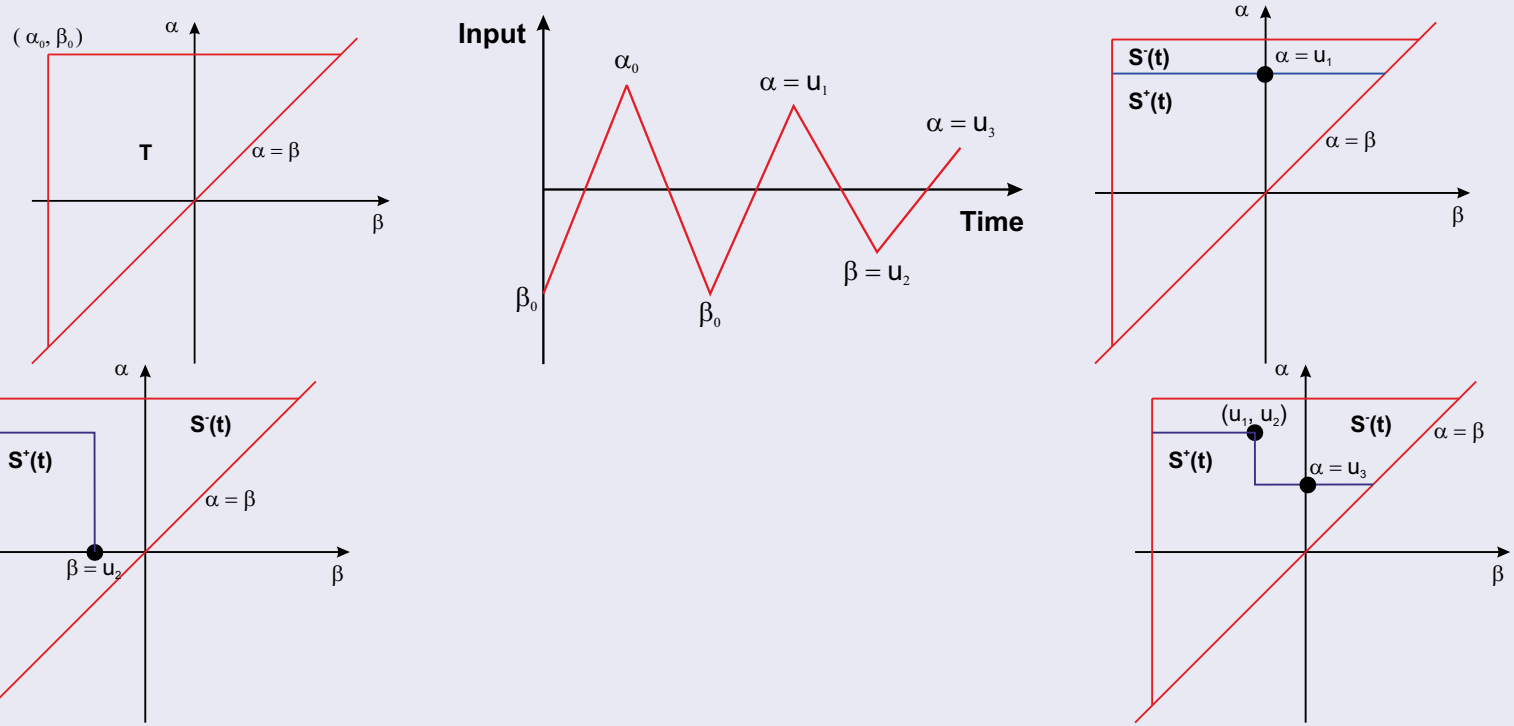
$$f(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta)\gamma_{\alpha\beta}u(t)d\alpha d\beta$$

where, hysteresis operators - $\gamma_{\alpha\beta}u(t)$, Preisach weighting function - $\mu(\alpha, \beta)$, α and β represent switching values of the input

