

ITMO UNIVERSITY

Linear Optical Quantum Computation Algorithm Based on Directed Graphs or Transform Matrices

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Outline

- Introduction
- Goals and objectives
- Method
- Results and discussion
- Conclusion

Linear optical quantum circuits

Quantum calculations

- Basic quantum logical elements
- Realization of different quantum algorithms

Quantum communiacation

- Basic quantum logical elements
- Possibility of using for networks

Goal

- To derive the linear optical quantum computation algorithm based on directed graphs or transform matrices

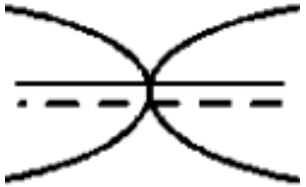
Objectives

- To obtain the relation between directed graphs and linear optical quantum circuits
- Calculation and formation of arbitrary linear optical quantum circuits with different elements and configurations
- Analysis of the performance of the linear optical quantum circuits with arbitrary deviation of parameters

Linear optical elements

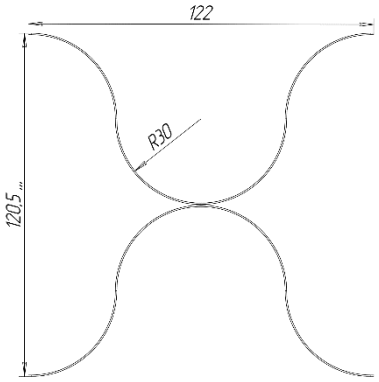
Beamsplitter:

$$\begin{pmatrix} \sqrt{\eta} & \sqrt{1-\eta} \\ \sqrt{1-\eta} & \sqrt{\eta} \cdot e^{i\pi} \end{pmatrix}, 0 \le \eta \le 1$$



Planar beamsplitter:

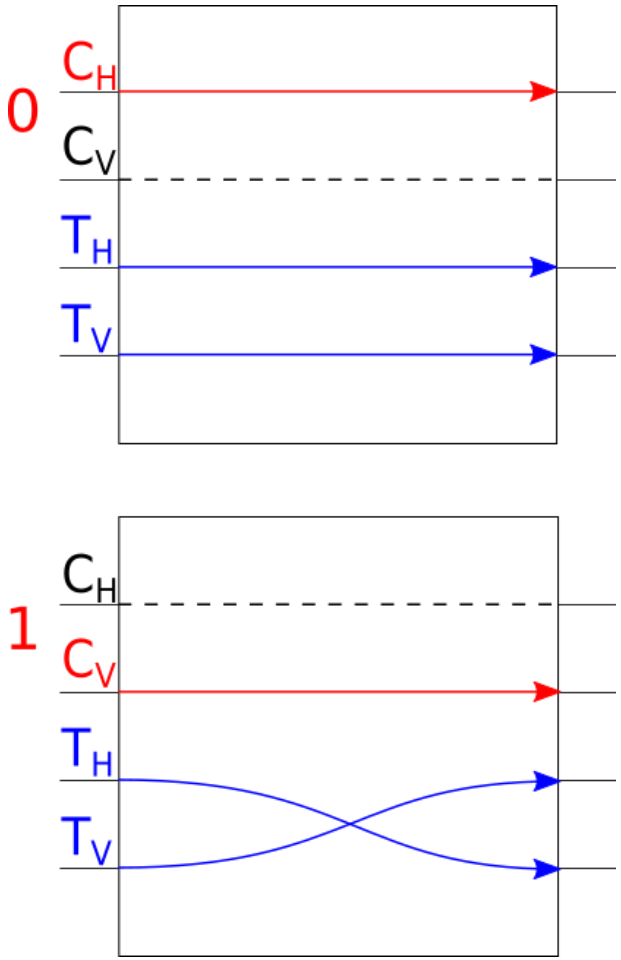
$$\begin{pmatrix} \sqrt{\eta} & \sqrt{1-\eta} \cdot e^{i\frac{\pi}{2}} \\ \sqrt{1-\eta} \cdot e^{i\frac{\pi}{2}} & \sqrt{\eta} \end{pmatrix}, 0 \le \eta \le 1$$

