The ABCD matrix for parabolic reflectors and its application to astigmatism free four-mirror cavities





Outline

- Motivations
- Geometrical compensation of ellipticity (with spherical mirrors involved)
  - Symmetry considerations
  - Numerical solutions
- Compensation of ellipticity with mirror shape (with parabolic mirrors)
  - ABCD matrix for parabolic mirror
  - Parabolic mirror cavities example(4-mirror cavities)

Recent Developments

- Increase of the cavity stacked power more than 670 kW *(H. Carstens OL 39(2014)9)*
- Burst mode development *(K. Sakaue NIMA 637 (2011) S107-S111)*
- Increase of laser beam power up to 1J@100Hz for passive cavity *(B.A. Reagan OL 37(2012)17)*
- Increase interest on (Compton) X/γ-ray machine with optical cavity
  - X-ray for material science, medical, etc.
  - γ-ray machine for photonuclear physics, particle physics, etc.

Optical considerations for Compton γ-ray beam production

Requirements

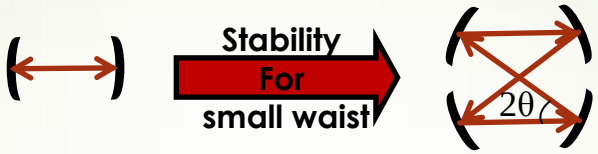
- Polarization switching (P/S)
- High γ-ray flux
  - ➔ High intensity laser beam
- High laser-cavity coupling
- Reasonable cavity length (few meters: ~100MHz)
- Mechanically stable (mode and beam path)

Constraints

- Even number of reflective surfaces
- Small waist (~30µm)
- large beam size nearly collimated at the injection
- No ellipticity (on mirrors)
- Large laser beam area on optics => avoid Laser Damage Threshold

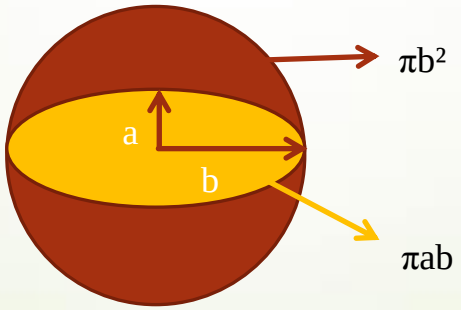
Starting point

- Angle θ ≠ 0 on a spherical mirror => ellipticity (astigmatism)



Consideration for optical cavities:

- Smaller waist => higher ellipticity (due to θ) **(if no compensation)**
- Higher Ellipticity => smaller beam spot area on optics
- Smaller beam spot => higher fluence



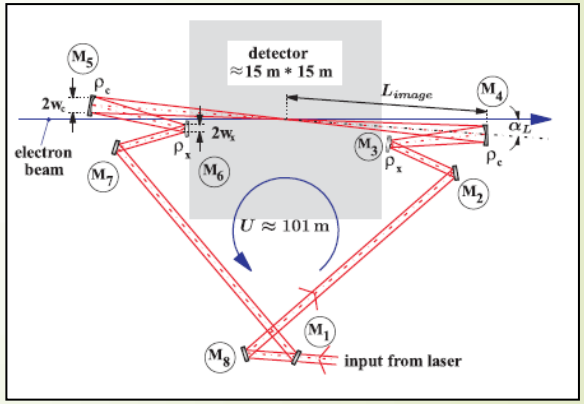
$$Ellipticity = \max_{mirrors} \frac{|a - b|}{a + b}$$

