International Conference and Exhibition on Mesoscopic and Condensed Matter Physics-2015

Three-component gyrotropic metamaterial

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Outline

- Introduction
- The model
- Calculation of effective permeability
- Calculation of effective permittivity
- Results of computer simulations
- Conclusions

According to W. Cai and V. Shalaev, Optical Metamaterials: Fundamentals and Applications (Springer-Verlag, Berlin, 2010), the term “metamaterials” can be used in a more general,as well as in a more specific sense. In the more general sense, these are materials possessing “properties unlike any naturally occurring substance” or simply “not observed innature.” More specifically, these are the materials with a negative refractive index, whose existence and properties
were discussed for the first time by V. Veselago.

Wenshan Cai, V. Shalaev, Optical Metamaterials,Springer 2010

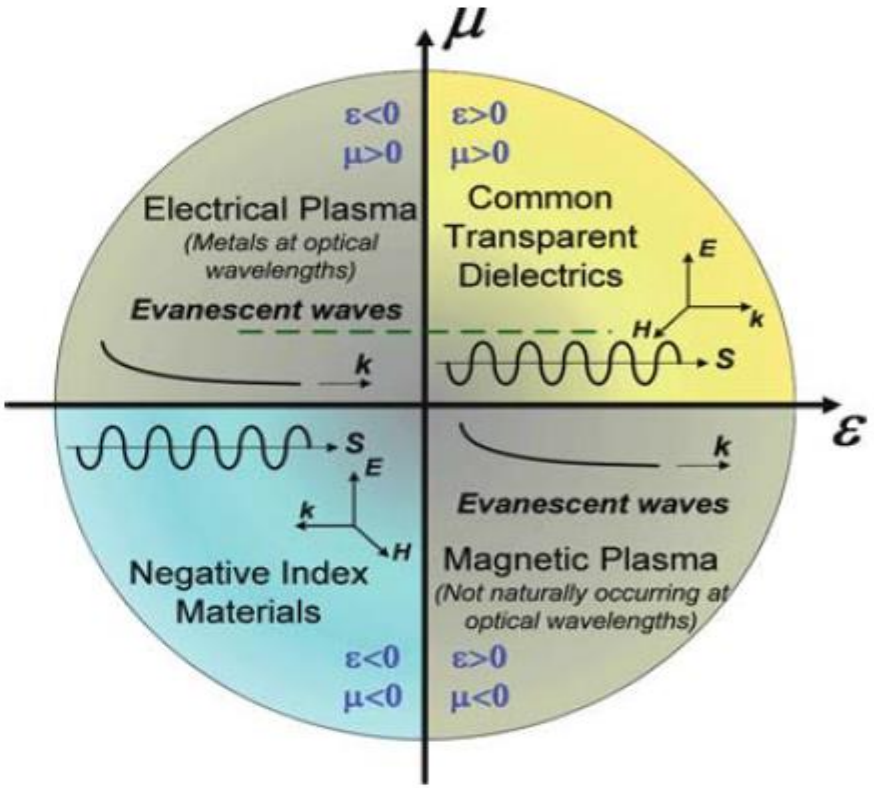
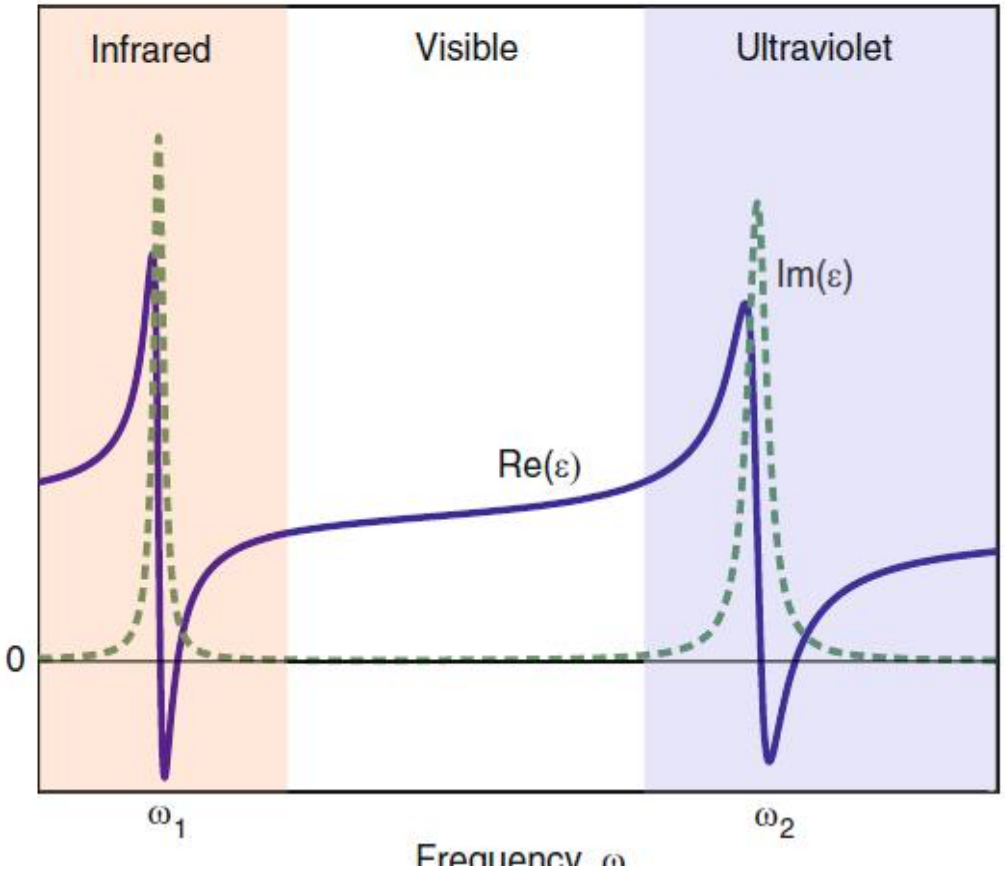
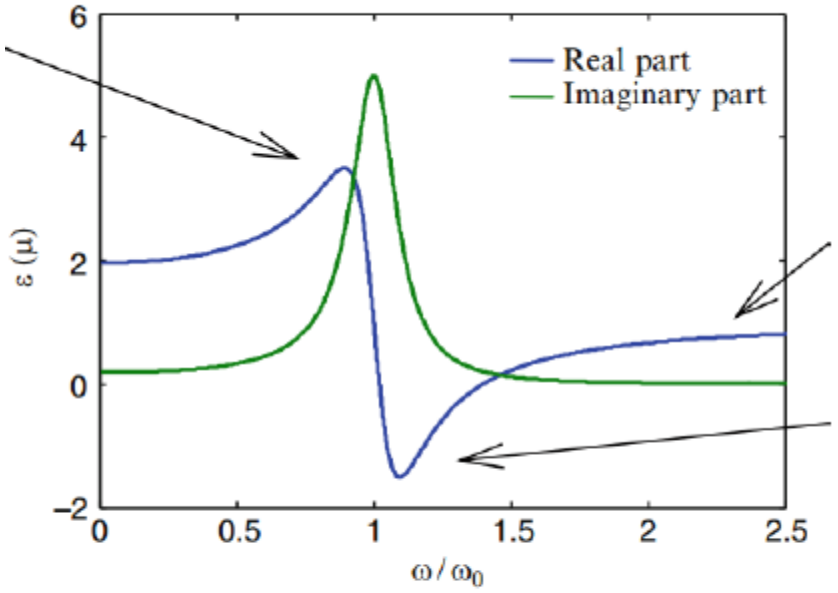