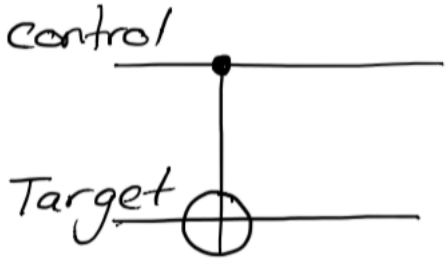


Wave plate:

$$\begin{pmatrix} e^{i\phi}\cos^2\alpha + \sin^2\alpha & \left(e^{i\phi}-1\right) cos\alpha\, sin\alpha \\ \left(e^{i\phi}-1\right) cos\alpha\, sin\alpha & e^{i\phi}\sin^2\alpha + \cos^2\alpha \end{pmatrix}, \begin{matrix} 0 \le \phi \le 2\pi \\ 0 \le \alpha \le 2\pi \end{matrix}$$

CNOT gate



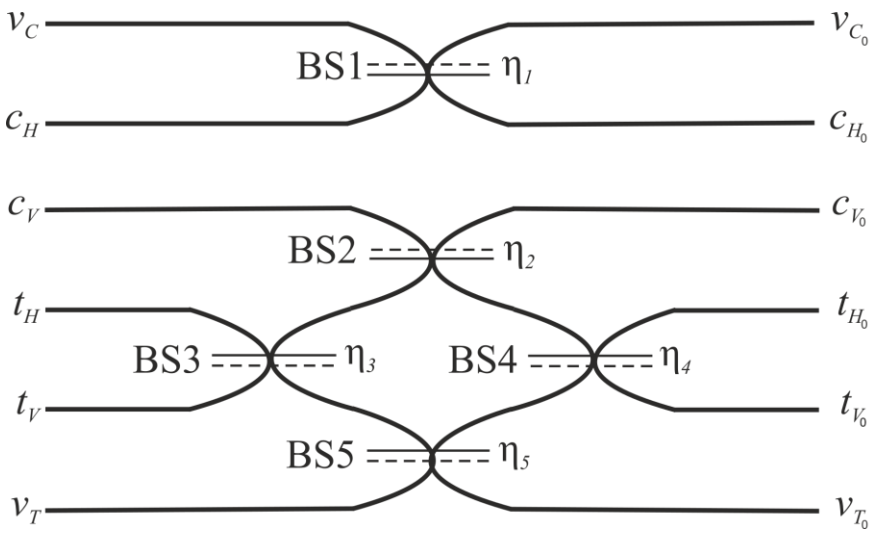


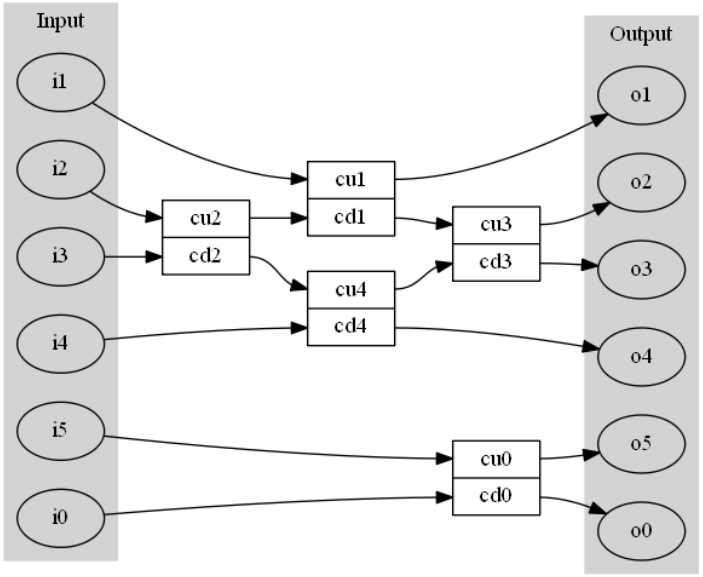
CNOT can represented by the matrix:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Pic.1 CNOT gate