➔ **Smaller waist ➔ higher ellipticity ➔ higher fluence on optics**

Solutions

➔ Ellipticity free cavity

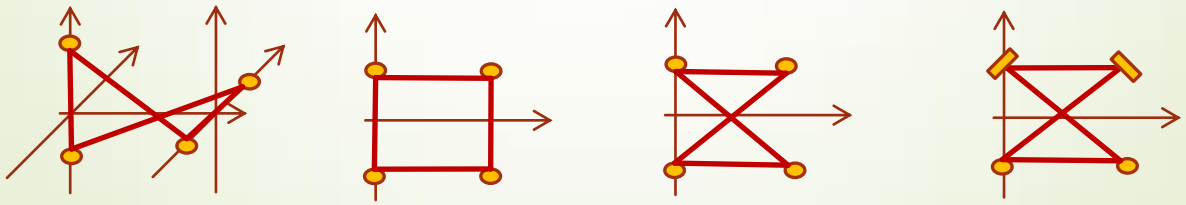
- **Geometrical compensation of ellipticity**
  - *(e.g. T. Skettrup: J.Opt.A 7(2005)7)*
- **Compensation of ellipticity with mirror shape**
  - Telescope system *(e.g. K. Mönig: NIMA 564(2006)212)*
    - Many optical surfaces
    - Stability to be studied
  - 2 Cylindrical mirrors (4-mirror cavity)
    - Tolerance on fabrication
    - Adjustable ?
  - 2 Parabolic mirrors (4-mirror cavity)
    - Intrinsically not astigmatic



K. Mönig: NIMA 564(2006)212

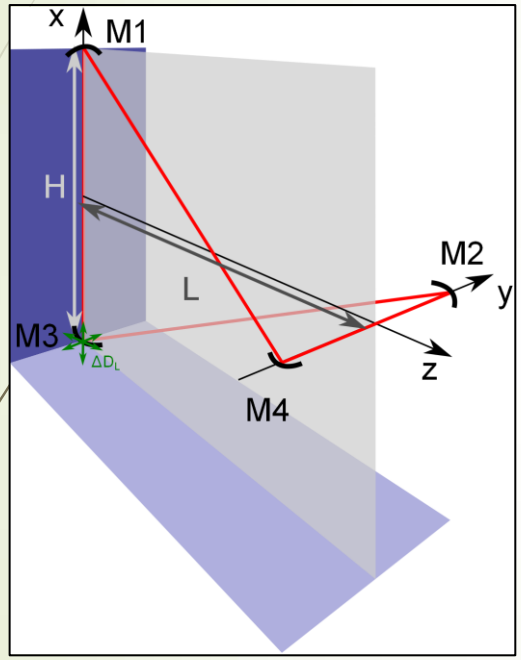
Geometrical compensation of ellipticity:

Studies of spherical mirror cavities



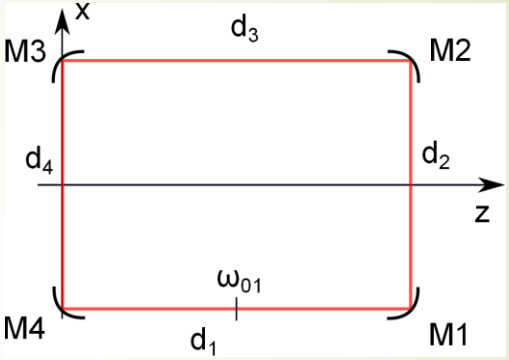
Symmetry considerations (with 4 spherical mirrors)

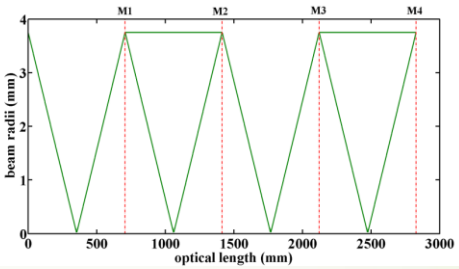
tetrahedron configuration *(I. Pupeza)*

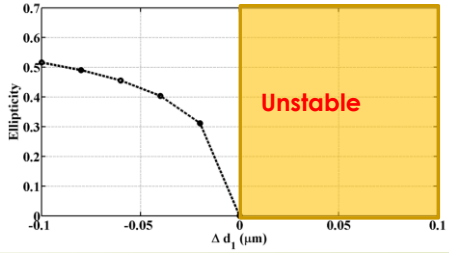


- **No ellipticity** by construction
- **Mechanically highly unstable**
- Polarization effect (only circular polarization) *(F. Zomer: Appl. Opt. 48(2009)35)*