Fig. 1.2 The parameter space for ε and μ. The two axes correspond to the real parts of permittivity and permeability, respectively. The *dashed green line* represents non-magnetic materials with $\mu = 1$

Wenshan Cai, V. Shalaev, Optical Metamaterials,Springer 2010





$$\varepsilon(\omega) = \varepsilon_r(\omega) + i\varepsilon_i(\omega)$$

$$\mu(\omega) = \mu_r(\omega) + i\mu_i(\omega)$$

We consider medium whose structural components sizes are of the order of or greater than the wavelength λ of waves propagating in the medium. In an isotropic medium, the frequency ω depends only on the absolute value of the wave vector k=|**k**|, and therefore the group velocity of the wave packet $\vec{v}_g = \frac{d\omega(\vec{k})}{d\vec{k}} = \frac{\vec{k}}{k}\frac{d\omega(k)}{dk}$ is co-directed with either **k** or -**k**, depending on the sign of $\frac{d\omega(k)}{dk}$.

"Let these conditions be satisfied and, hence, the energy propagate with the group velocity. But we know that the group velocity can be negative. This means that the group (and the energy) propagates in the direction opposite to the propagation direction of the phase of the wave. Is this possible in reality?

One considers a sinusoidal plane wave incident at an angle φ on the interface plane $y = 0$,

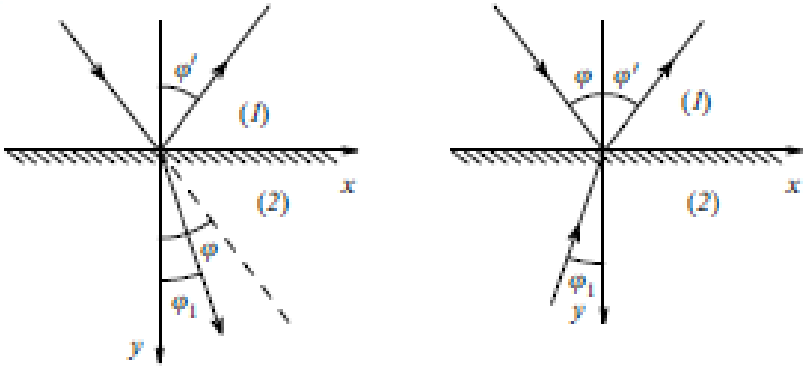
$$E_{\text{inc}} = \exp\{\mathrm{i}[\omega t - k(x\sin\varphi + y\cos\varphi)]\}$$

and, in addition, two other waves: the reflected one

$$E_{\text{refl}} = \exp\{\mathrm{i}[\omega t - k(x\sin\varphi' - y\cos\varphi')]\}$$

and the refracted one

$$E_{\text{refr}} = \exp\{\mathrm{i}[\omega t - k_1(x\sin\varphi_1 + y\cos\varphi_1)]\}\,.$$



gure 2. Reflection and refraction of an incident plane wave. (The picture taken from Mandelstam's lectures [1, 3].)

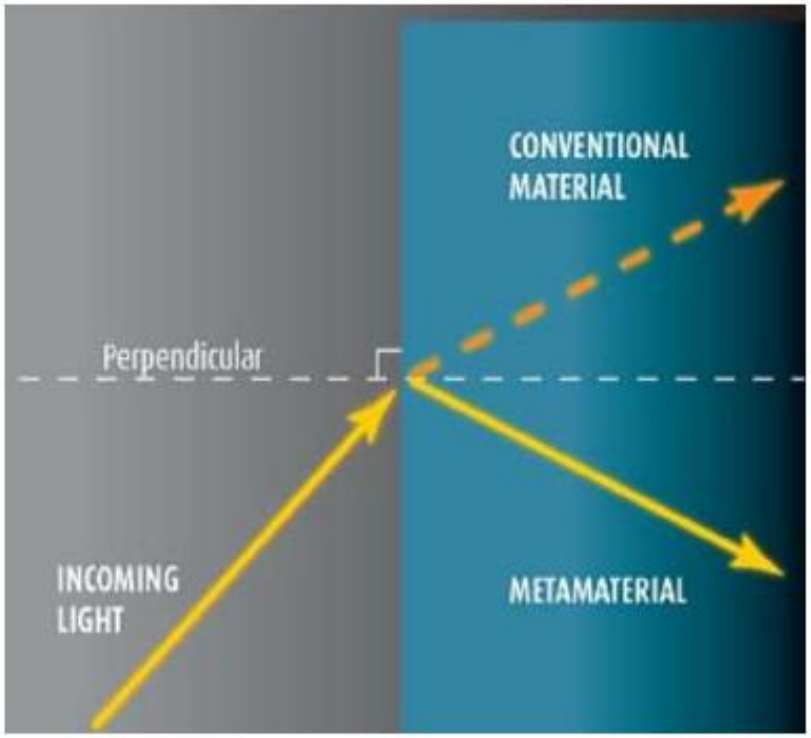
The boundary conditions immediately imply the laws of reflection

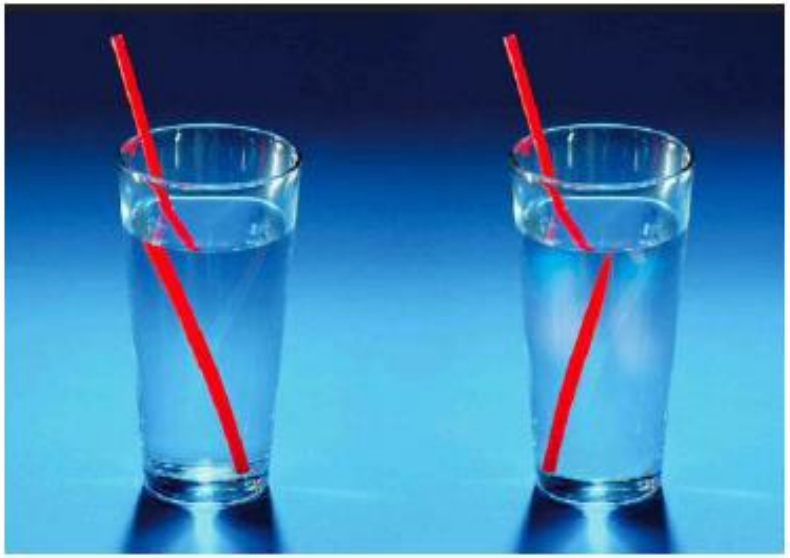
$$\sin\varphi = \sin\varphi'\,, \quad \varphi = \varphi'$$

