International Conference and Exhibition on Mesoscopic and Condensed Matter Physics-2015

Undulator-like radiation and N^2 cooperative phenomena in micro-structures with grating

University of Rzeszów

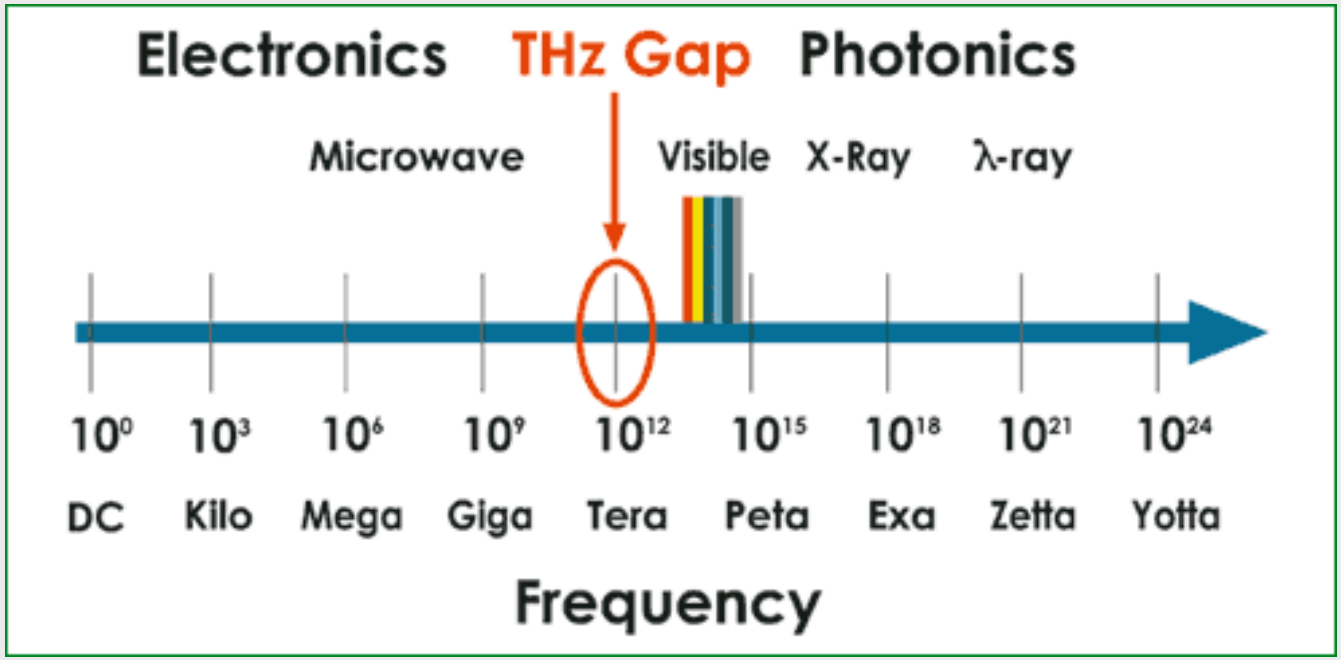
Igor Tralle

22-24 June, 2015 Boston, USA

Outline

- Short introduction
- Cooperative N^2-effect in a system of classical oscillators
- Undulator radiation in a semiconductor microstructure with grating
- Gunn-effect and cooperative N^2-efekt
- Superradiance in a semiconductor microstructure with grating
- Summary

THz radiation



źródło: RPI Center for Terahertz Reseach

The limits:
0.1 THz– 30 THz
3 mm - 0.01mm

Some properties of THz radiation

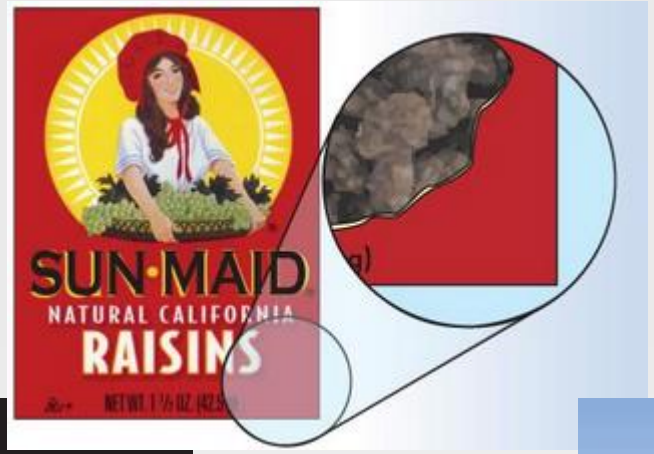
- **Penetrates paper, cardboard , wood, walls, plastic, clothing**
- **To some extent penetrates fog and clouds,**
- **Does not penetrate water and metals**

- Terahertz radiation is non-ionizing submillimeter microwave radiation and shares with microwaves the capability to penetrate a wide variety of non-conducting materials. Terahertz radiation can pass through clothing, paper, cardboard, wood, masonry, plastic and ceramics. It can also penetrate fog and clouds, but cannot penetrate metal or water.

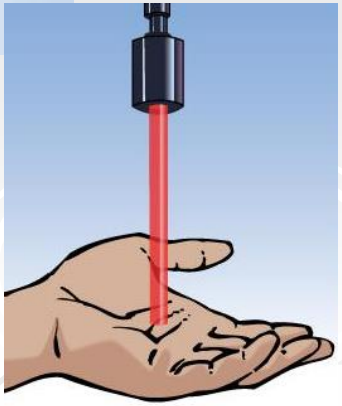
Possible applications of THz radiatiom

- medicine, public safety, astronomy, museum service, communication systems, manufacturing









THz-radiation

A passive outdoor image of an individual holding a metal knife concealed in a newspaper obtained at 94 GHz. Although there is little spectral information available over this frequency range, differences in the electromagnetic properties of skin, explosives, metal, etc., can allow the detection of weapons or concealed packages



(Courtesy of Roger Appleby, QinetiQ, UK.)

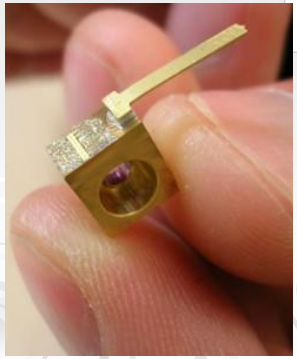
THz-radiation sources

- Gyrotron
- Far infrared laser
- Free eletcron lasers
- Cascade laser
- Synchrotron







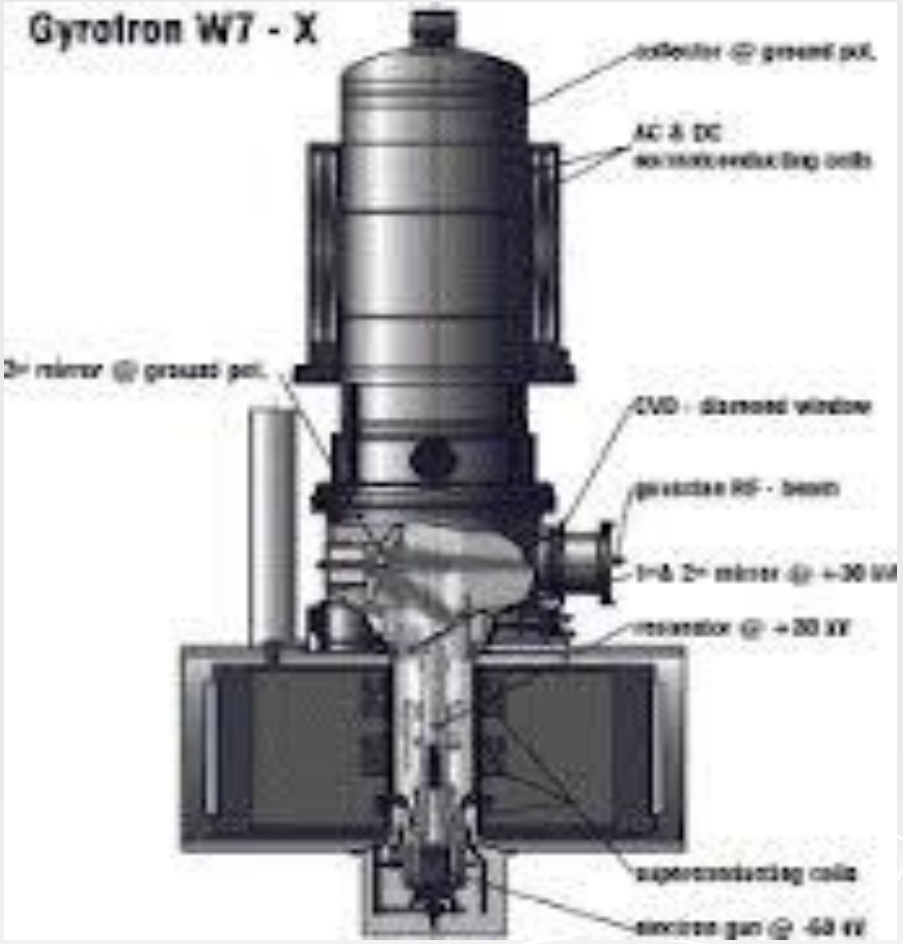




Gyrotron

- **Gyrotrons** are high powered vacuum tubes which emit millimeter-wave beams by bunching electrons with cyclotron motion in a strong magnetic field. Output frequencies range from about 20 to 250 GHz, covering wavelengths from microwave to the edge of the terahertz gap. Typical output powers range from tens of kilowatts to 1-2 megawatts. Gyrotrons can be designed for pulsed or continuous operation.

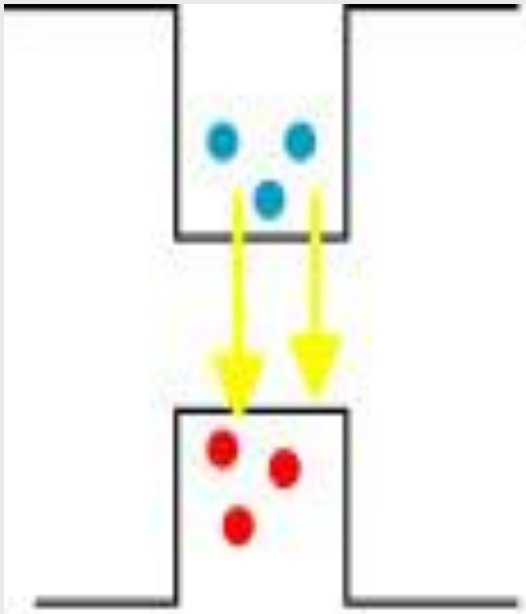



Gyrotron W7 - X
collector @ ground pot.
AC & DC
2nd mirror @ ground pot.
CVD - diamond window
gaussian RF - beam
1st & 2nd mirror @ +30 kV
resonator @ +20 kV
superconducting coils
electron gun @ -60 kV

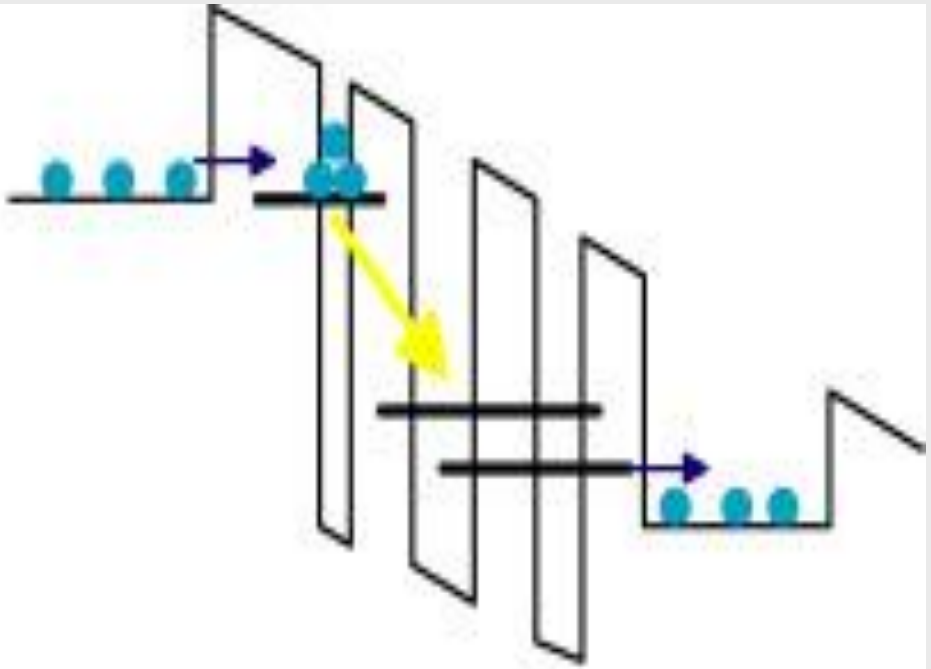
- The gyrotron has high power at millimeter wavelengths because its dimensions can be much larger than the wavelength, unlike conventional vacuum tubes, and it is not dependent on material properties, as are conventional masers. The bunching depends on a relativistic effect called the Cyclotron Resonance Maser instability. The electron speed in a gyrotron is slightly relativistic (comparable to but not close to the speed of light). This contrasts to the free electron laser that work on different principles and which electrons are highly relativistic.