planar ring configuration *(T. Skettrup: J.Opt.A 7(2005)645)*







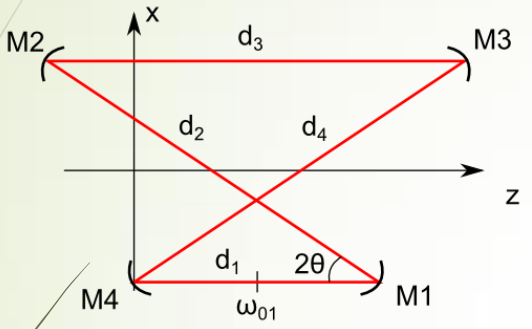
With $d_1 = d_3$, $d_2 = d_4$

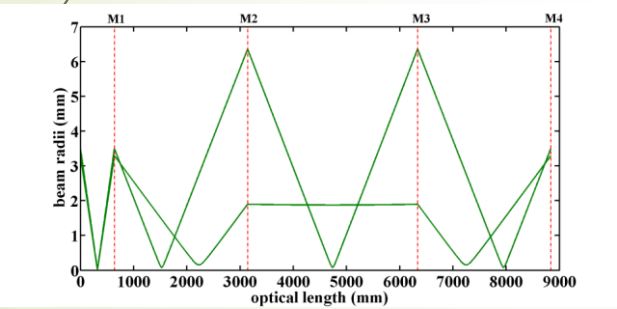
- **Mechanically unstable**
- polarization effects *(High incident angle: 45°)*
- **Not adjustable**

Numerical solutions

Solutions of the equations introduced in T. Skettrup (J.Opt.A 7(2005)7)

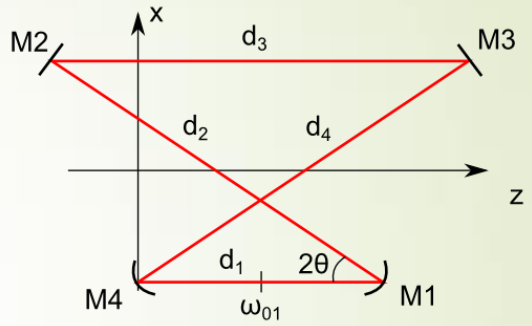
4 spherical mirrors Bow Tie Cavity configuration (BTC)

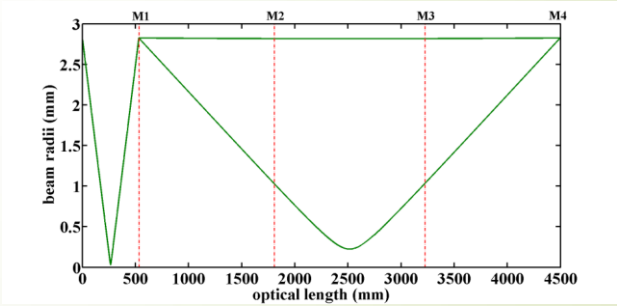




- **High ellipticity** on Mirrors
- **Very low coupling efficiency** (no collimated beam + untypical beam mode)
- Long cavity

2 spherical + 2 flat mirrors BTC configuration





- **High ellipticity** on Mirrors
- **Low coupling efficiency** (no collimated beam)
- Long cavity

Summary of cavities compound of spherical mirrors

- Always circular beam waist (even using spherical mirrors with non vanishing incident angle)
- Ring and tetrahedron geometry mechanically unstable
- Bow Tie Configuration :
  - 4 spherical mirrors
    - coupling issues
      - Difficult to inject through spherical mirror = diverging lens
      - Beam mode
    - long cavity
  - 2 spherical mirrors
    - coupling issues
    - long cavity
  - High ellipticity on mirrors