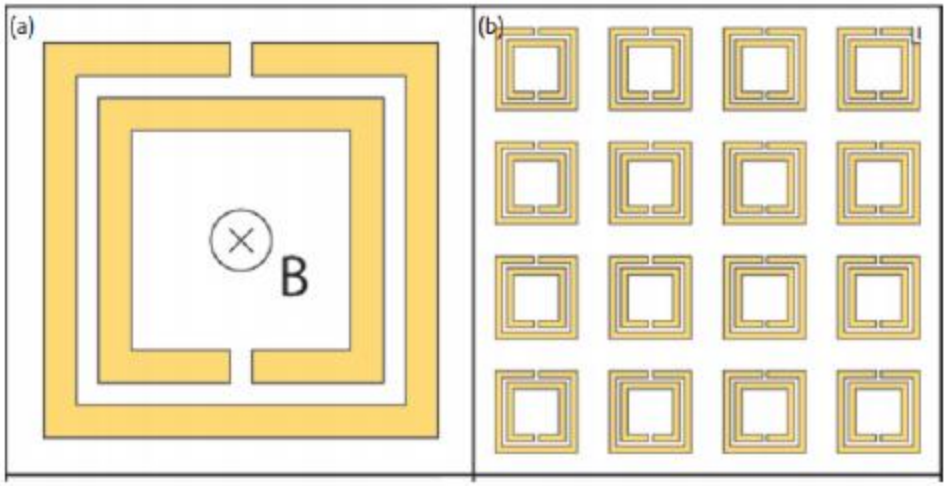
and of refraction

$$k\sin\varphi = k_1\sin\varphi_1\,.$$

The last equation is, however, satisfied not only by the angle φ_1 but also by the angle $\pi - \varphi_1$. The wave corresponding to φ_1 propagates in the second medium away from the interface (left panel in Fig. 2). On the contrary, the wave corresponding to $\pi - \varphi_1$ propagates towards the interface

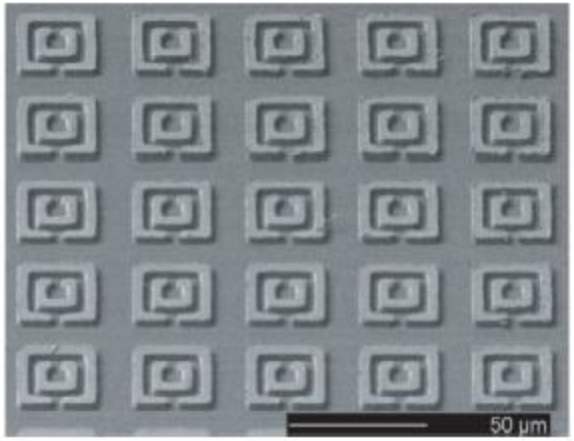

CONVENTIONAL
MATERIAL
Perpendicular
INCOMING
LIGHT
METAMATERIAL





$$\mu(\omega) = 1 + \frac{F\omega^2}{\omega_0^2 - \omega^2 - i\Gamma\omega}$$

$$\varepsilon(\omega) = 1 - \frac{\omega_P^2}{\omega_0^2 - \omega^2 - i\Gamma\omega}$$