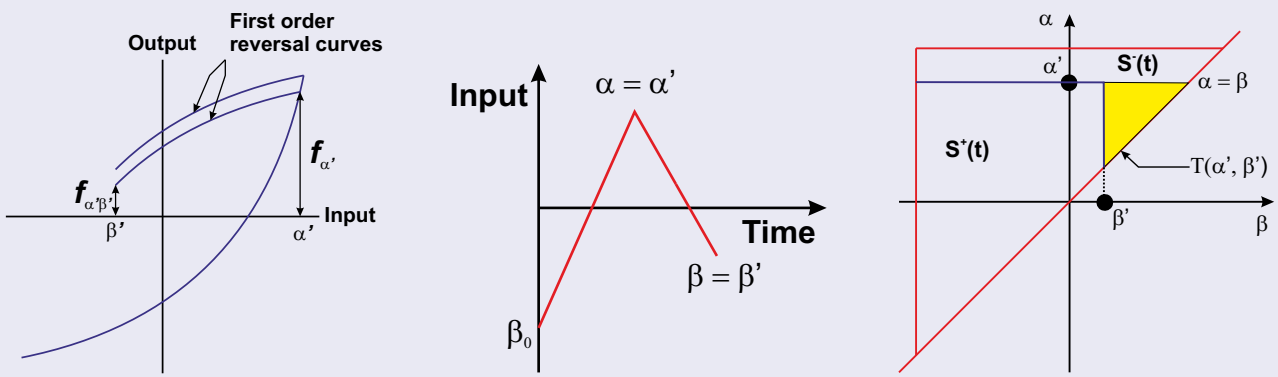
Hysteretic operator



Geometric Interpretation of Preisach Model

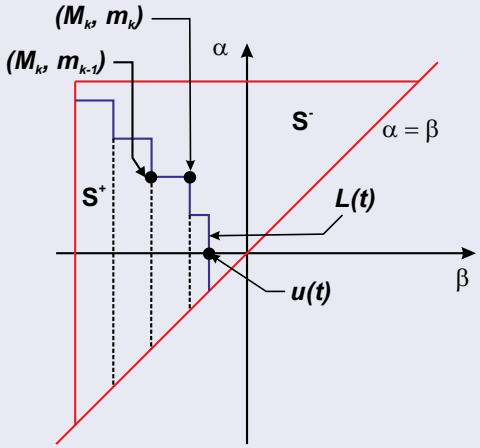


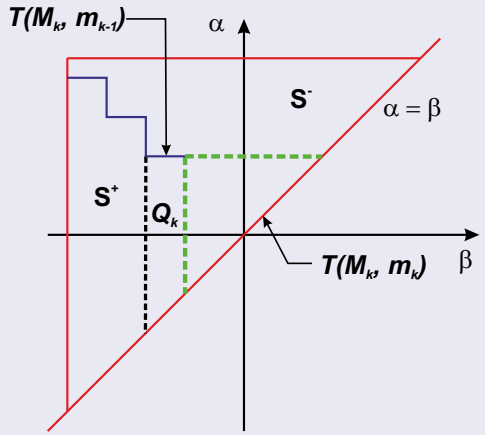
Experemental procedure



Evaluation of Preisach weighing function Cont.

Numerical formulation





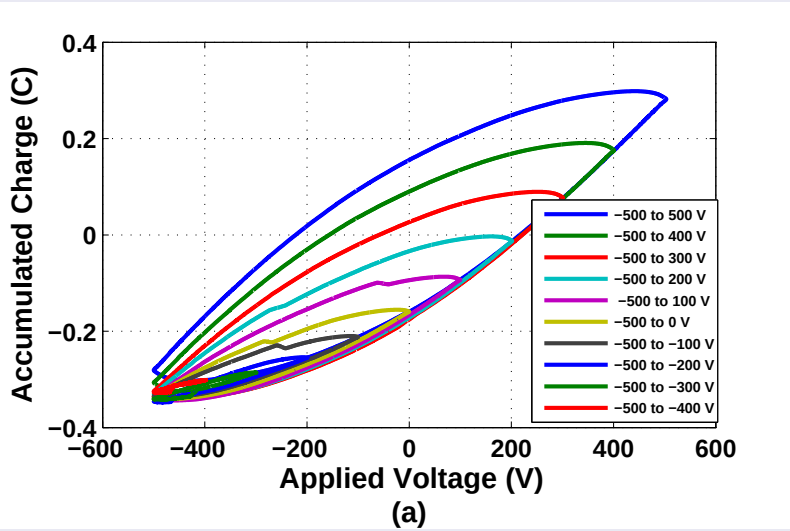
$$f(t) = 2\iint_{S^+(t)} \mu(\alpha,\beta)d\alpha d\beta - \iint_{T} \mu(\alpha,\beta)d\alpha d\beta \tag{9}$$