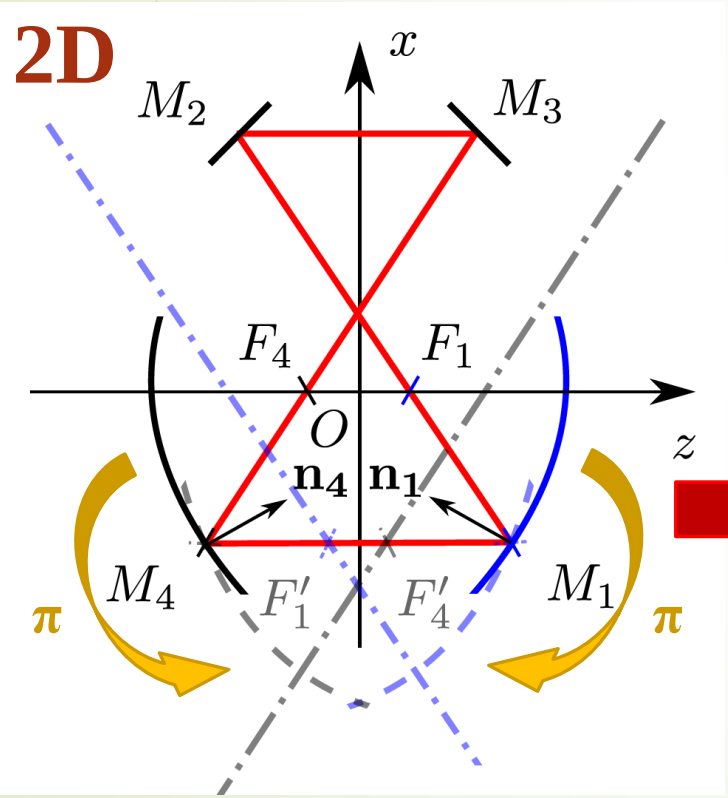
➔ **Use of stigmatic mirrors** (e.g. parabolic mirrors) **in BTC configuration with 2 concave mirrors + 2 flat mirrors**

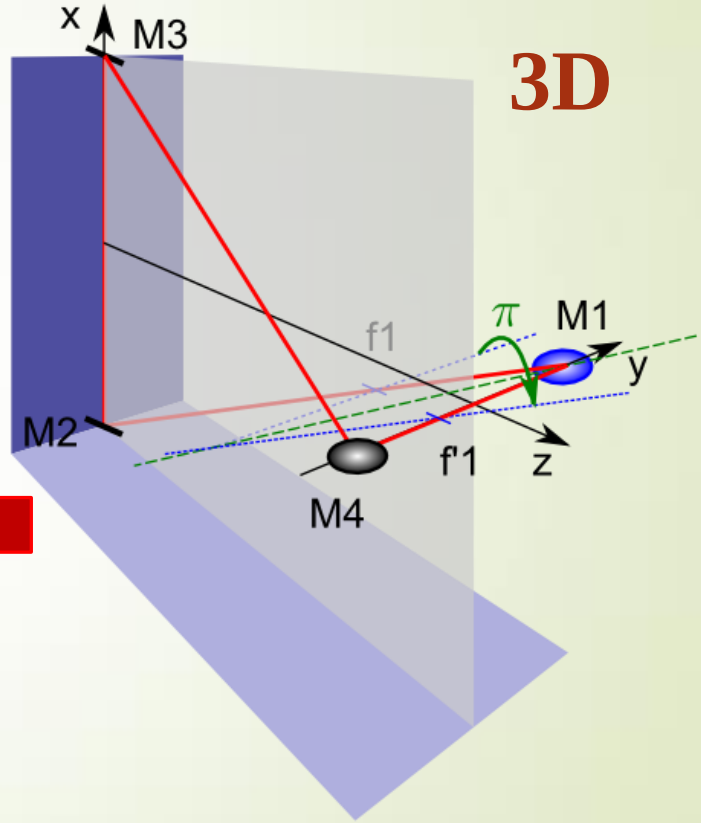
Compensation of ellipticity with mirror shape :