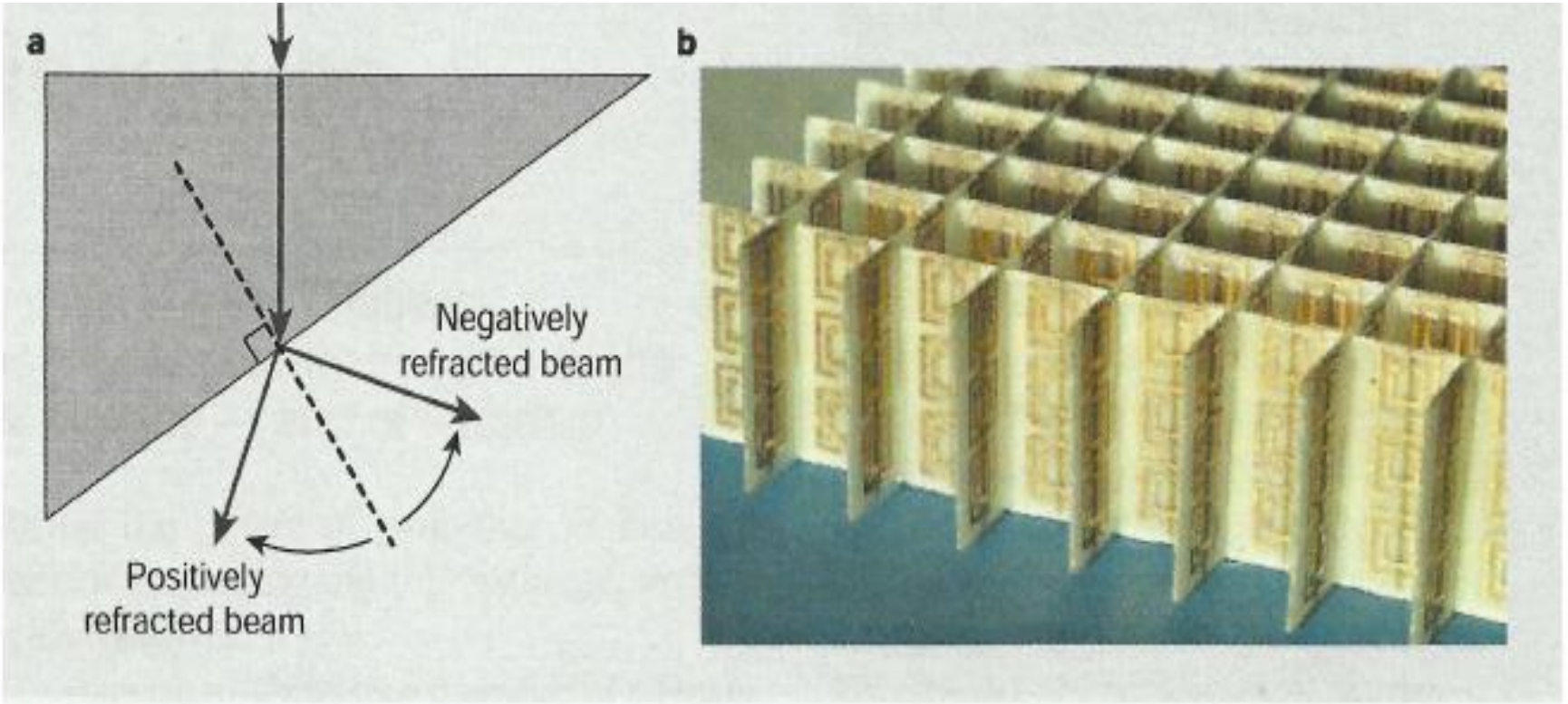


Figure 1 Negative refraction. a, Light incident on a normal material refracts at a positive angle (blue), but in a negative-index material the refraction angle is negative (red). Negative refraction occurs only in specially engineered materials. b, This arrangement of fibreglass sheets (1 cm high) in which is embedded an array of copper loops and wires was the first 'material' for which negative refraction was seen[2].

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J. B. PENDRY

Negative refraction

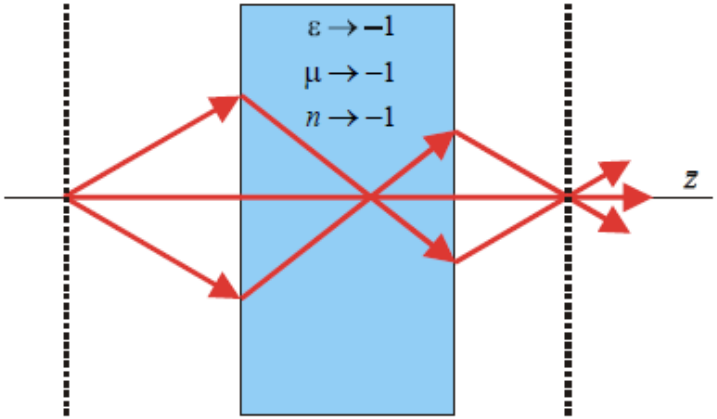


Figure 5. A negative refractive index medium bends light to a negative angle relative to the surface normal. Light formerly diverging from a point source in the object plane is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time in the image plane.

First a few words on the wavelength limit to resolution. In figure 5 an object emits electromagnetic waves of frequency ω. Note that because of dispersion we can only define $\varepsilon \to -1, \mu \to -1$ at a single frequency. Each wave has by a wave vector, $\mathbf{k}$, where,

$$k_z = \sqrt{\omega^2/c_0^2 - k_x^2 - k_y^2}$$

is responsible for driving the wave from object to image, and k_x, k_y define the Fourier components of the image.