Quantum Cascade Laser

- **Quantum cascade lasers** (QCLs) are semiconductor lasers that emit in the mid- to far-infrared part of the electromagnetic spectrum and were first demonstrated by Jerome Faist, Federico Capasso, Deborah Sivco, Carlo Sirtori, Albert Hutchinson, and Alfred Cho at Bell Laboratories in 1994.
- Unlike typical interband semiconductor lasers that emit electromagnetic radiation through the recombination of electron–hole pairs across the material band gap, QCLs are unipolar and laser emission is achieved through the use of intersubband transitions in a repeated stack of semiconductor multiple quantum well heterostructures,

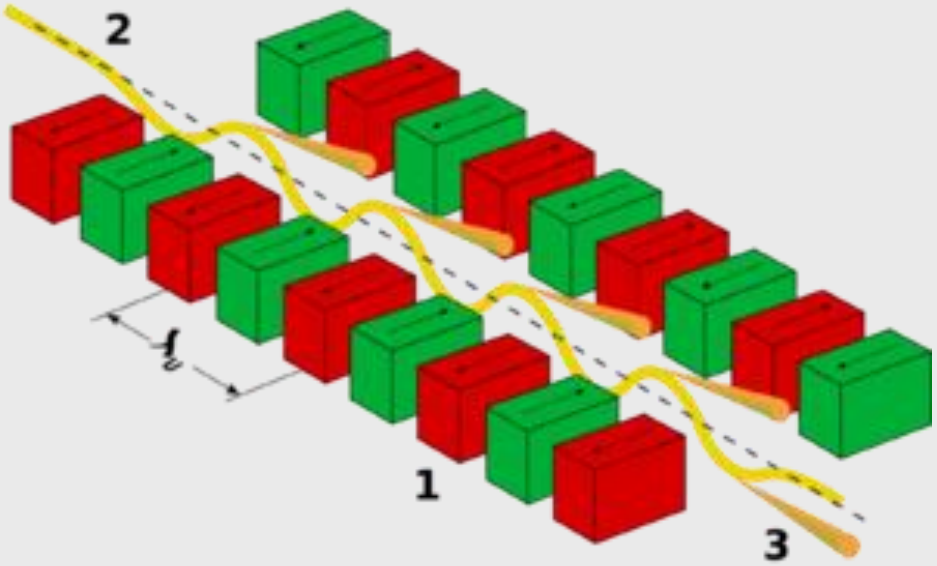


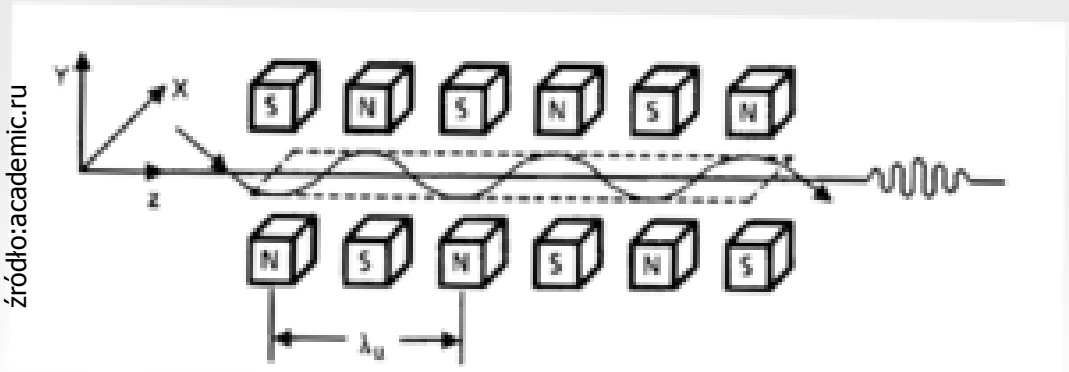
Interband
Laser



Intersubband
Laser

Undulator radiation





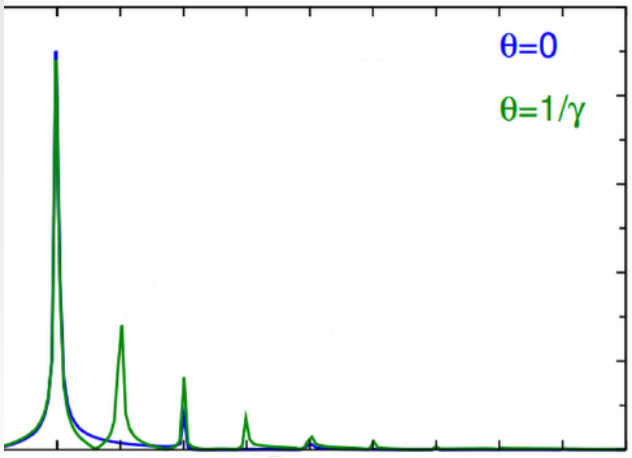
źródło:academic.ru

$$\vec{B} = [0, B_0 \sin(k_u z), 0]$$

Undulator radiation

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \theta^2\gamma^2\right)$$

$$\omega = \frac{2\pi n}{T}\left(1 - \frac{L}{cT}\cos\theta\right)^{-1}$$



Ensemble of classical oscillators

If the ensemble of *N* identical oscillators (the sources of radiation) emit waves, the amplitudes of generated fields can add together in a different way depending on their relative phases.

Consider N identical oscillators, each emitter radiates with its own phase; phases are completely random with respect to one another.

The total emitted field is $\mathbf{E}_t = \mathbf{E}_t^+ + \mathbf{E}_t^-,$

where

$$\mathbf{E}_t^+ = \sum_{r=1}^{N} \mathbf{E}_\mathbf{r}^+ \exp(i\varphi_r)$$

and the sum runs over all emitters in the system.

Total radiated power - cont.

Assuming all electrons have the same velocity v

$$\left|\sum_{i=1}^{N} e^{i\omega t_i}\right|^2 = \left|\sum_{i=1}^{N} e^{i\omega z_i / v}\right|^2 = \sum_{i=1}^{N} e^{i\omega z_i / v} \sum_{j=1}^{N} e^{-i\omega z_j / v} =$$

$$= N + \sum_{i=1}^{N} e^{i\omega z_i / v} \sum_{j=1, i\neq j}^{N} e^{-i\omega z_j / v}.$$

Total radiated power - cont.

If $S(z)$ ($S(z)$) is the density distribution of electrons along z axis is normalized i.e. $\int_{-\infty}^{+\infty} S(z)\,dz = 1$, we can rewrite

$$\sum_{i=1}^{N} e^{i\omega z_i / v} \sum_{j=1, i\neq j}^{N} e^{-i\omega z_j / v} \approx$$

$$\approx N(N-1)\left(\int_{z=-\infty}^{\infty} S(z)\, e^{i\omega z / v} dz \right)\left(\int_{z=-\infty}^{\infty} S(z)\, e^{-i\omega z / v} dz \right) =$$

$$= N(N-1)\left| \int_{z=-\infty}^{\infty} S(z)\, e^{i\omega z / v} dz \right|^2$$

and since $N >> 1$, $P_N \propto N^2 P_1$

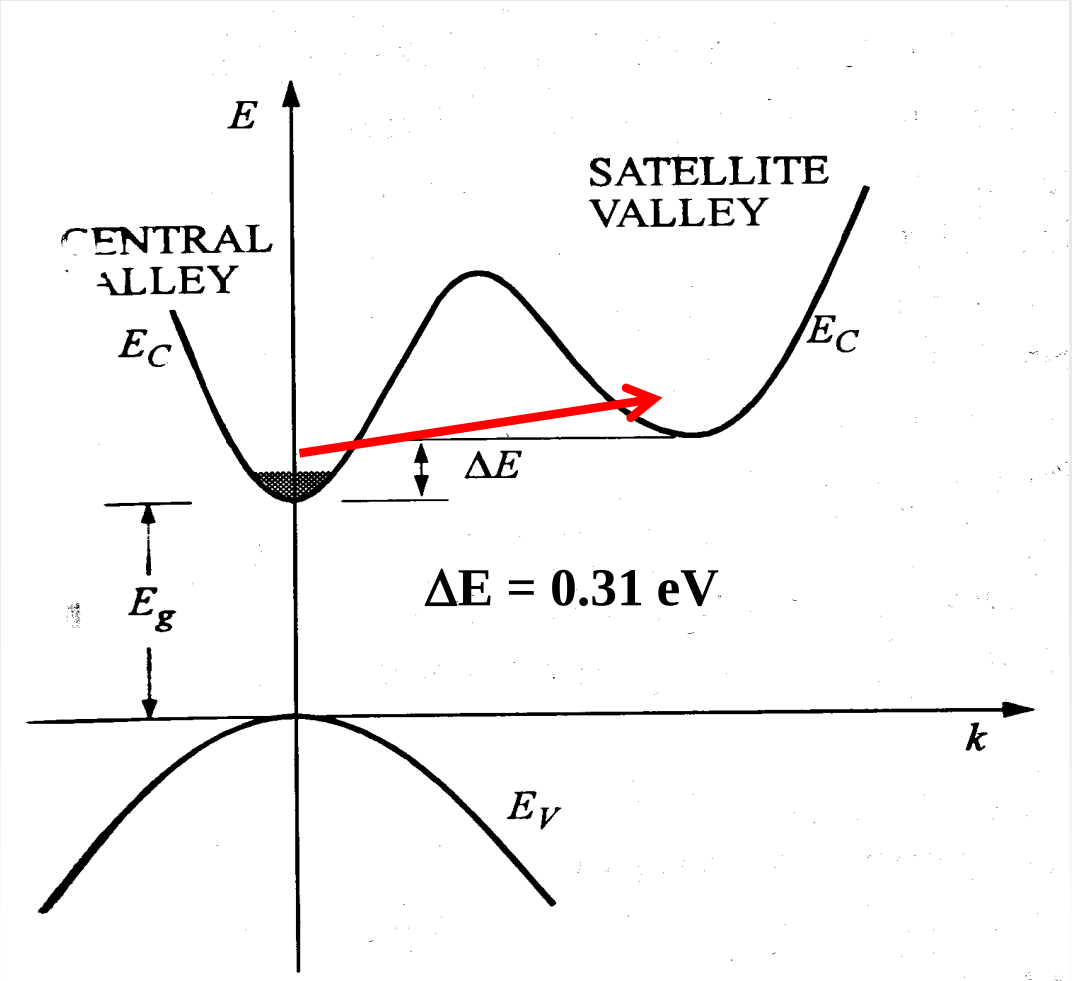
- It was suggested that specific phase coherence which leads to $\sim N^2$ – effect can be achieved, if the linear sizes l of the electron bunch in FEL is much smaller than the emitted wavelength λ .

- If $l << \lambda$, the electron bunch generate waves as a single point source and all electrons in a bunch generate waves of the same phase.

- Since the total charge is equal *Ne*, the generated power is $\sim N^2$.