Study of parabolic mirrors cavity

Ellipticity free cavity *(with parabolic mirrors)*

➔ Circular beam spot ⇔
optical path pass through the 2 focal points of parabolic mirrors
(stigmatic configuration)





= ∞ ellipticity free configurations

ABCD matrix for parabolic mirror

Details in K. Dupraz (Opt. Com. 353(2015)178-183)

- M. Sieber *(Nonlinearity 11 (1998) 1607–1623)* gives for any ellipsoidal surface:

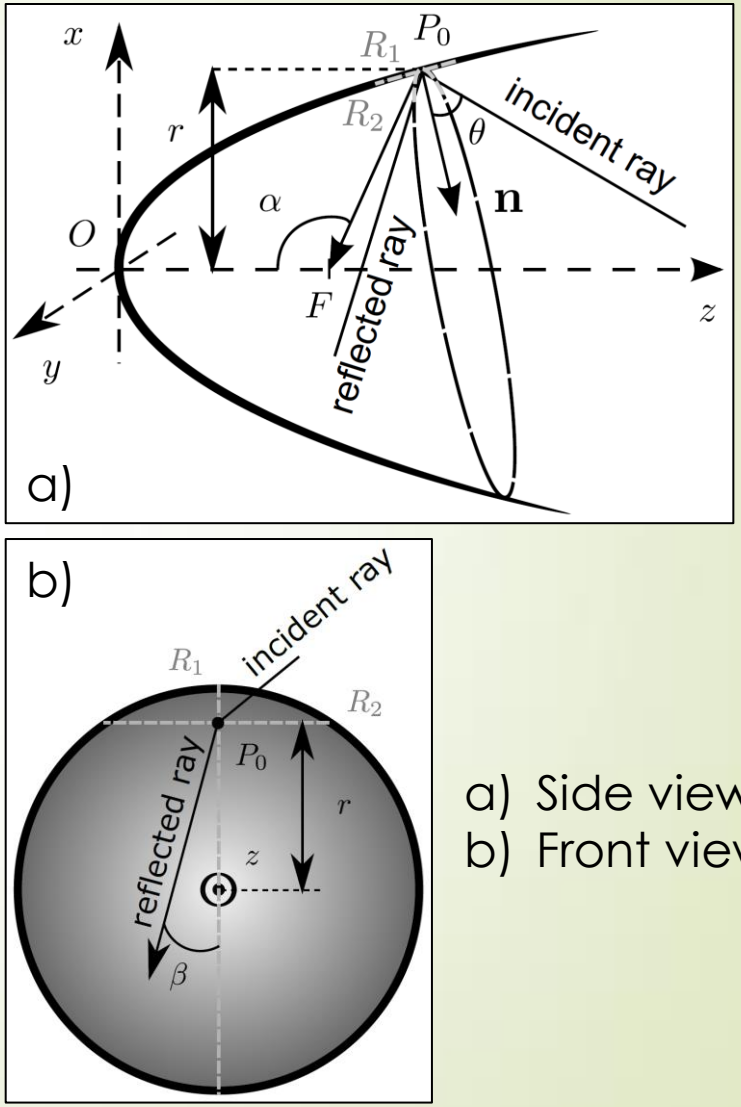
$$M = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{2}{R_a \cos\theta} & \frac{2}{R_c} & -1 & 0 \\ -\frac{2}{R_c} & -\frac{2\cos\theta}{R_b} & 0 & 1 \end{pmatrix}$$

- With:

$$\begin{aligned} \frac{1}{R_a} &= \frac{\cos^2\beta}{R_1} + \frac{\sin^2\beta}{R_2}, \\ \frac{1}{R_b} &= \frac{\sin^2\beta}{R_1} + \frac{\cos^2\beta}{R_2}, \\ \frac{1}{R_c} &= \frac{R_2 - R_1}{R_1 R_2}\cos\beta\sin\beta. \end{aligned}$$

- Where R_1 and R_2 the main radii of curvature of the surface and β the angle made between the reflection plane and the main curvature R_1.