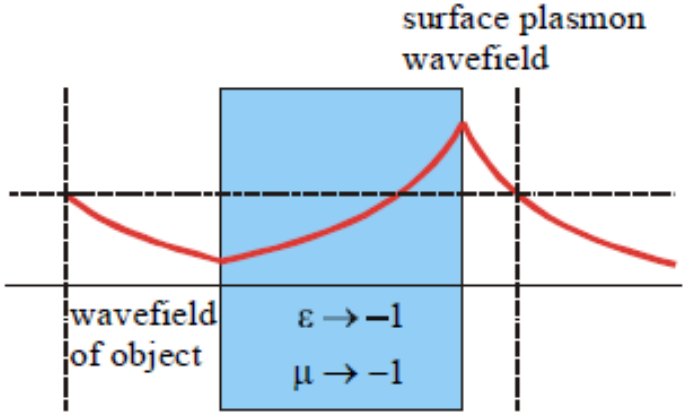
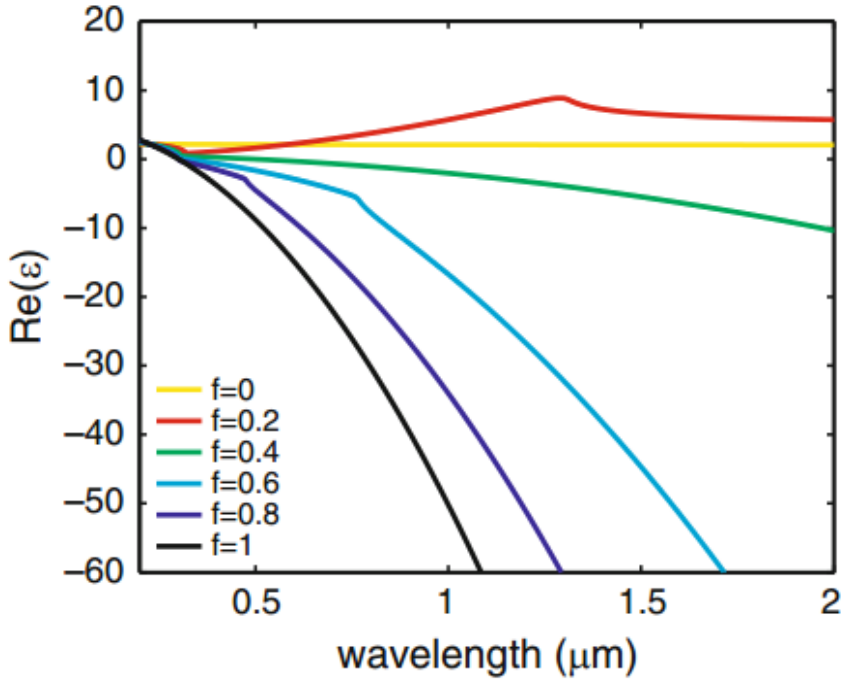
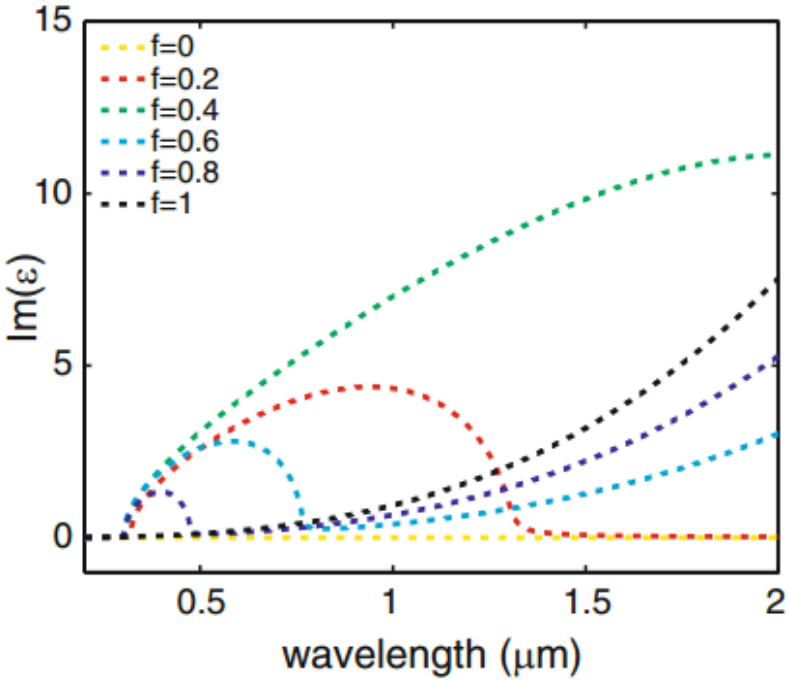