$$\iint_{T} \mu(\alpha,\beta)d\alpha d\beta = F(\alpha_0,\beta_0) \tag{10}$$

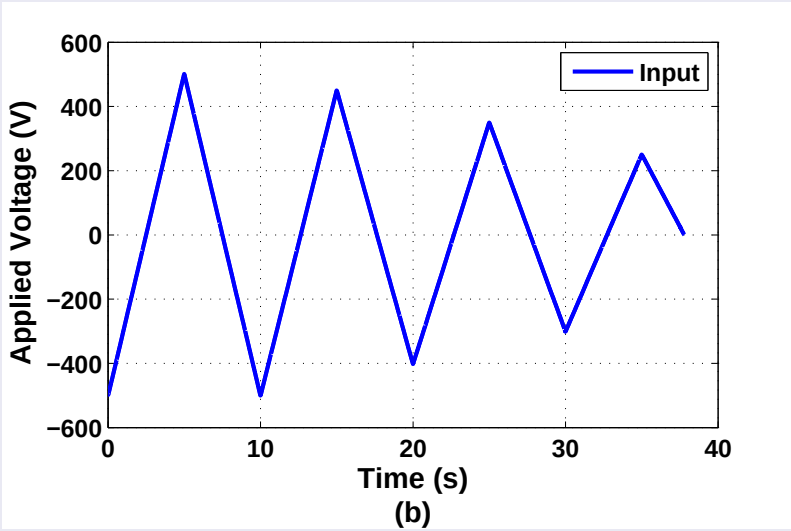
$$\iint_{S^+(t)} \mu(\alpha,\beta)d\alpha d\beta = \sum_{k=1}^{n(t)} \iint_{Q_k(t)} \mu(\alpha,\beta)d\alpha d\beta = \sum_{k=1}^{n(t)} \iint_{T(M_k,m_{k-1})} \mu(\alpha,\beta)d\alpha d\beta - \iint_{T(M_k,m_k)} \mu(\alpha,\beta)d\alpha d\beta \tag{11}$$

$$f(t) = -F(\alpha_0,\beta_0) + 2\sum_{k=1}^{n(t)} [F(M_k,m_{k-1}) - F(M_k,m_k)] \tag{12}$$

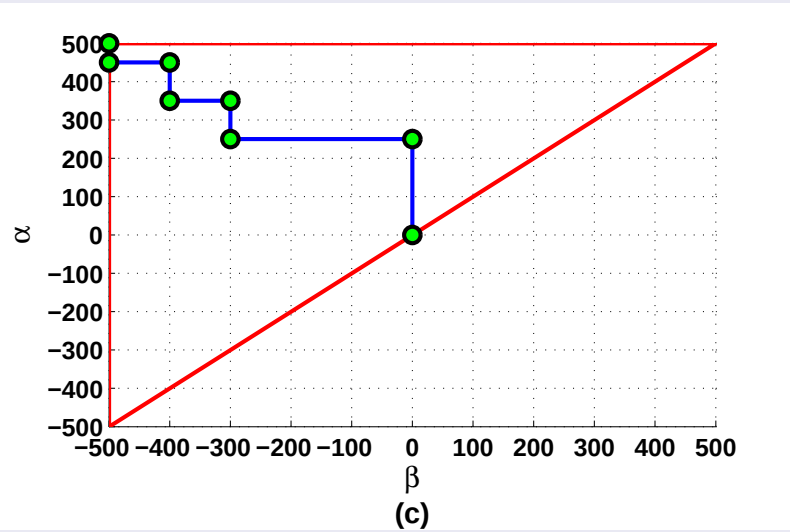
Derivation of Preisach Model Cont.