- ➔ It remains to calculate the two main radii R_1, R_2 and the angle β.



a) Side view
b) Front view

ABCD matrix for parabolic mirror

Details in K. Dupraz (Opt. Com. 353(2015)178-183)

Normal vector to the surface:

$$\mathbf{n} = \frac{\frac{\partial \mathbf{P_0}}{\partial \mathbf{r}} \wedge \frac{\partial \mathbf{P_0}}{\partial \phi}}{\sqrt{\det\left(\underline{\underline{\mathbf{A}}}\right)}}$$

From Geometry Analysis:

- A point P_0 on a parabolic surface is expressed by

$$\mathbf{P_0} \equiv \overrightarrow{OP_0}(r,\phi) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\phi \\ r\sin\phi \\ \frac{r^2}{2p} \end{pmatrix}$$

With $p = 2f$.

- The two first metric tensor are:

$$\underline{\underline{A}} = \begin{pmatrix} \frac{\partial \mathbf{P_0}}{\partial r}^T \frac{\partial \mathbf{P_0}}{\partial r} & \frac{\partial \mathbf{P_0}}{\partial r}^T \frac{\partial \mathbf{P_0}}{\partial \phi} \\ \frac{\partial \mathbf{P_0}}{\partial \phi}^T \frac{\partial \mathbf{P_0}}{\partial r} & \frac{\partial \mathbf{P_0}}{\partial \phi}^T \frac{\partial \mathbf{P_0}}{\partial \phi} \end{pmatrix} = \begin{pmatrix} \left(1+\frac{r^2}{p^2}\right) & 0 \\ 0 & r^2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\underline{\underline{B}} = \begin{pmatrix} \mathbf{n}^T \frac{\partial^2 \mathbf{P_0}}{\partial \mathbf{r}^2} & \mathbf{n}^T \frac{\partial^2 \mathbf{P_0}}{\partial \mathbf{r} \partial \phi} \\ \mathbf{n}^T \frac{\partial^2 \mathbf{P_0}}{\partial \phi \partial \mathbf{r}} & \mathbf{n}^T \frac{\partial^2 \mathbf{P_0}}{\partial \phi^2} \end{pmatrix} = \begin{pmatrix} \frac{1}{p\sqrt{1+\frac{r^2}{p^2}}} & 0 \\ 0 & \frac{r^2}{p\sqrt{1+\frac{r^2}{p^2}}} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

- As the Tensor are diagonal we get the two main radii of curvature:

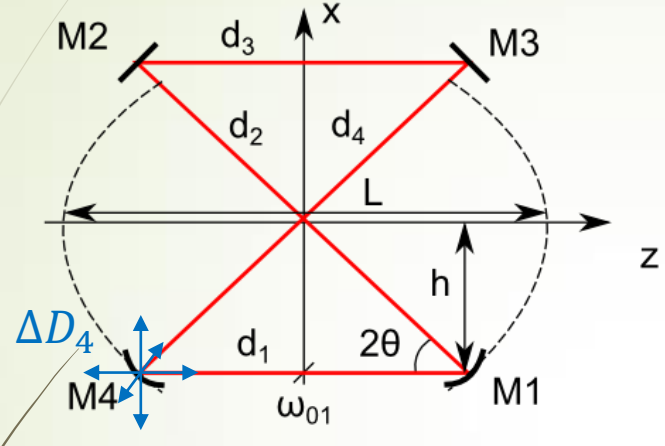
$$R_1 = \frac{a_{11}}{b_{11}} = p\left(1+\frac{r^2}{p^2}\right)\sqrt{1+\frac{r^2}{p^2}} = \frac{2f}{\cos^3\frac{\alpha}{2}}$$

$$R_2 = \frac{a_{22}}{b_{22}} = p\sqrt{1+\frac{r^2}{p^2}} = \frac{2f}{\cos\frac{\alpha}{2}}$$