Linear optical CNOT gate





Pic.2 Linear optical CNOT gate circuit*

Pic.3 Directed graph for linear optical CNOT gate circuit

*T. C. Ralph, N. K. Langford, T. B. Bell, and A. G. White Linear optical controlled-NOT gate in the coincidence basis // Physical Review A, V. 65, I. 062324, 2002

Algorithm

- Creating the directed graph
- Obtaining the transform matrix
- Checking all elements
- Checking the unitary of the matrix
- Analyzing deviations from original logical description
- Analyzing error probability for different deviation of circuit’s parameters
- Creating an optical circuit

Transform matrix for linear optical CNOT gate

$$\begin{pmatrix}
\sqrt{\eta_0}\cdot e^{i\pi} & 0 & 0 & 0 & 0 & \sqrt{1-\eta_0} \\
0 & \sqrt{\eta_1} & \sqrt{1-\eta_1}\cdot\sqrt{\eta_3} & \sqrt{1-\eta_1}\cdot\sqrt{1-\eta_3} & 0 & 0 \\
0 & \sqrt{\eta_2}\cdot\sqrt{1-\eta_1} & \begin{matrix}\sqrt{\eta_2}\cdot\sqrt{\eta_1}e^{i\pi}\cdot\sqrt{\eta_3}+\\ +\sqrt{1-\eta_2}\cdot\sqrt{\eta_4}\cdot\sqrt{1-\eta_3}\end{matrix} & \begin{matrix}\sqrt{\eta_2}\cdot\sqrt{\eta_1}e^{i\pi}\cdot\sqrt{1-\eta_3}+\\ +\sqrt{1-\eta_2}\cdot\sqrt{\eta_4}\cdot\sqrt{\eta_3}e^{i\pi}\end{matrix} & \sqrt{1-\eta_2}\cdot\sqrt{1-\eta_4} & 0 \\
0 & \sqrt{1-\eta_2}\cdot\sqrt{1-\eta_1} & \begin{matrix}\sqrt{1-\eta_2}\cdot\sqrt{\eta_2}e^{i\pi}\cdot\sqrt{\eta_3}+\\ +\sqrt{\eta_2}e^{i\pi}\cdot\sqrt{\eta_4}\cdot\sqrt{1-\eta_3}\end{matrix} & \begin{matrix}\sqrt{1-\eta_2}\cdot\sqrt{\eta_1}e^{i\pi}\cdot\sqrt{1-\eta_3}+\\ +\sqrt{\eta_2}e^{i\pi}\cdot\sqrt{\eta_4}\cdot\sqrt{\eta_3}e^{i\pi}\end{matrix} & \sqrt{\eta_2}e^{i\pi}\cdot\sqrt{1-\eta_1} & 0 \\
0 & 0 & \sqrt{1-\eta_4}\cdot\sqrt{1-\eta_3} & \sqrt{1-\eta_4}\cdot\sqrt{\eta_3}e^{i\pi} & \sqrt{\eta_4}\cdot e^{i\pi} & 0 \\
\sqrt{1-\eta_0} & 0 & 0 & 0 & 0 & \sqrt{\eta_0}
\end{pmatrix}$$

$$\boldsymbol{A}=\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66}
\end{pmatrix}$$