Gunn effect

Band structure of GaAs



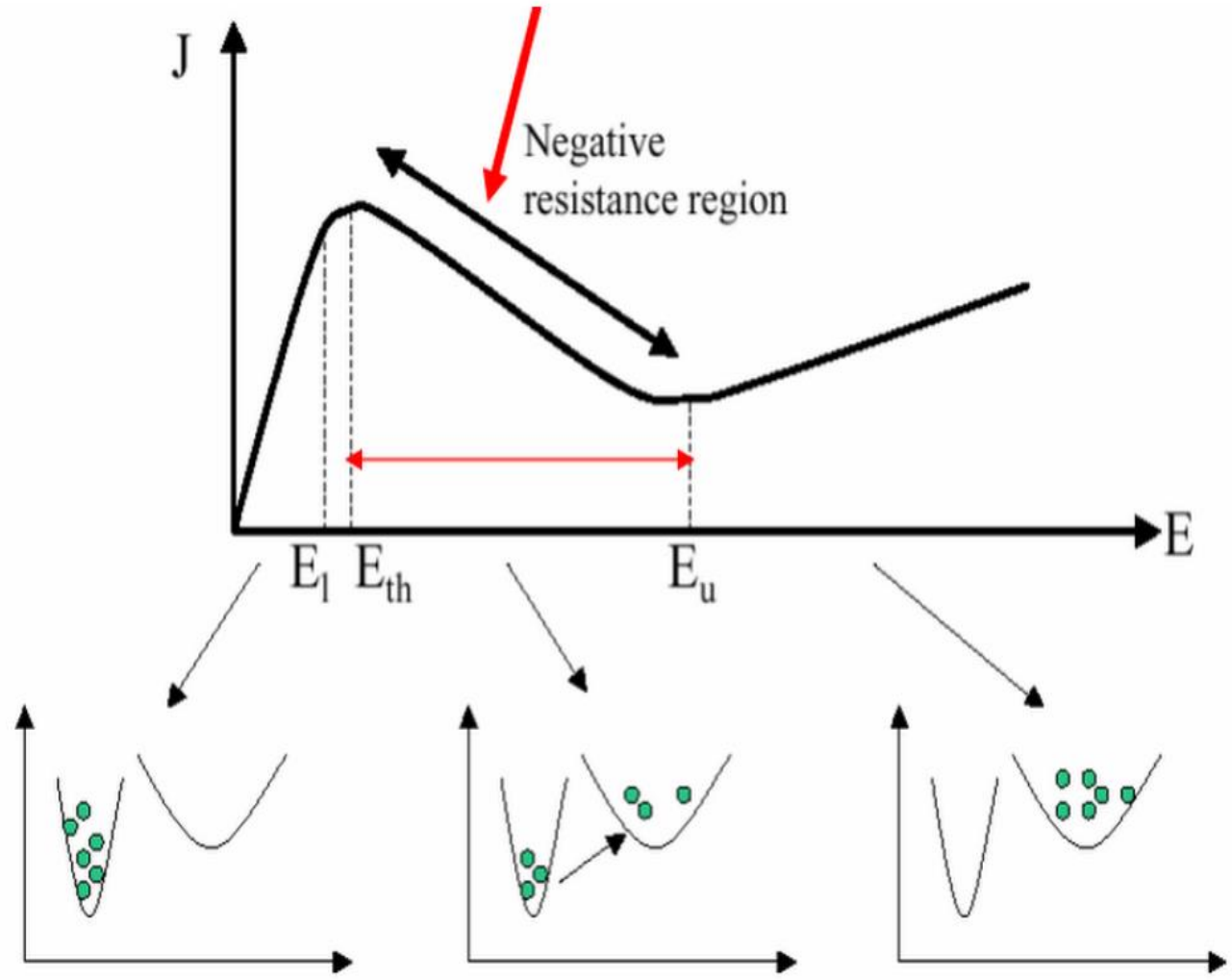
effective mass

$$m^\star = \frac{\hbar}{\frac{d^2 E}{dk^2}}$$

mobility

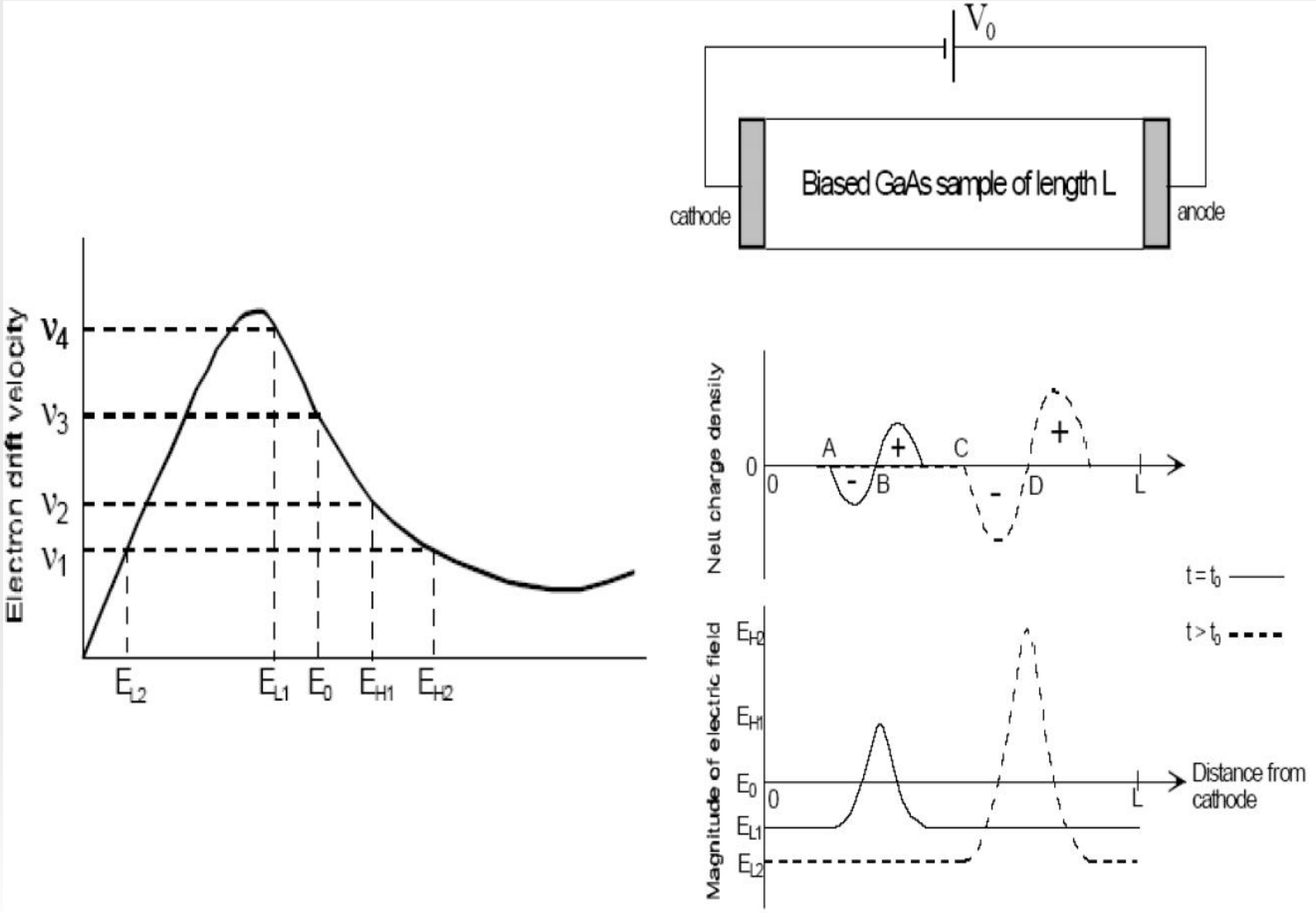
$$\mu \propto \frac{1}{m^\star}$$

Valley	effective mass [me]	mobility [cm^2/(V x s)]
Γ	0.068	8000
L	1.2	180

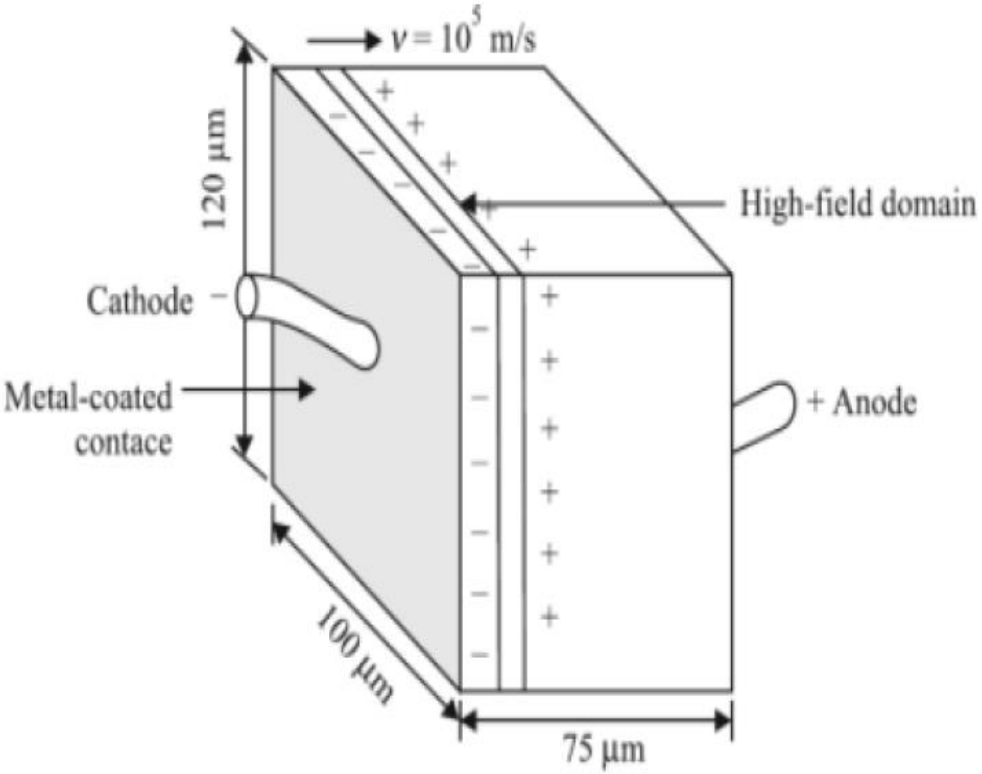


$$j = \sigma \times E$$

$$E_{th} = 3.3kV/cm$$

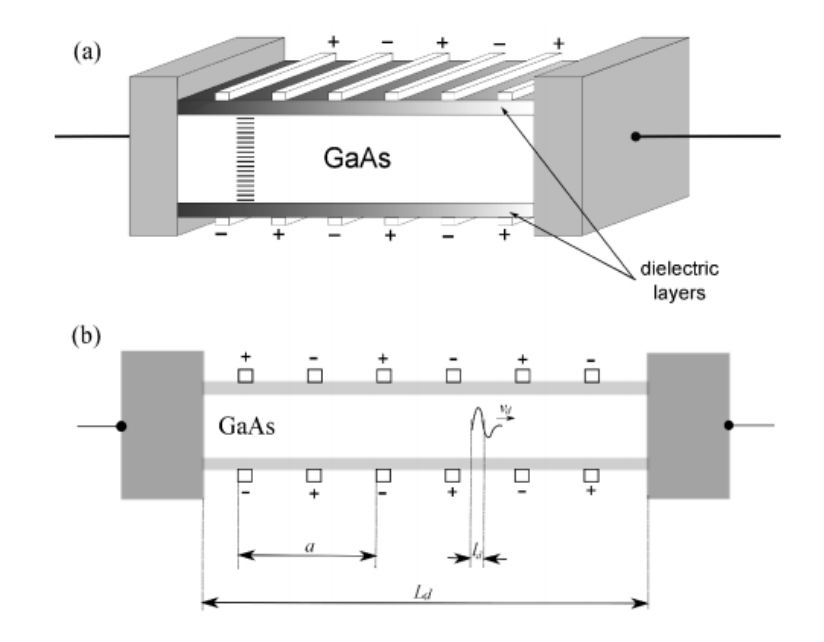

V_0
Biased GaAs sample of length L
cathode
anode
Electron drift velocity
v_4
v_3
v_2
v_1
E_{L2}
E_{L1}
E_0
E_{H1}
E_{H2}
Nett charge density
0
A
B
C
D
L
+
-
$t = t_0$
$t > t_0$
Magnitude of electric field
E_{H2}
E_{H1}
E_0
E_{L1}
E_{L2}
Distance from cathode

Gunn-effect diode





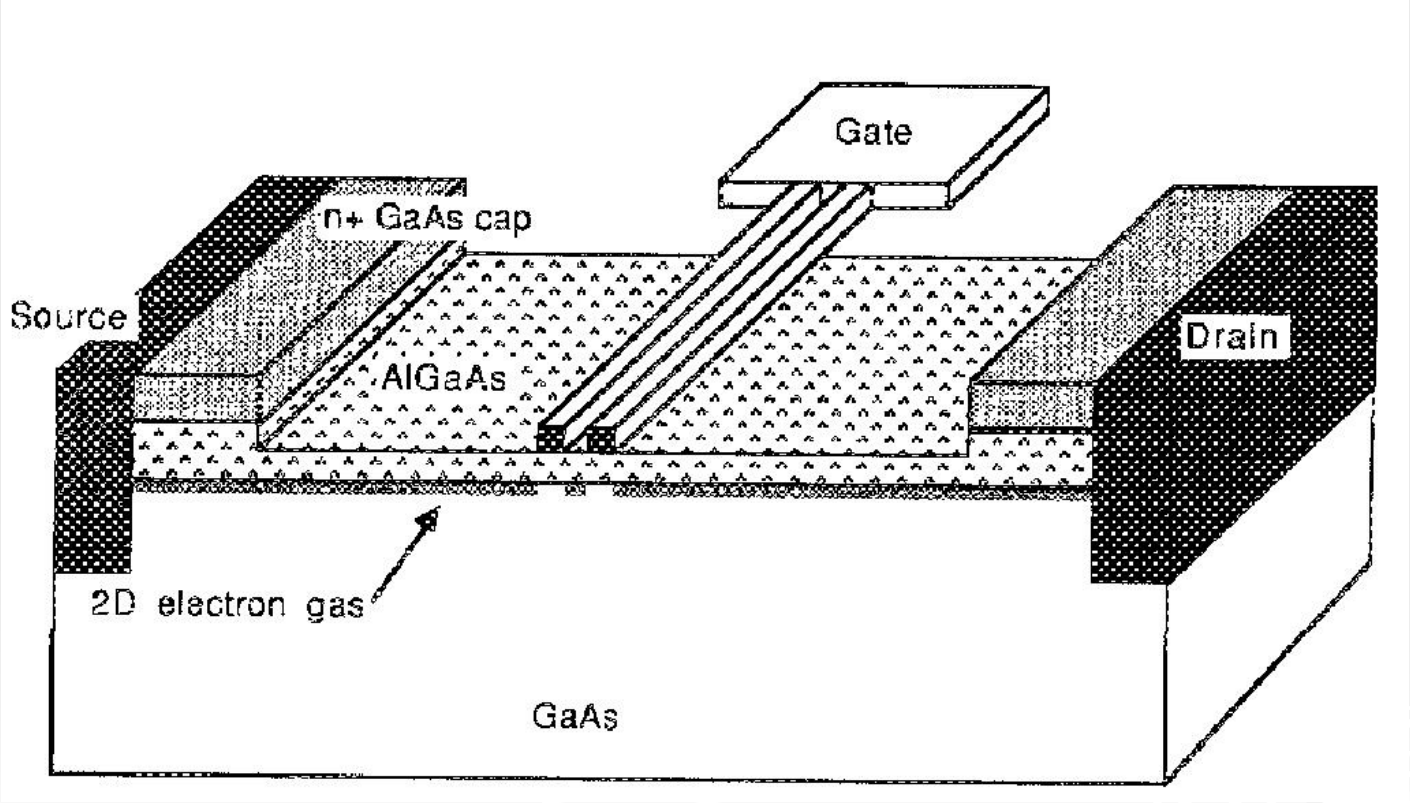
Let us consider the hypothetical device, which is the Gunn-effect diode with the gratings on its both surfaces.



- The total length of the device ranges from 10^{-3} cm to 10^{-2} cm.
- The period of grating a is about 2×10^{-4} cm; thickness of a „bunch” l_d ~3×10^{-5}
- Electron drift velocity v_d ~10^7 cm $\cdot s^{-1}$; the frequency of an undulator-like
- radiation $\omega_0 = \dfrac{2\pi v_d}{a\left(1-(v_d/c)\cos\vartheta\right)}$ ~ 3.14×10^{11} Hz; λ ~ 0.18 cm.
- Obviously, $l_d << \lambda$ and $l_d << a$

S.Y.Chou, D.R. Allee, R.F.W. Pease and J.S. Harris, Appl. Phys. Lett., **55**(2), p. 176 (1989)

Electrical Engineering Department, Stanford University



The model

$$\ddot{x}_i + \omega^2 x_i = f(x_i, \dot{x}_1, \ldots, \dot{x}_N, t)$$

$$f(x_i, \dot{x}_1, \ldots, \dot{x}_N, t) = \frac{1}{6}\frac{\omega^2}{a^2}x_i^3 - \frac{2e^2\omega^2}{3mc^3}\sum_{j=1}^{N}\dot{x}_j + \frac{eE}{m}\cos(\nu t)$$

- Here ω is the oscillator eigen-frequency, ν – the field frquecy, a is the oscillation amplitude associated with the preiod of grating, E is the electric field and N is the number of oscillators

Method of slowly varying amplitudes and phases

We seek a solution to (1) by the method of slowly varying amplitudes and phases. We assume $x_i(t)$ has the form

$$x_i(t) = a A_i(t) \cos(\nu t + \theta_i(t)), \tag{2}$$

where by hypothesis $A_i(t)$ and $\theta_i(t)$ are the functions whose rate of variation is small compared with the angular frequency ν.