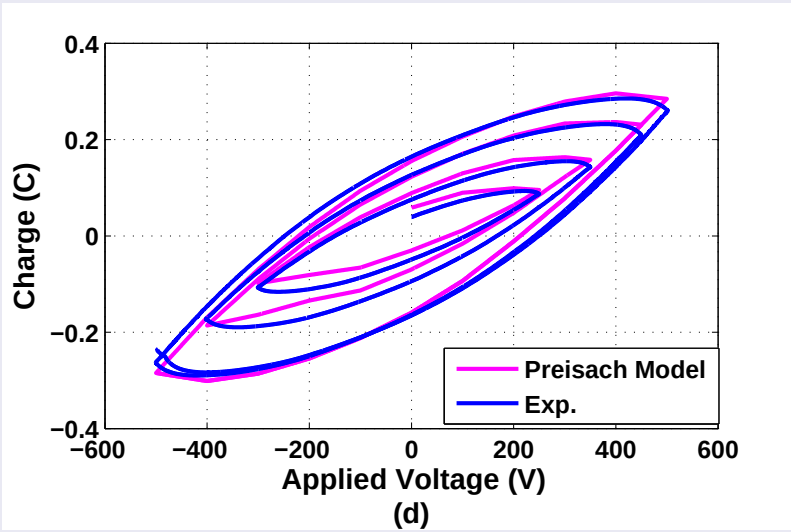
(a)



(b)



(c)



(d)

Summary

- An analytical model based on equivalent layered approach is devised. To account geometric properties and non-uniform electric field a FE approach with PBC is proposed. Experiments are performed on commercially available MFCs.
- A comparative study is carried out between proposed models and the experimental results which are in good agreement.
- The simulated results show that the effect of temperature is significant on the coupling and electrical constants comparative to mechanical constants.
- Based on the devised numerical model 10^0C temperature rise results in $\approx 3.5\%$ increase in coupling constants.
- In addition to the linear characterization, a classical Preisach model is developed to predict the output of a MFC subjected to a cyclic excitation voltage.

References


High J and Wilkie W 2003 Method of fabricating NASA-standard macro-fiber composite piezoelectric actuators NASA/TM-2003-212427, ARL-TR-2833


Bent A A 1997 Active fiber composites for structural applications (PhD Thesis, MIT, USA)


Deraemaeker A, Nasser H, Benjeddou A and Preumont A 2009 Mixing rules for the piezoelectric properties of Macro Fiber Composites J. Intell. Mat. Sys. Struct. 20 1475-82


Deraemaeker A and Nasser H 2010 Numerical evaluation of the equivalent properties of macro fiber composite (MFC) transducers using periodic homogenization Int. J. Solids Struct. 47 3272-85


Suquet P 1987 Elements of homogenization theory for inelastic solid mechanics ed E Sanchez-Palencia and A Zaoui (Homogenization techniques for composite media, Springer-Verlag, Berlin)


Williams R B 2004 Non-linear mechanical and actuation characterization of piezoceramic fiber composites (PhD Thesis, Blacksburg (VA), USA)


Sakthivel M and Arockiarajan A 2012 Thermo-electro-mechanical response of 1-3-2 piezoelectric composites: effect of fiber orientations Acta Mech. 223 1353-69


Mayergoyz ID 1991 Mathematical Models of Hysteresis. New York: Springer.


Yu Y, Naganathan N, Dukkipati R 2002 Preisach modeling of hysteresis for piezoceramic actuator system. Mech Mach Theory 37 4959.

Thank you