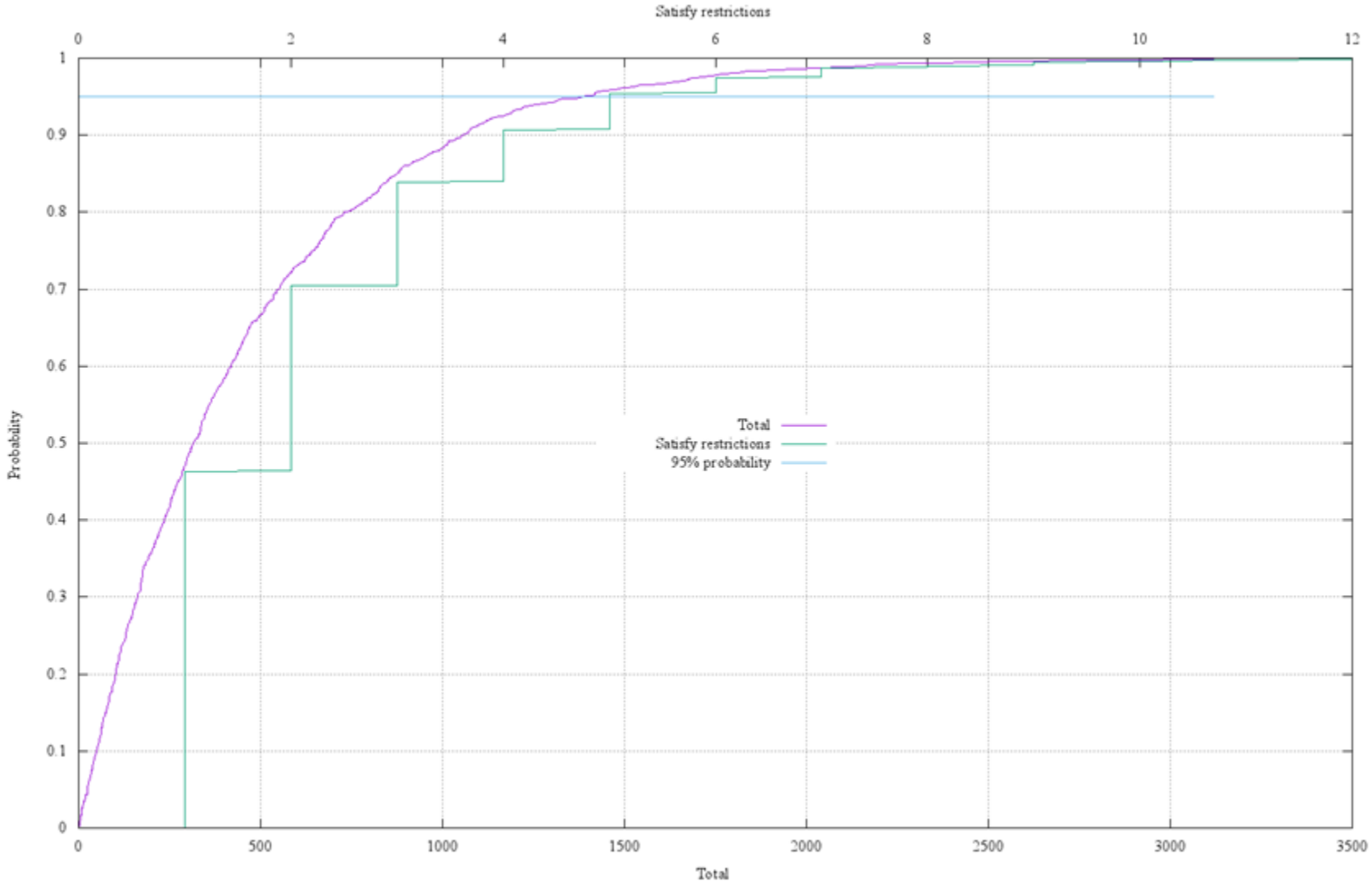
$$\boldsymbol{M}=\begin{bmatrix}
\begin{matrix}a_{22}a_{44}+\\ +a_{24}a_{42}\end{matrix} & \begin{matrix}a_{22}a_{34}+\\ +a_{24}a_{32}\end{matrix} & \begin{matrix}a_{12}a_{44}+\\ +a_{14}a_{42}\end{matrix} & \begin{matrix}a_{12}a_{34}+\\ +a_{14}a_{32}\end{matrix} \\
\begin{matrix}a_{21}a_{43}+\\ +a_{23}a_{42}\end{matrix} & \begin{matrix}a_{22}a_{33}+\\ +a_{24}a_{32}\end{matrix} & \begin{matrix}a_{12}a_{43}+\\ +a_{13}a_{42}\end{matrix} & \begin{matrix}a_{12}a_{33}+\\ +a_{13}a_{32}\end{matrix} \\
\begin{matrix}a_{21}a_{44}+\\ +a_{24}a_{41}\end{matrix} & \begin{matrix}a_{21}a_{34}+\\ +a_{24}a_{31}\end{matrix} & \begin{matrix}a_{11}a_{44}+\\ +a_{14}a_{41}\end{matrix} & \begin{matrix}a_{11}a_{34}+\\ +a_{14}a_{31}\end{matrix} \\
\begin{matrix}a_{21}a_{43}+\\ +a_{23}a_{41}\end{matrix} & \begin{matrix}a_{21}a_{33}+\\ +a_{23}a_{31}\end{matrix} & \begin{matrix}a_{11}a_{43}+\\ +a_{13}a_{41}\end{matrix} & \begin{matrix}a_{11}a_{33}+\\ +a_{13}a_{31}\end{matrix}
\end{bmatrix}$$