Method of slowly varying amplitudes and phases - cont.

Differentiating (2) we get

$$\dot{x}_i = a[\dot{A}_i \cos(\nu t + \theta_i) - A_i(\nu + \dot{\theta}_i)\sin(\nu t + \theta_i)]$$

If the functions A_i, θ_i were constants, then the derivatives $\dot{x}_i$ would be

$$\dot{x}_i = a\nu A_i \sin(\nu t + \theta_i). \qquad (3)$$

Comparing the last two equations gives

$$\dot{A}_i \cos(\nu t + \theta_i) - A_i \dot{\theta}_i \sin(\nu t + \theta_i) = 0 \qquad (4)$$

Method of slowly varying amplitudes and phases - cont.

Differentiating (2) twice we compute x_i

$$\ddot{x}_i = -a\nu[\dot{A}_i \sin(\nu t + \theta_i) + A_i(\nu + \theta_i)\cos(\nu t + \theta_i)]. \qquad (5)$$

Inserting (2), (3) and (5) into (1) we get

$$\nu \dot{A}_i \sin(\nu t + \theta_i) + A_i[(\nu^2 - \omega^2) + \nu\dot{\theta}_i]\cos(\nu t + \theta_i) = \qquad (6)$$

$$= f(x_i, \dot{x}_1, ..., \dot{x}_N, t)/\nu$$

Method of slowly varying amplitudes and phases - cont.

From (4), (6) we compute $\dot{A}_i$, $\dot{\theta}_i$

$$\dot{A}_i = -\frac{1}{\nu^2}[A_i(\nu^2 - \omega^2)\cos(\nu t + \theta_i) - f(x_i, \dot{x}_1, ..., \dot{x}_N, t)]\sin(\nu t + \theta_i), \quad (7)$$

$$\dot{\theta}_i = -\frac{\nu^2 - \omega^2}{\nu}\cos^2(\nu t + \theta_i) + \frac{f(x_i, \dot{x}_1, ..., \dot{x}_N, t)}{\nu^2 A_i}. \quad (8)$$

Because $\dot{A}_i$ and $\dot{\theta}_i$ by assumption are slowly varying functions, we substitute the right-hand sides of the last two equations by their values averaged over the period $2\pi/\nu$

Method of slowly varying amplitudes and phases - cont.

Finally, the system (1) takes the form

$$\frac{dA_i}{d\tau} = H\left[\sum_{j=1}^{N} A_j(\tau)\cos(\theta_i(\tau)-\theta_j(\tau))\right] - G\sin\theta_i(\tau) \qquad (9)$$

$$\frac{d\theta_i}{d\tau} = -\frac{1}{2}\frac{\nu^2-\omega^2}{\nu^2} - \frac{1}{16}\frac{\omega^2}{\nu^2}A_i^{\,2}(\tau) - \frac{G}{A_i(\tau)}\cos\theta_i(\tau) \qquad (10)$$

where

$$\tau = \nu t, \qquad H = \frac{e^2\omega^2}{3\nu mc^3}, \qquad G = \frac{eE}{2am\nu^2}.$$

Measure of phasing

$P(\tau)$—is a measure of phasing of the electrons in a bunch

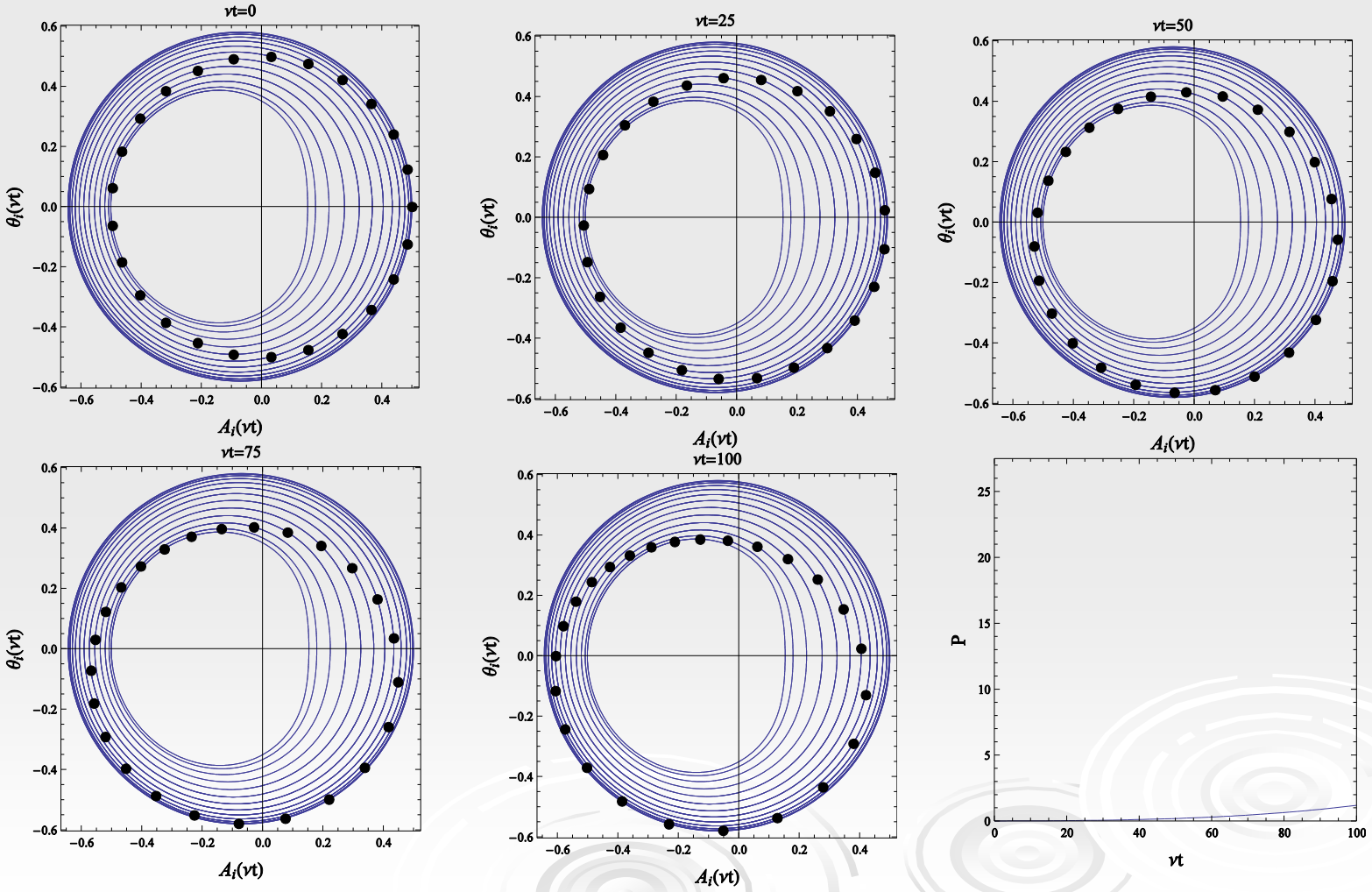
$$P(\tau) = \frac{1}{N}\left|\sum_{j=1}^{N} \exp\, \theta_j(\tau)\right|^2 .$$

It ranges from 0 when all oscillators have completely random phases, to N when all of them oscillate with roughly the same phase.

Numerical solution - Equally spaced phases

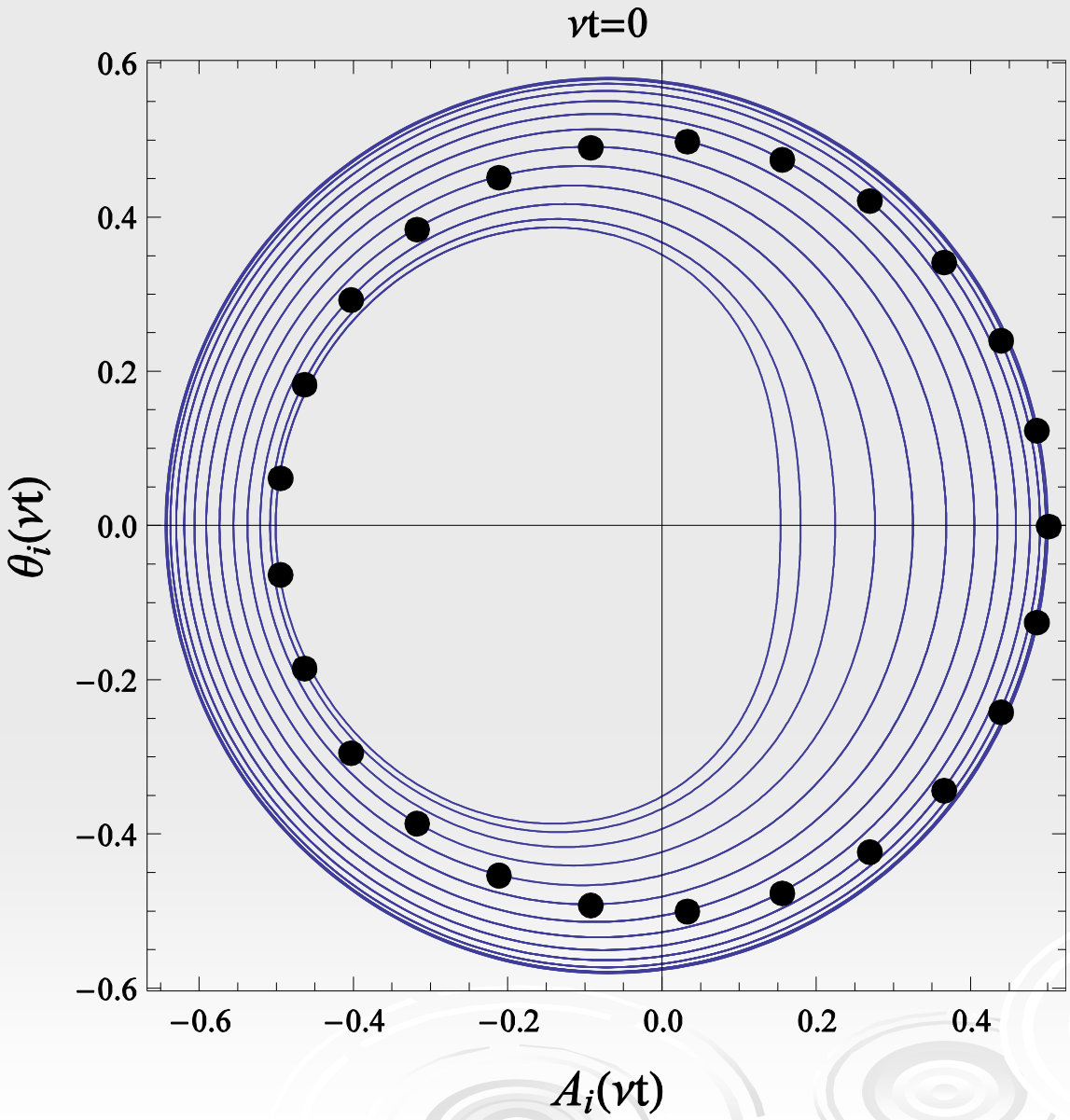
$$l_d \ll a$$

Phase portrait in case of initial phases of individual oscillators uniformly distributed in the interval from 0 to 2π.