With $r = p\left|\tan\frac{\alpha}{2}\right|$

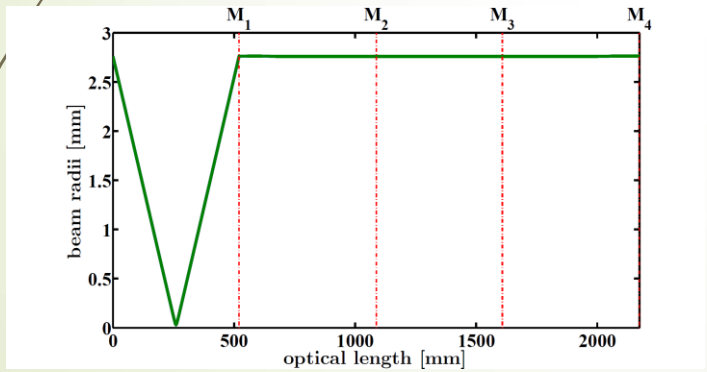
2 parabolic mirrors + 2 flat mirrors cavity (design)

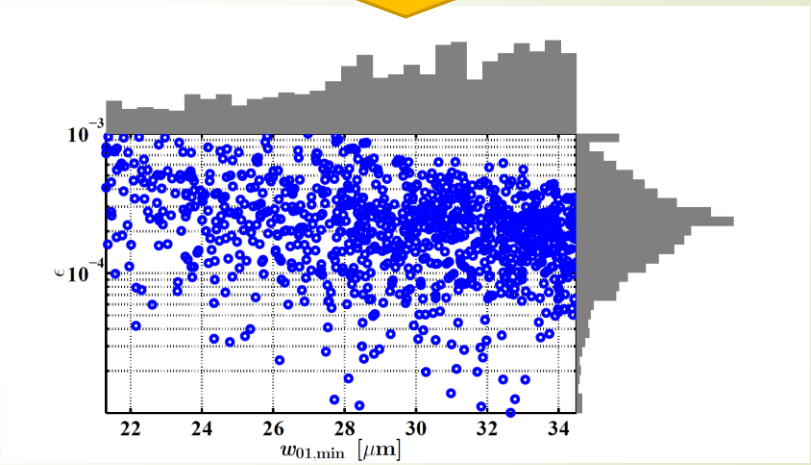
Details in K. Dupraz (Opt. Com. 353(2015)178-183)



Parameters	Value
L (mm)	541,75
h (mm)	102
R (mm)	250
ω0 (µm)	30

$$\Delta D_4 \in [-0.1\ ;\ 0.1]\ mm$$





- Optically perfect
- Mechanically Stable
- Cavity length can be chosen

$$\varepsilon = Ellipticity$$

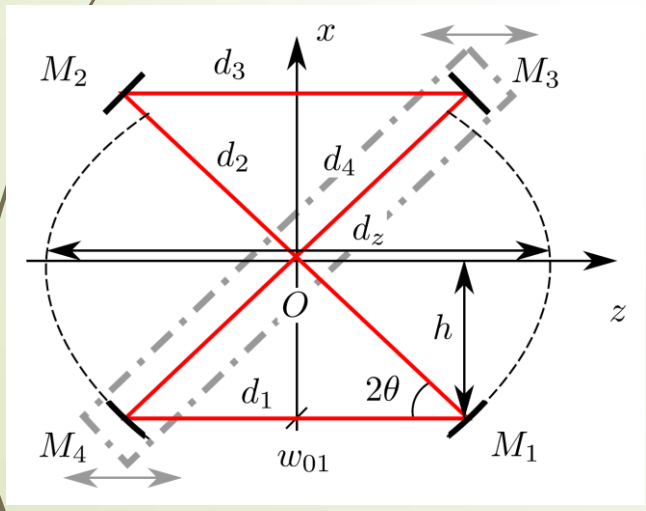
2 parabolic mirrors + 2 flat mirrors cavity (Alignment)