Monte Carlo method

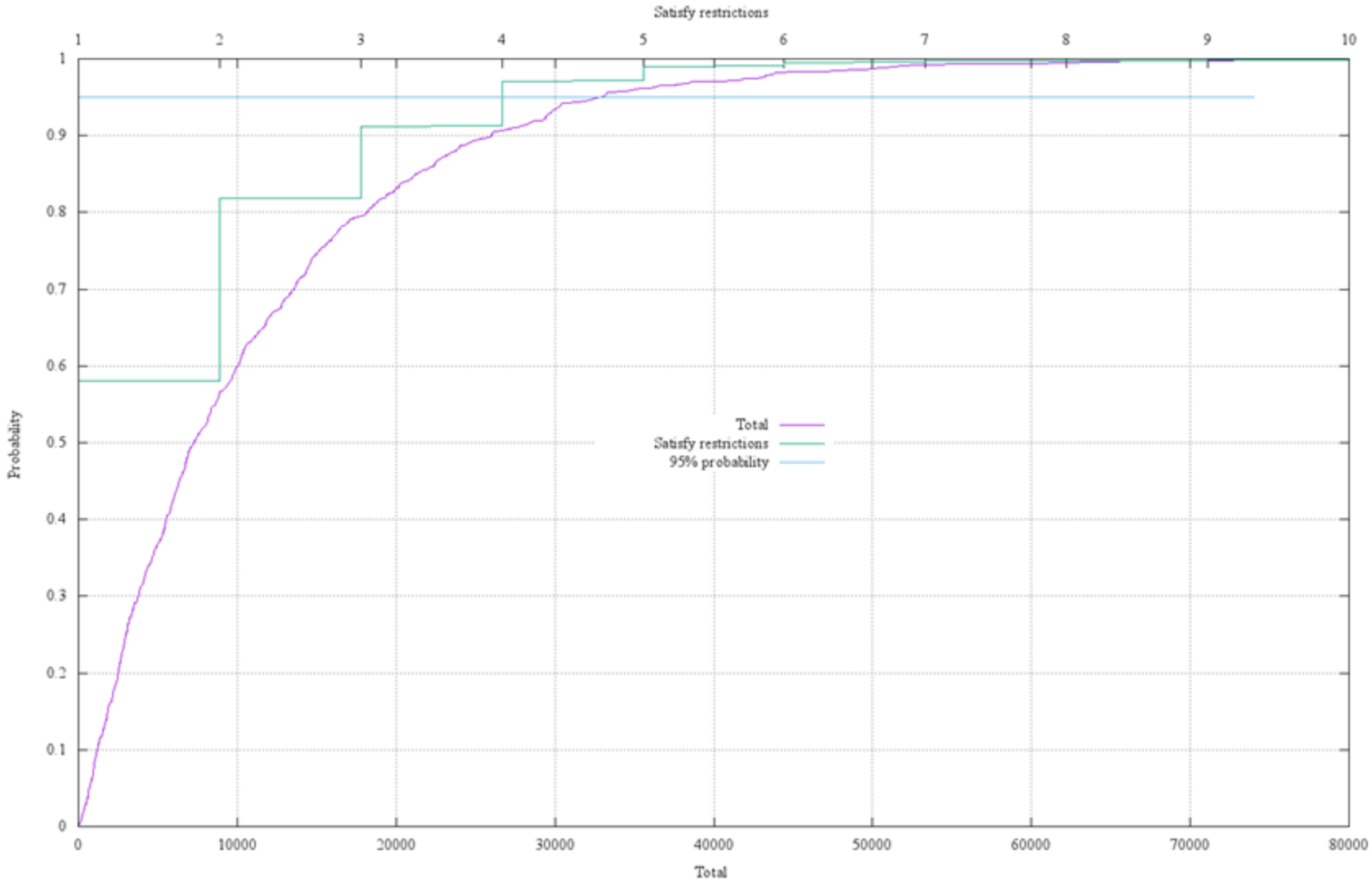
- Randomly choose values of parameters for different elements
- Checking whether all the conditions for this values are equal to each other within eps