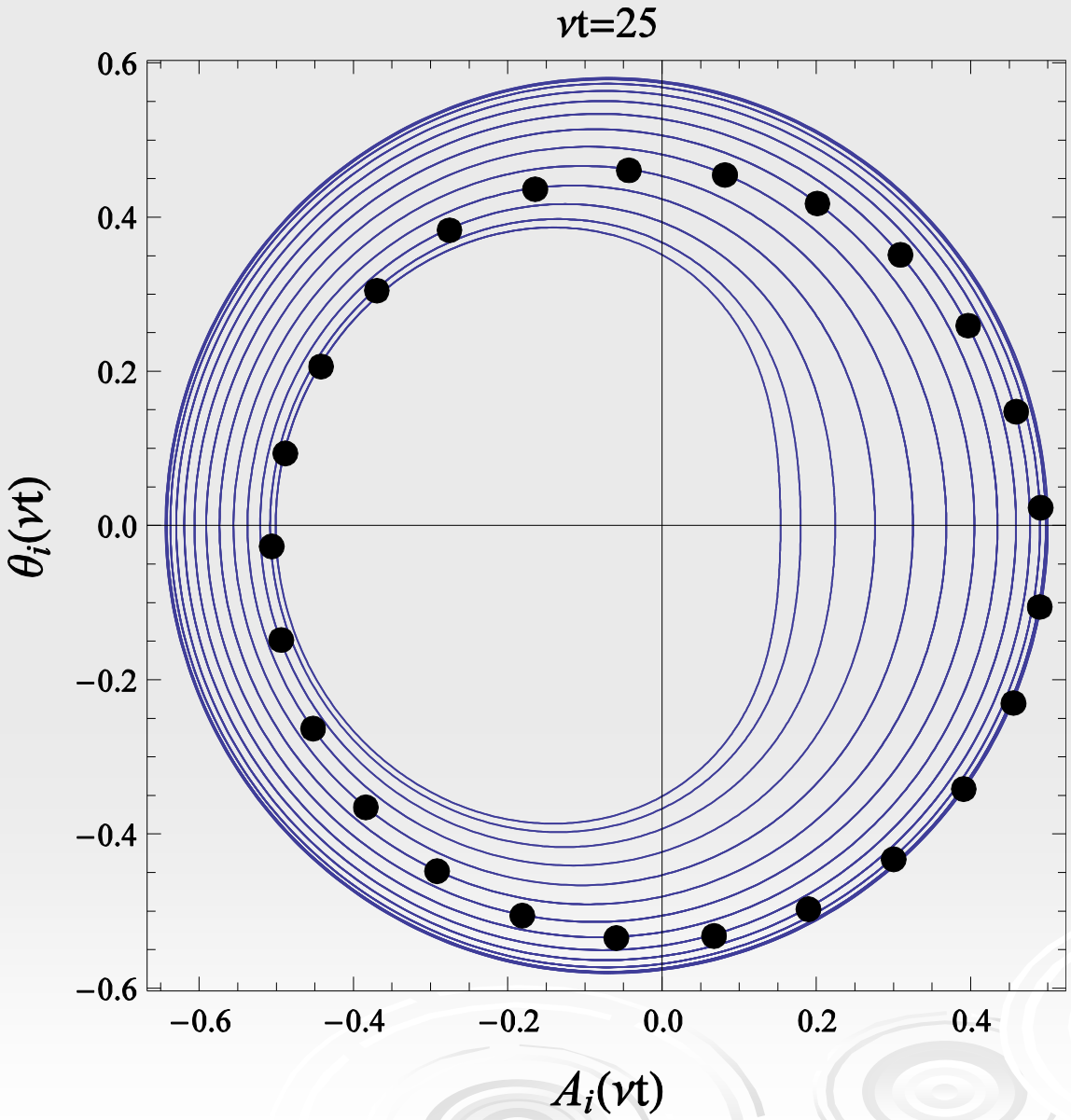
Equally spaced phases vt = 0

$$l_d \ll a$$



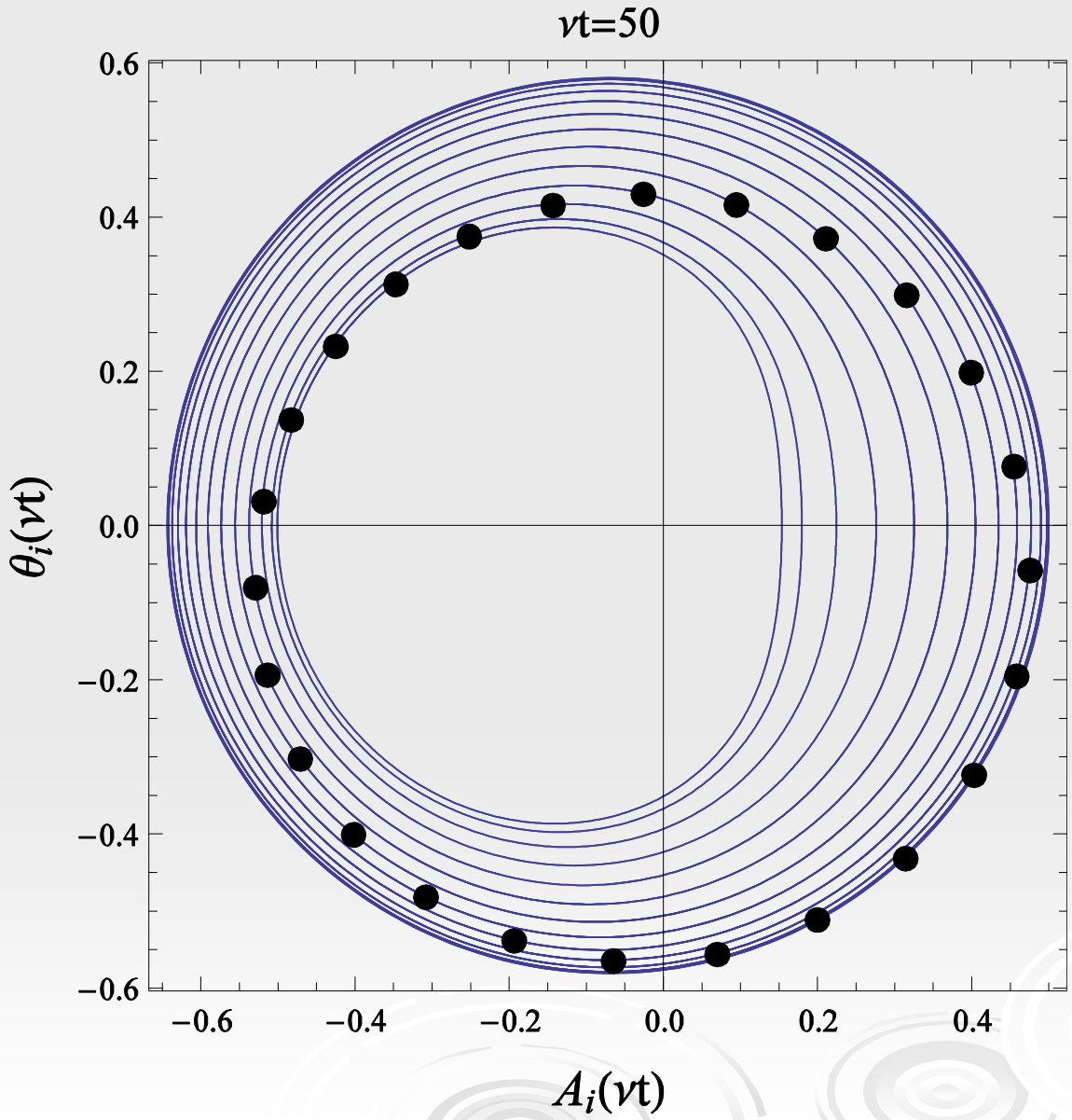
Equally spaced phases vt = 25

$$l_d \ll a$$

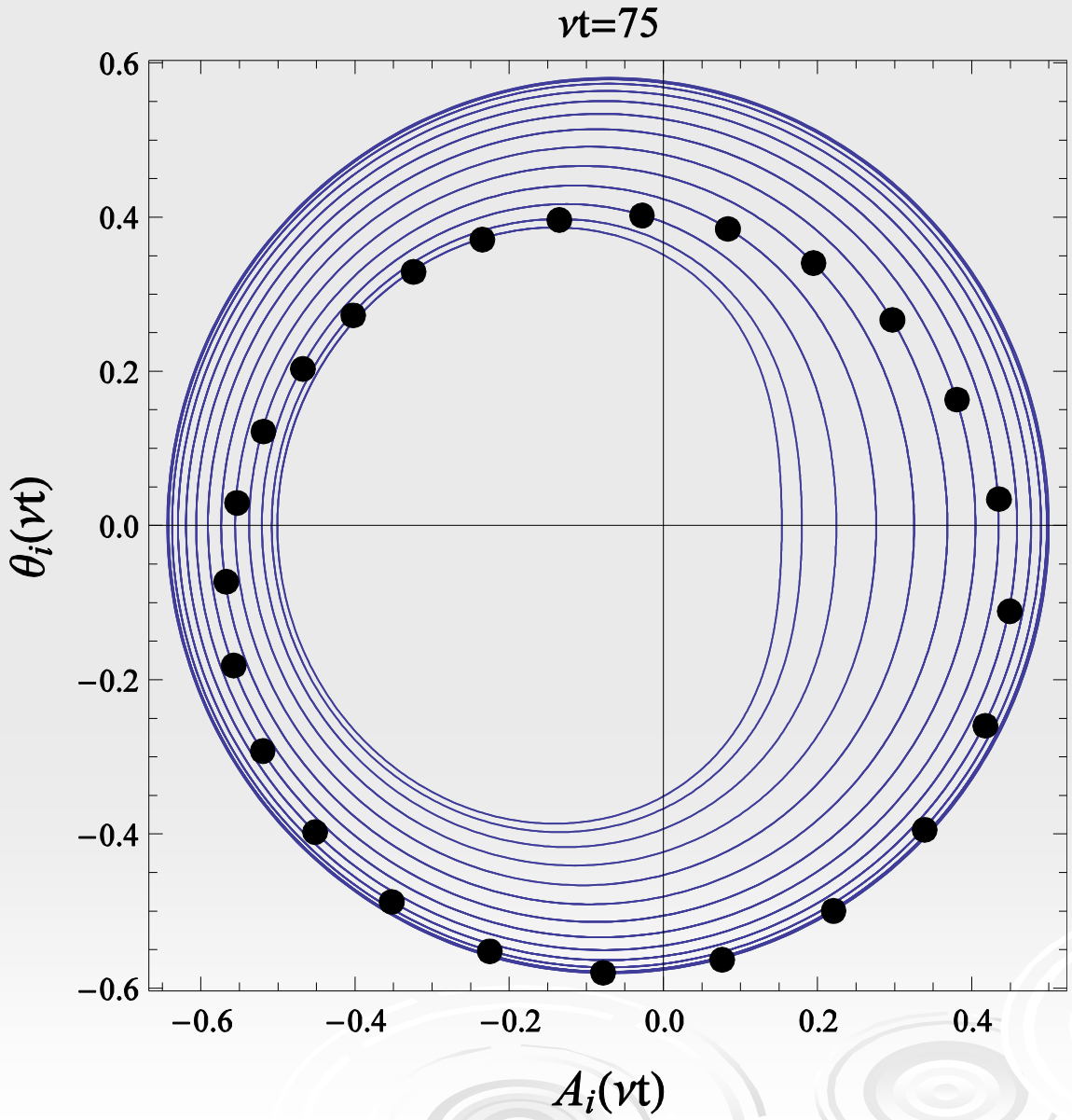


Equally spaced phases vt = 50

$$l_d \ll a$$

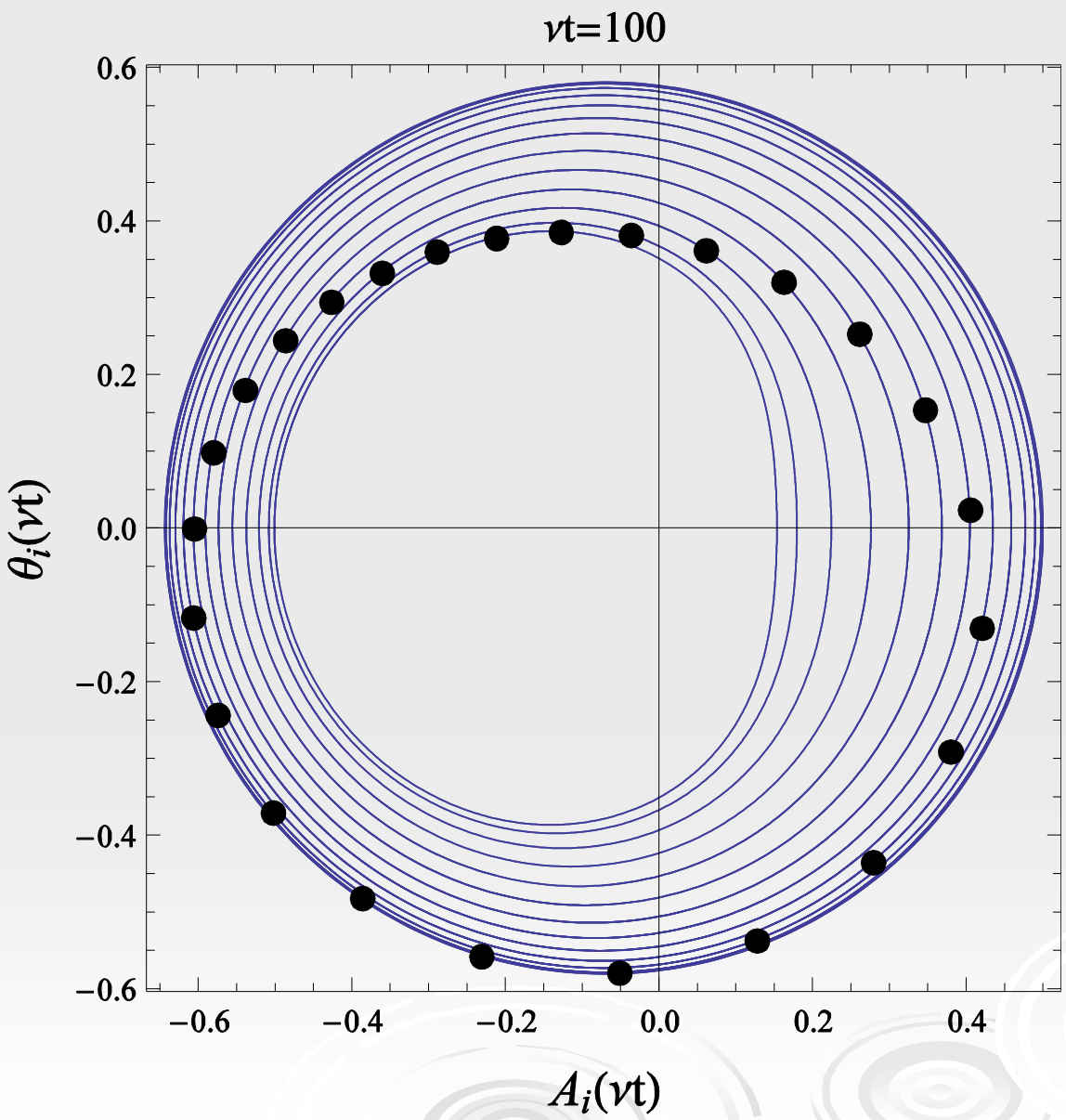


Equally spaced phases vt = 75



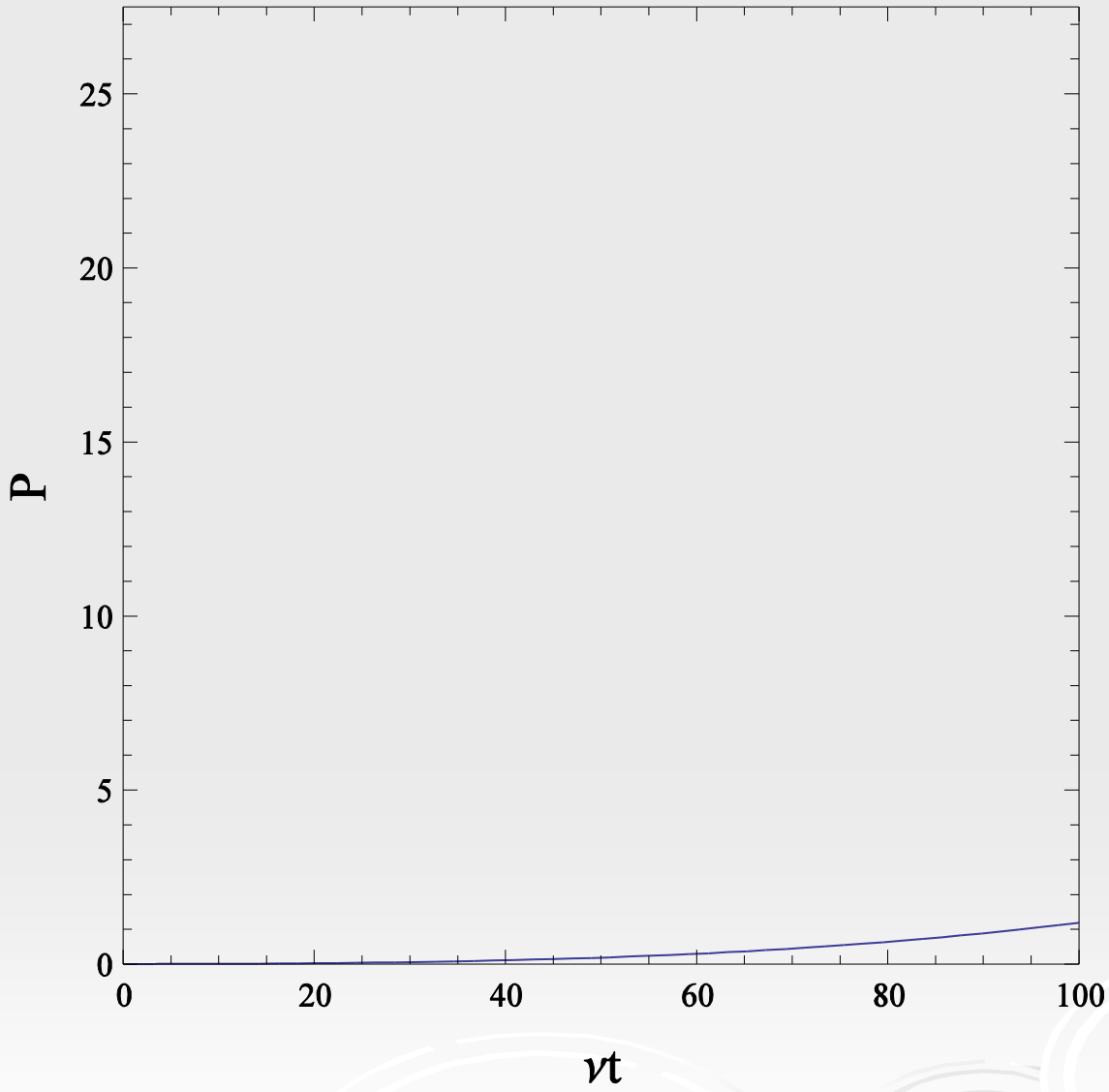
Equally spaced phases vt = 100