Details in K. Dupraz (Opt. Com. 353(2015)178-183)

- Difficulty to bring the cavity to the working point (many configurations available)



➔ alignment algorithm + observables (constraints on the cavity geometry)

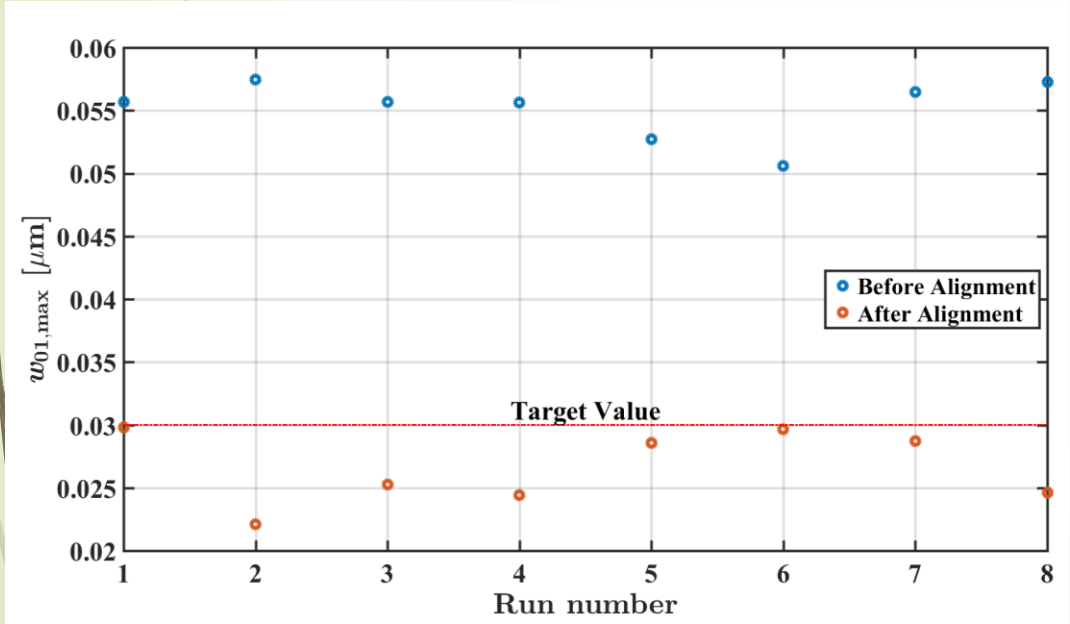


Start with large beam spot size w_{01} (easy to align manually), then:

Iteration

- Act on the tilts (Tx,Ty) of M_2, M_3 and on the tilts (Tx,Ty,Tz) and the position (Dx,Dy) of M_4, to reach non elliptic beam mode (and maintaining the same optical plane)
- Act on the translation Dz of M_3 and M_4 simultaneously to reduce the beam spot size w_{01}

2 parabolic mirrors + 2 flat mirrors cavity (results)



Constraints:

- ✓ Even number of reflective surfaces
- ✓ Small waist (~30µm)
- ✓ No ellipticity (stigmatic mode with always ellipticity < 1%)
- ✓ Large laser beam area on optics => to avoid Laser Damage Threshold

General summary

- In the way to 1MW stacked inside cavity
  - already ~700 kW stacked *(H. Carstens OL 39(2014)9)*.
- New consideration of the ellipticity for small waists in cavity compound of spherical mirrors
- New study on ellipticity free cavities with parabolic mirrors
  - Good numerical results are obtained for 2 parabolic mirrors + 2 flat mirrors cavities
  - Experiment assembly in progress

Thank you