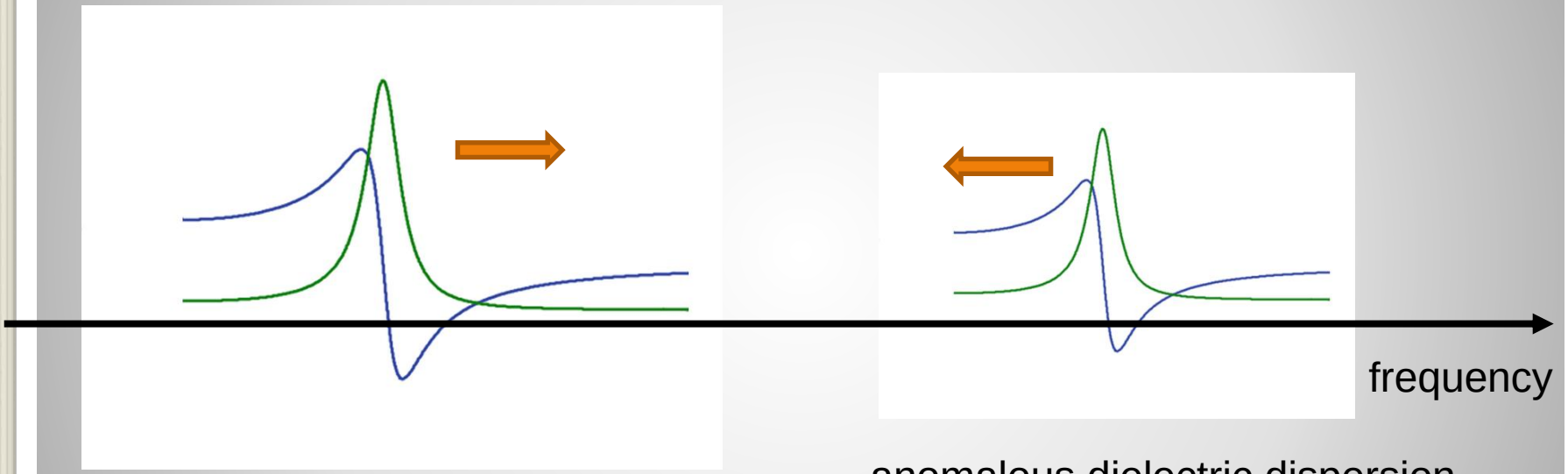
Figure 16. The new lens works by excitation of surface plasmons. Matching the fields at the boundaries selectively excites a surface plasmon on the far surface thus reproducing the same amplitude in the image plane as in the object plane.

Dielelctric function of the mixture Ag + dielectric (silica)





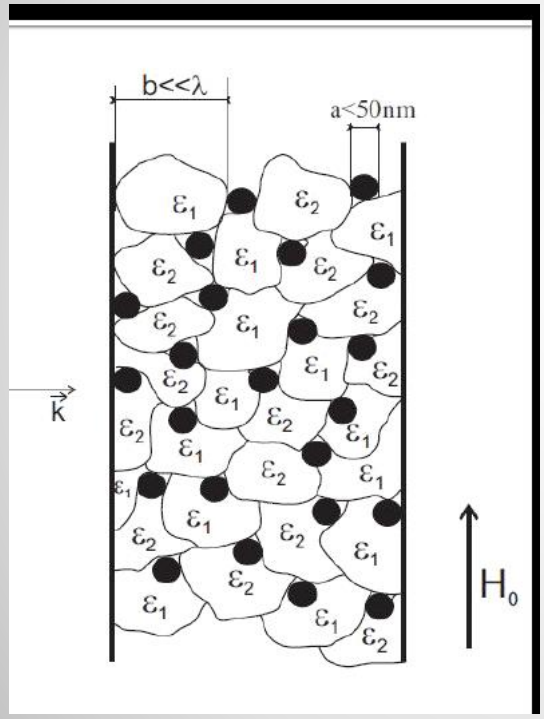
The model



ferromagnetic resonans

anomalous dielectric dispersion

The model



- We consider the mixture of the three ingedients:
1. Silver
2. Hg(1x)Cd(x)Te
3. Ferromagnetic nanoparticles

Effective maganetic permeability of the composite: Bruggemann theory

$$f_1 \frac{\mu_1 - \mu}{\mu_1 + 2\mu} + f_2 \frac{\mu_2 - \mu}{\mu_2 + 2\mu} + f_3 \frac{\mu_3 - \mu}{\mu_3 + 2\mu} = 0$$

Since $\mu_1, \mu_2 \approx 1$

$$\mu = f_1\mu_1 + f_2\mu_2 + f_3(\chi + 1) \approx f_{12} + (1 - f_{12})(\chi_+ + 1)$$

where $f_{12} = f_1 + f_2$

The model

The total magnetization of a single metallic particle can be define as:

$$\vec{m} = \mu_0 \vec{e}_0 + \alpha \vec{H}$$

where:

- μ_0 - the magnetic moment of the grain,
- $\vec{e}_0$ - the unit vector along the direction of $\vec{\mu}_0$
- α - the magnetic polarizability of the particle,
- $\vec{H}$ - the external magnetic field.