$$l_d \ll a$$



Equally spaced phases - phasing factor

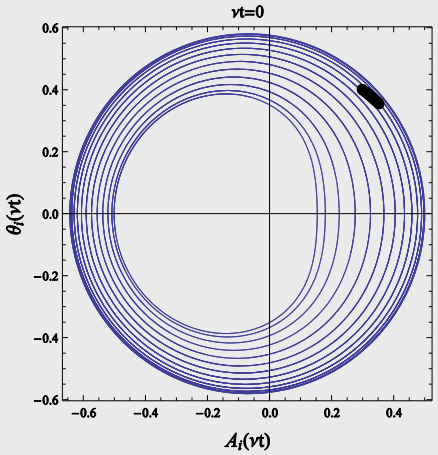
$$l_d \ll a$$

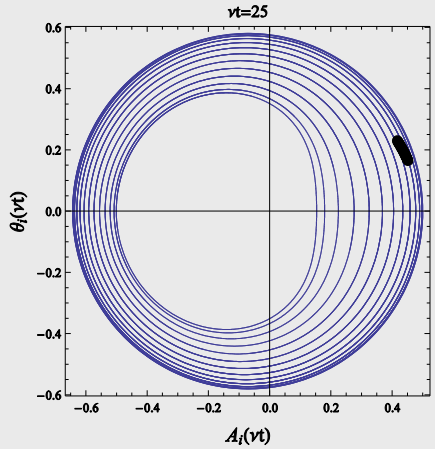


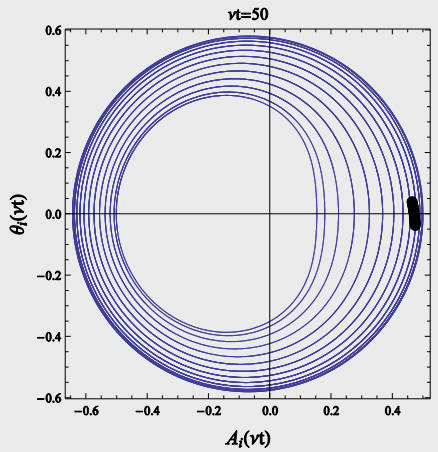
Numerical solution - Clustered phases

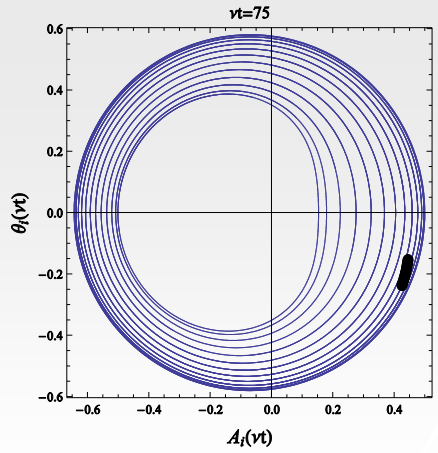
$$l_d \ll a$$

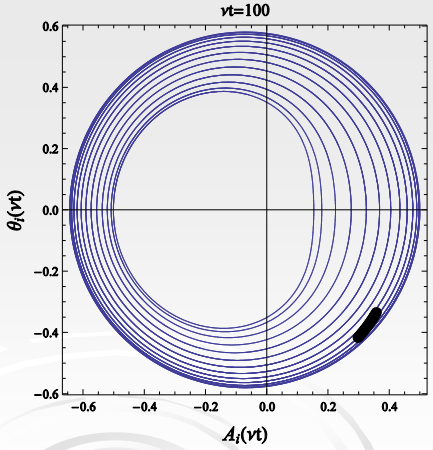
Phase portrait in case of initial phases clustered around π/4 with the dispersion 0.15.