$$\begin{bmatrix} a_{22}a_{44}+ & a_{22}a_{34}+ & a_{12}a_{44}+ & a_{12}a_{34}+ \\ +a_{24}a_{42} & +a_{24}a_{32}=0 & +a_{14}a_{42}=0 & +a_{14}a_{32}=0 \\ a_{22}a_{43}+ & a_{22}a_{33}+ & a_{12}a_{43}+ & a_{12}a_{33}+ \\ +a_{23}a_{42}=0 & +a_{24}a_{32} & +a_{13}a_{42}=0 & +a_{13}a_{32}=0 \\ a_{21}a_{44}+ & a_{21}a_{34}+ & a_{11}a_{44}+ & a_{11}a_{34}+ \\ +a_{24}a_{41}=0 & +a_{24}a_{31}=0 & +a_{14}a_{41}=0 & +a_{14}a_{31} \\ a_{21}a_{43}+ & a_{21}a_{33}+ & a_{11}a_{43}+ & a_{11}a_{33}+ \\ +a_{23}a_{41}=0 & +a_{23}a_{31}=0 & +a_{13}a_{41} & +a_{13}a_{31}=0 \end{bmatrix}$$

- If satisfied - then check whether the objective function is in the permissible range



Pic.1. The dependence of the probability of finding the global maximum of the number of random points with eps = 0.1.



Pic.1. The dependence of the probability of finding the global maximum of the number of random points with eps = 0.05.

Results and discussion

- Derived the relation between directed graphs and linear optical quantum circuits
- Calculated arbitrary linear optical quantum circuits with different elements and configurations
- Obtained and plotted the most efficient linear optical quantum circuit
- Developed the linear optical quantum computation algorithm based on directed graphs or transform matrices

Conclusion

- It was derived that there is only several apropriative possible combinations of parameters of circuit

- it was investigated, that with

$$\eta_1 = \eta_2 = \eta_5 = 0.33 \pm 0.03,$$

$$\eta_3 = \eta_4 = 0.5 \pm 0.05$$

the circuit operates good and the error probability is near 1 per cent



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Thank you for your attention!

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