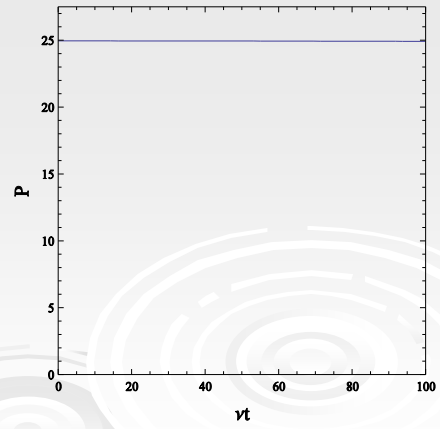






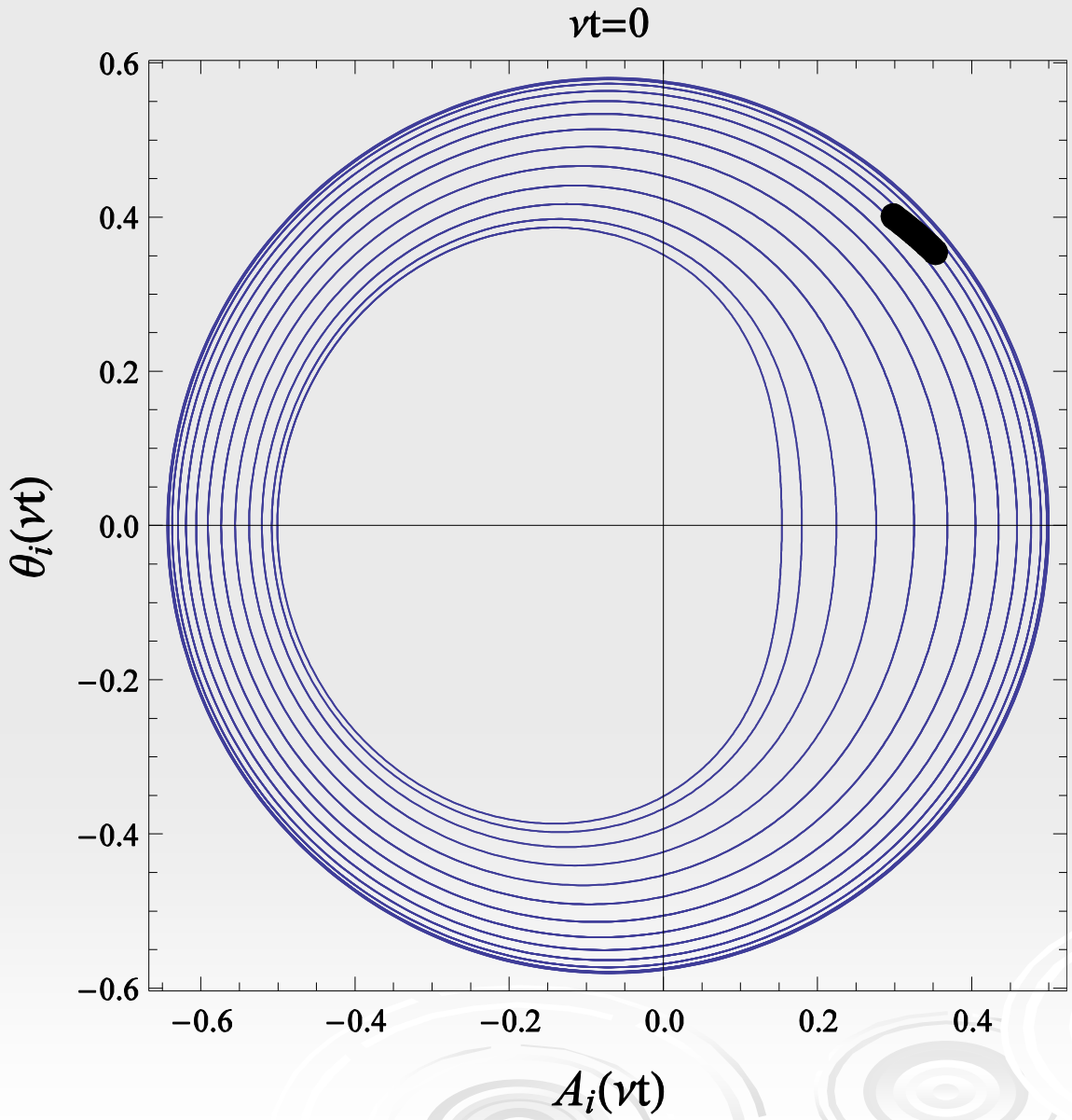






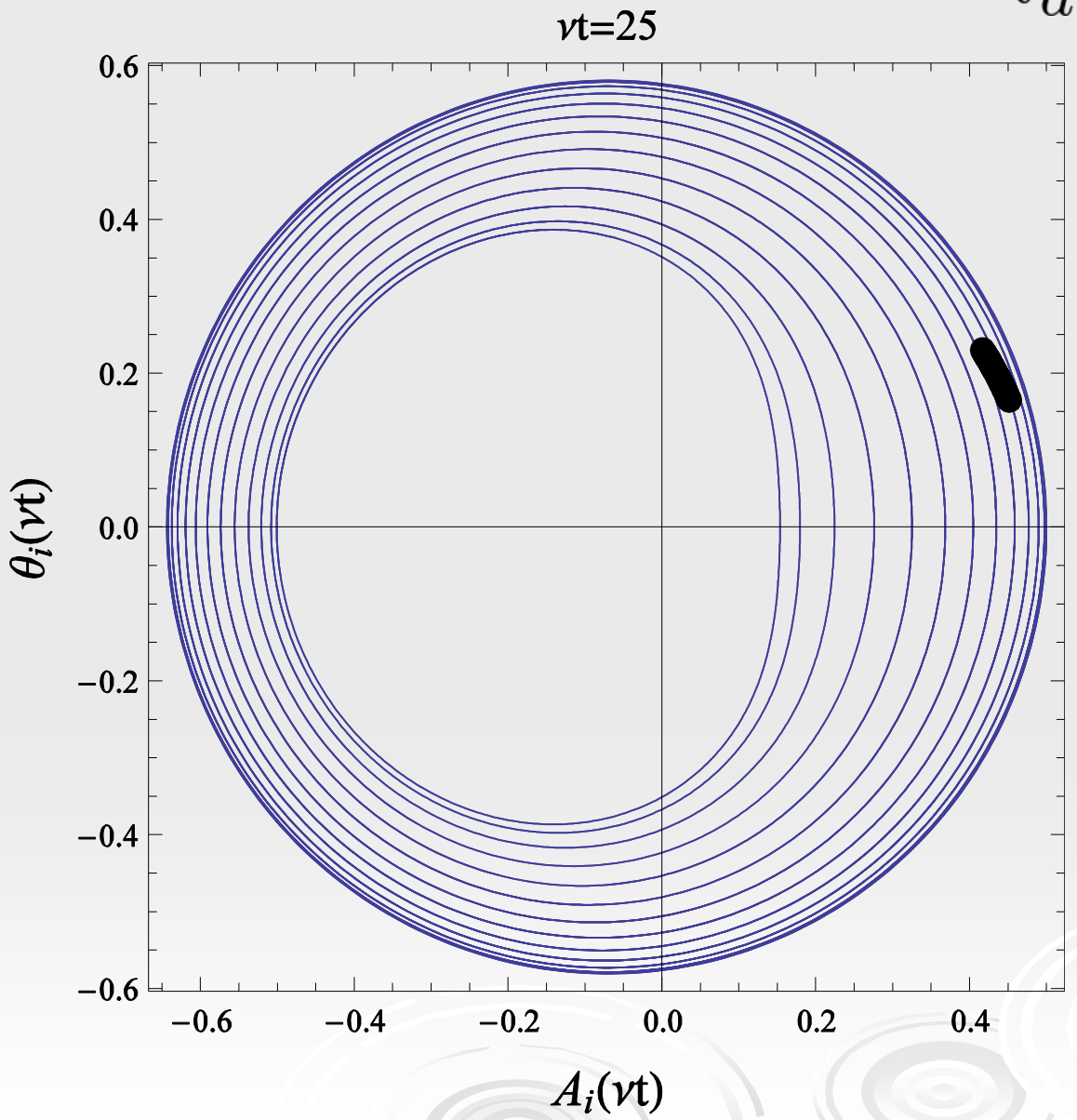
Clustered phases vt = 0

$$l_d \ll a$$



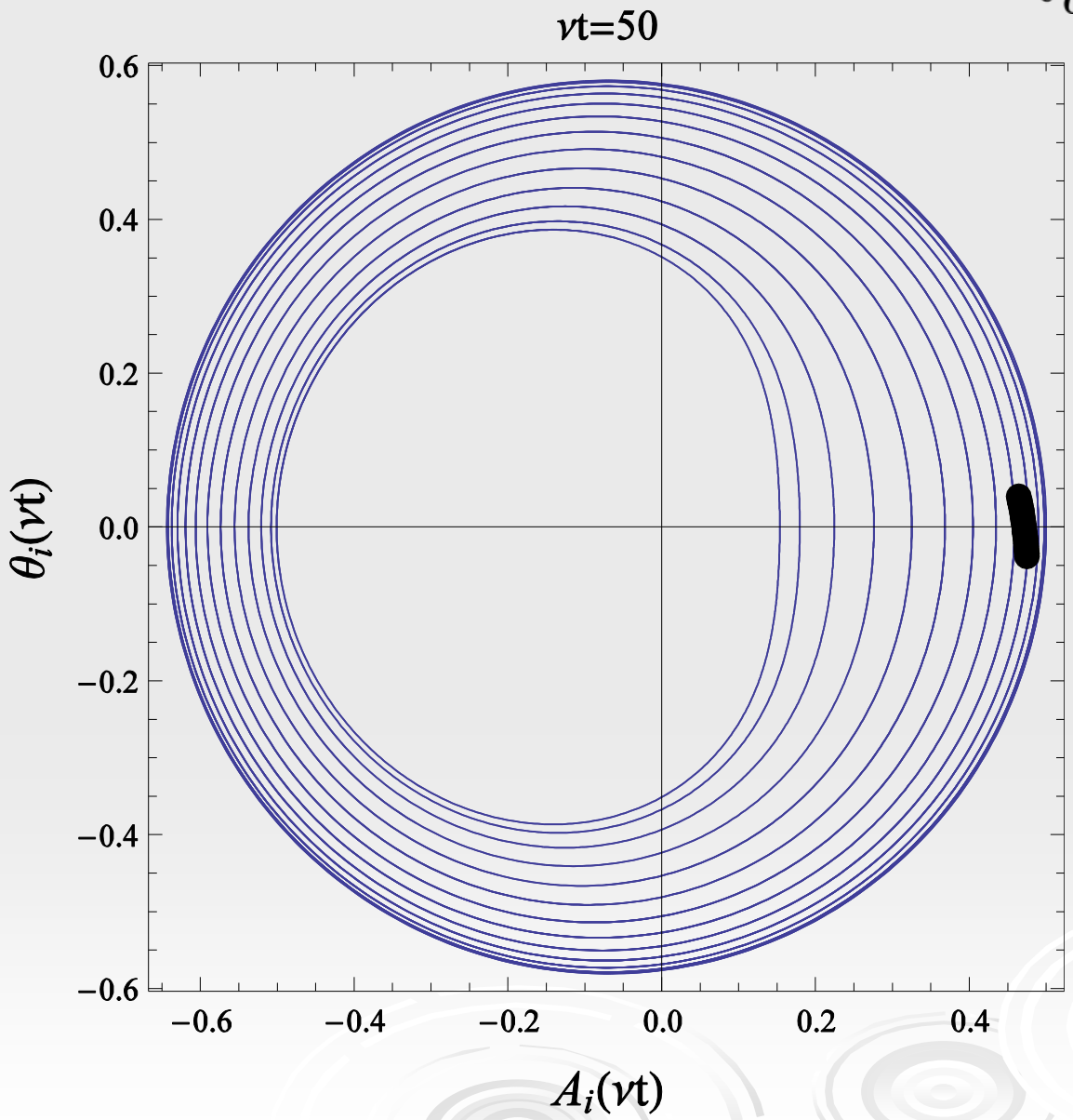
Clustered phases vt = 25

$$l_d \ll a$$



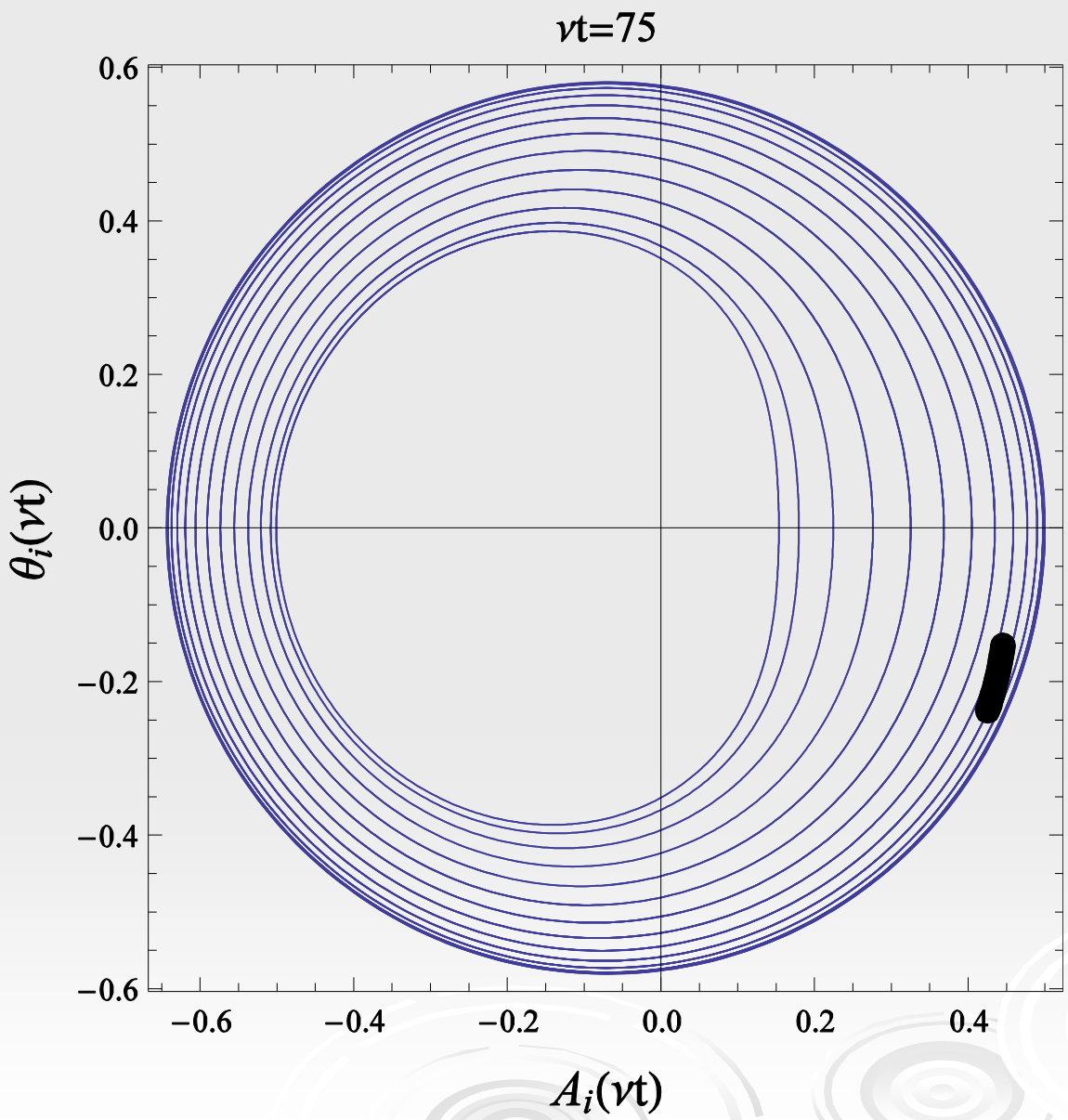
Clustered phases vt = 50

$$l_d \ll a$$



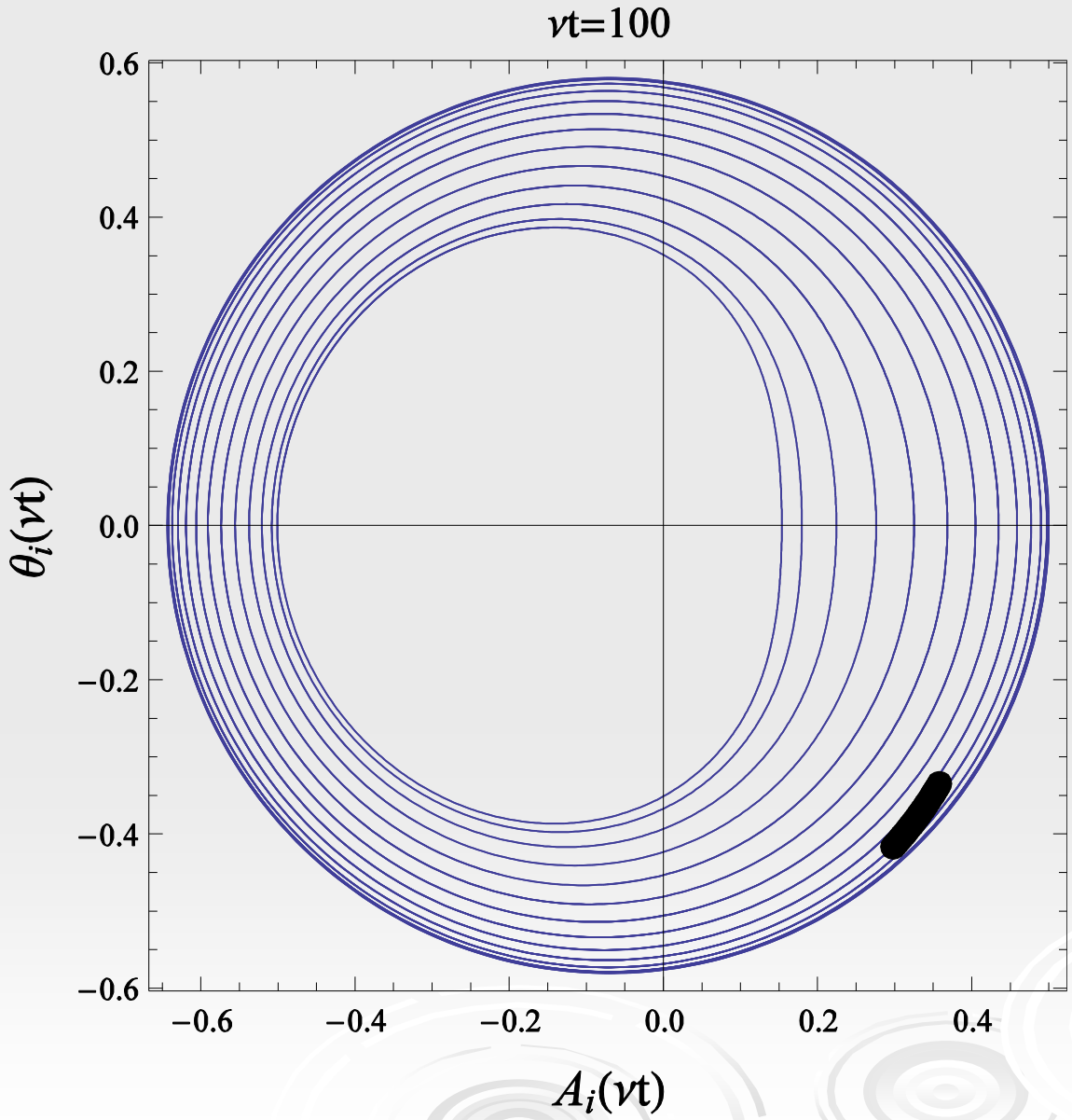
Clustered phases vt = 75

$$l_d \ll a$$



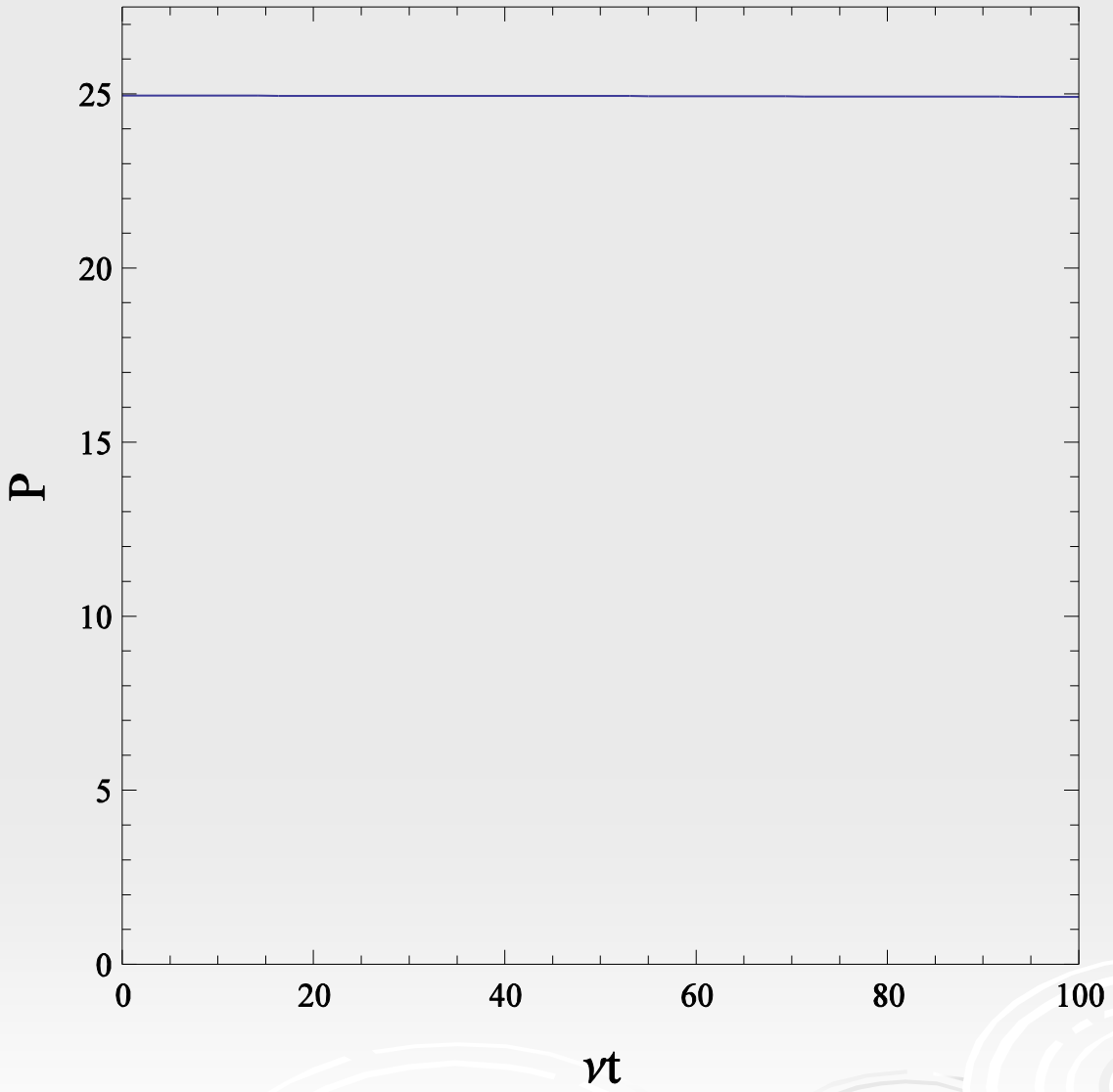
Clustered phases vt = 100

$$l_d \ll a$$



Clustered phases - phasing factor

$$l_d \ll a$$



I.Tralle and P. Zięba, Acta phys. polon. A, vol. 21, N 2 (2012)

Dicke model

- Consider a two-level atom with the energy levels $\hbar\omega_a$ and $\hbar\omega_b$ interacting with the radiation mode $\hbar\omega$

$$H = H_0 + H_1$$

$$H_0 = \hbar \sum_i (\omega_a a_i^\dagger a_i + \omega_b b_i^\dagger b_i)$$

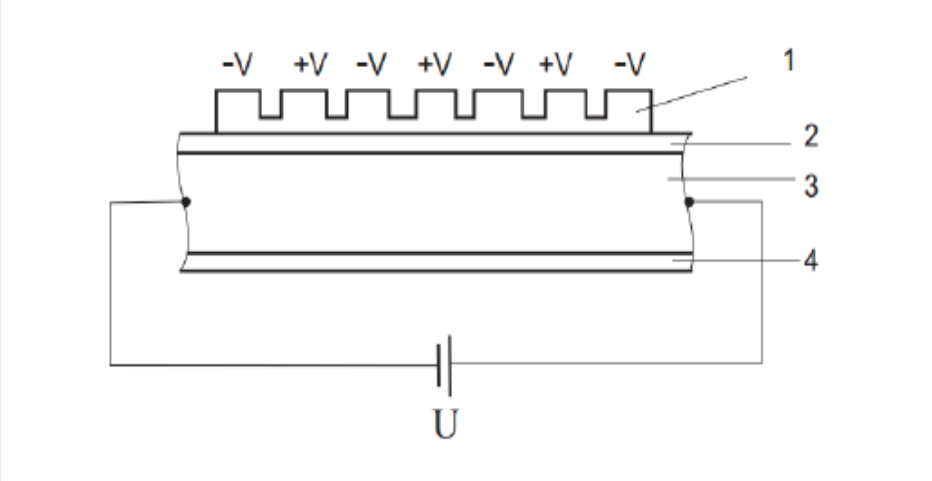
$$H_1 = \hbar g \sum_i (a_i^\dagger b_i B^\dagger e^{i\omega t} + b_i^\dagger a^i B e^{-i\omega t})$$

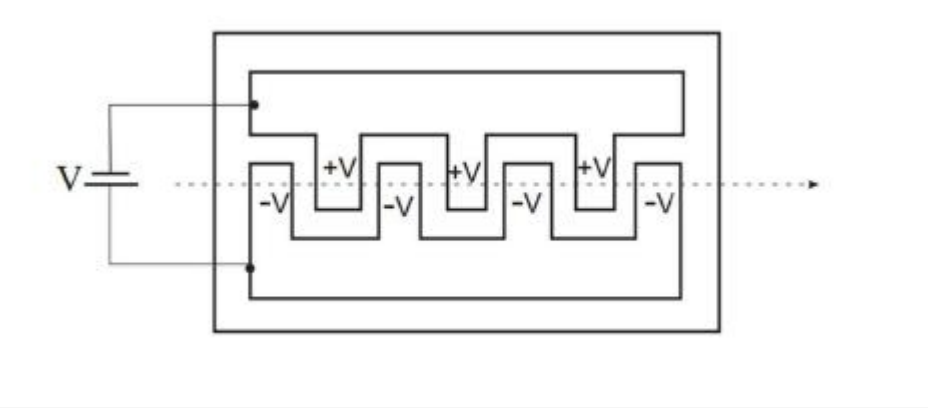
Superradiance

- Superradiance is a cooperative radiation arises due to the self-induced and amplified correlations of initially independent oscillators („atoms). This process is called phasing.
- The effect was predicted by R. Dicke in 1954.
- Dicke himself did not consider the phasing but only a system of N>>1 oscillators, interacting through the emitted electromagnatic fields (Dickie' model)
- The other characteristic feature of the superradiance is that the radiation power is proportional to ~ N^2

- Dicke himself did not consider the phaisng but only the symmetry of the many-particles wavefucntion of two-level atoms with respect to the permutataion of the variables of arbitray pair of atoms. This enabled him to guess the wavefunction of the total ensemble, not considering the difficult issue of phasing.
- Dike's inquiry related the system whose characteristic length $L << \lambda$, where λ is the wavelength.
- Later on the superadiance was predicted theoretically also for the case when $L >> \lambda$ (*D.C. Burnham*, *R.Y. Chiao*, *Phys Rev.* **188**, 667 (1969)) and observed experimentally (*N. Skribanowitz*, *et al.*, *Phys Rev Lett* **30**, 309 (1973). In this experiment also the oscillations of radiated power was observed.
- In accordance with the modern point of view, it is namely the phasing which plays the key-role in the superradiance.

The model of the generator





- 1 – grating
- 2 – insulator layer
- 3 – semiconductor (GaAs)
- 4 - substrate

New equations of motion

$$\ddot{x}_i + \omega_i^2 x_i = f_i(x_i, \dot{x}_1, \ldots, \dot{x}_N, t)$$

$$f_i(x_i, \dot{x}_1, \ldots, \dot{x}_N, t) = \frac{1}{6}\frac{\omega_i^2}{a^2}x_i^3 - \frac{2e^2\omega_i^2}{3mc^3}\sum_{j=1}^{N}\dot{x}_j + \frac{e}{m}(E_0 + E(t))\cos(\nu t)$$

Solution

$$x_i(t) = a\, A_i(t) \cos(\nu t + \theta_i(t))$$

$$\frac{dA_i}{d\tau} = H_i \left[\sum_{j=1}^{N} A_j(\tau) \cos(\theta_i(\tau) - \theta_j(\tau)) \right] - G \sin\theta_i(\tau)$$

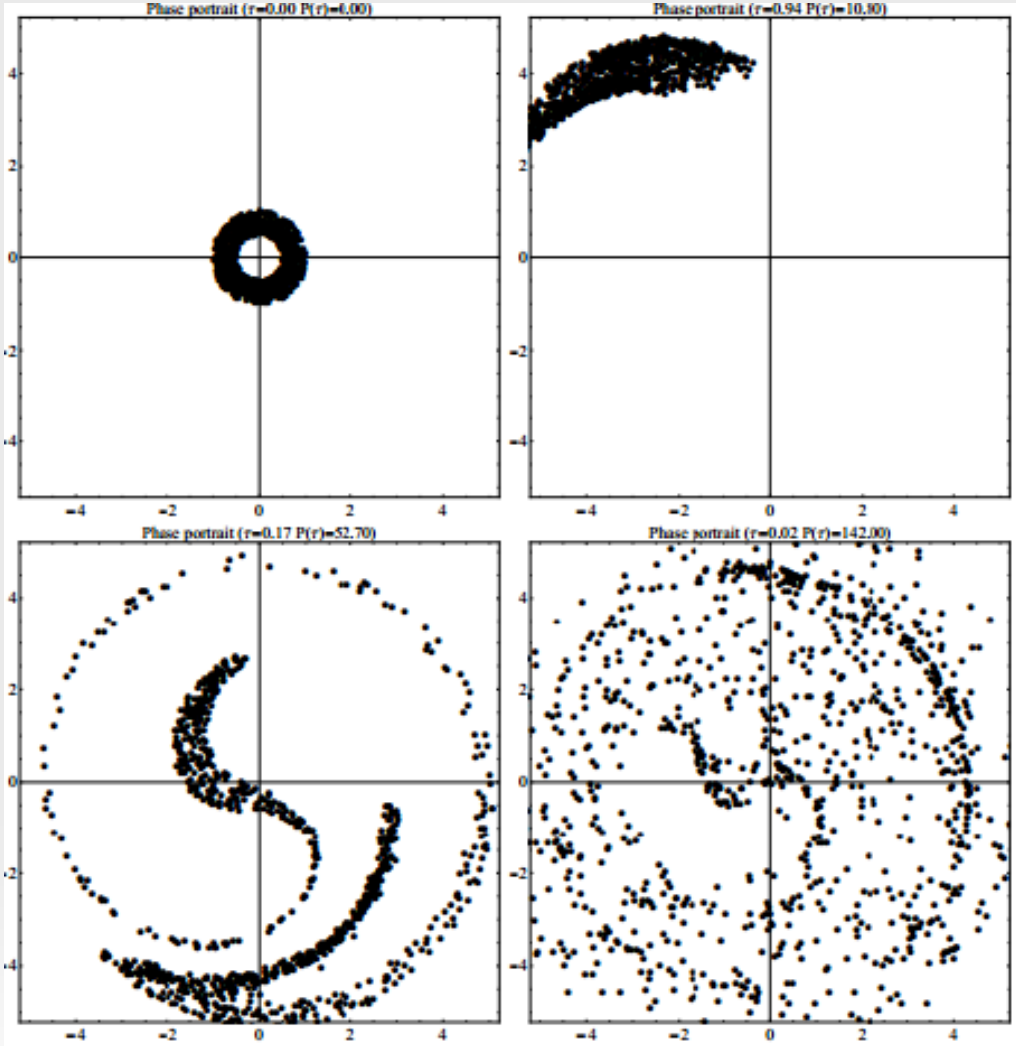
$$\frac{d\theta_i}{d\tau} = -\frac{1}{2}\frac{\nu^2 - \omega_i^2}{\nu^2} - \frac{1}{16}\frac{\omega_i^2}{\nu^2} A_i^2(\tau) - \frac{G}{A_i(\tau)} \cos\theta_i(\tau)$$

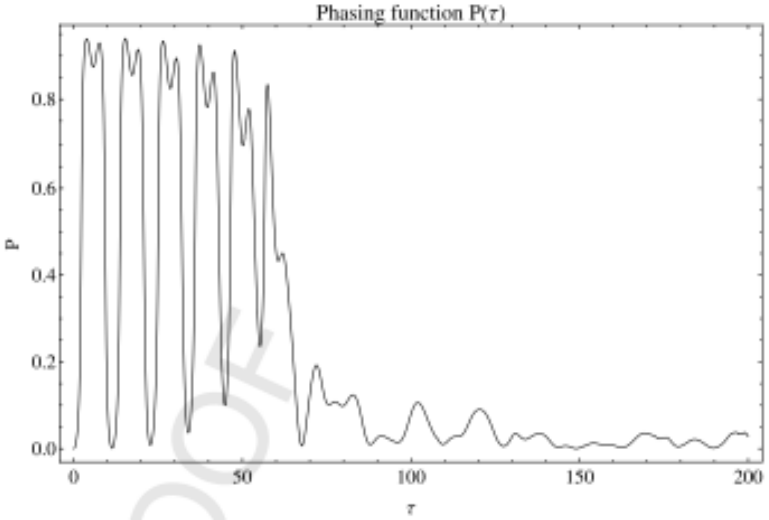
$$\tau = \nu t, \qquad H_i = \frac{e^2\omega_i^2}{3\nu m c^3}, \qquad G = \frac{e(E_0 + E(P))}{2am\nu^2},$$

Measure of phasing

$$P(\tau) = \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\theta_j(\tau)}\right|^2 \quad \Rightarrow \quad P(\tau) = \frac{1}{N^2}\left|\sum_{j=1}^{N} e^{i\theta_j(\tau)}\right|^2$$

- $P = 0$, when there is no any phasing (phase coherence)
- $P = 1$, correspondes to total coherence (100% phasing)





I. Tralle, P. Zięba, Phys Lett A, **378**, 1364-1368 (2014)

$$E \propto 4\sqrt{\frac{\pi n_0 L}{3}}\,\frac{el\omega_0^2}{c^2},\ \omega_0 = \frac{2\pi v_d}{a\left(1-\frac{v_d}{c}\cos\vartheta\right)},$$

where L-structure length, a –period of grating , ω_0 – the frequency of the first harmonics of undulator raiation, n_0 – electron concenrtaion and v_d – drift velocity.

At v_d of the order 10^5 cm·s^{-1}, $n_0 = 10^{16}$ cm^{-3}, and the frequency of the orer of THz , the threshold value of E should be about 10^{-3} V/cm, an hence , L has to be about 5 cm.

Thanks for your attention!

My special thanks to Dr Paweł Zięba,
my former PhD sudent