The model



where:

$$\alpha' = \frac{3}{8\pi}\left(1 - \frac{3\sin(2x) - 6x\cos(2x)}{2x^3 + 3x\sin(2x) - 6\cos(2x)}\right)$$

$$\beta' = \frac{9}{16x^2}\left(1 - x\frac{\sin(2x) + 2x}{\cos(2x) - 3}\right)$$

here $x = \frac{a}{\delta}, \delta = c/\sqrt{2\pi\sigma\omega}$ is the skin depth, c is the speed of light in vacuum, σ is the conductivity of a metallic grain.

The model

Assuming conductivity of the metallic nano-particles



and for the frequencies up to $\omega \sim 10^{14}$ Hz

we have for the α' and α'' the next expressions:

$$\alpha' = -\frac{4\pi a^4 \sigma^2 \omega^2}{10\varepsilon c^4}$$

$$\alpha'' = \frac{a^2 \sigma \omega}{10\varepsilon c^2}$$

Calculation of effective permeability

We consider the hypotetical material in an external magnetic field:

$$\vec{H} = \vec{H}_0 + \vec{h}(t)$$

Where $\vec{h}(t)$ is the time-dependent magnetic field of electromagnetic wave, propagating in the medium.

The equation of motion is of the form:

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H},$$

where γ is the gyromagnetic constant.

The model

The magnetic moments of single-domain nano-particles at room temperature are distributed at random and we can describe their behaviour in the framework of Langevin theory of paramagnetism.

In the external magnetic field $\vec{H}_0$ an averaged magnetic moment of the unite volume of such medium is:

$$\vec{M}_0 = \chi_0 \vec{H}_0,$$

where:

$$\chi_0 = \frac{(N_{mg-n} \mu_0^2)}{3 k_B T},$$

N_{mg-n} - the concetration of magnetic nano-particles,
k_B - Boltzmann constant,
T - temperature.

Calculation of effective permeability

$$\vec{m} = \chi \vec{h}_\perp - i G \vec{h}_\perp$$

where:

$$\chi = \chi_0 \left(\frac{1}{1 - \tilde{\omega}_1 / \omega} - \frac{1}{1 - \tilde{\omega}_2 / \omega} \right)$$

$$G = \chi_0 \left(\frac{\omega_0}{1 - \tilde{\omega}_1 / \omega} - \frac{\omega_0}{1 - \tilde{\omega}_2 / \omega} \right)$$

here:

$\vec{h}_\perp$ is the component of $\vec{h}(t)$ perpendicular to $\vec{H}_0$,

$$\vec{H}_0 = (0, 0, H_0) \qquad \vec{G} = (0, 0, G),$$

$$\Gamma = \tau^{-1} \qquad \tilde{\omega}_1 = \omega_0 - \sqrt{\omega_0^2 + \Gamma^2}, \qquad \tilde{\omega}_2 = \omega_0 - \sqrt{\omega_0^2 + \Gamma^2}, \qquad \omega_0 = \gamma H_0$$

$$\frac{d\mathbf{m}(t)}{dt} = \gamma \mathbf{m} \times \mathbf{H_0} + \gamma \mathbf{M}_0 \times \mathbf{h}(t) - \frac{\mathbf{m}(t)}{\tau} \quad \Leftarrow \quad \mathbf{h}(t) = \mathbf{h_0} \exp(-\mathbf{i}\omega \mathbf{t})$$

$$-i\omega \mathbf{m} = \gamma \mathbf{h} \times \mathbf{H}_0 + \gamma \chi_0 \mathbf{H}_0 \times \mathbf{h} - \frac{\mathbf{m}}{\tau}$$

$$\mathbf{m} = \chi \mathbf{h}_\perp - \mathbf{i}\mathbf{G} \times \mathbf{h}_\perp$$

$$\chi = \chi_0 \frac{\omega_0^2}{2i\Gamma} \left(\frac{1}{\tilde{\omega_1} - \omega} - \frac{1}{\tilde{\omega_2} + \omega} \right) \qquad G = \chi_0 \frac{\gamma\omega}{2i\Gamma} \left(\frac{1}{\tilde{\omega_1} - \omega} - \frac{1}{\tilde{\omega_2} + \omega} \right) H_0$$

$\mathbf{h}_\perp$ -is the component of $\mathbf{h}(t)$ perpendicular to $\mathbf{H_0}$

$$\mathbf{H_0} = (0, 0, H_0), \mathbf{G} = (0, 0, G), \Gamma = \tau^{-1}$$
$$\tilde{\omega}_1 = -i\Gamma + \sqrt{\omega_0^2 - 2\Gamma^2}, \tilde{\omega}_2 = -i\Gamma - \sqrt{\omega_0^2 - 2\Gamma^2}, \omega_0 = \gamma H_0$$

$$\chi_{\alpha\beta} = \begin{pmatrix} \chi & iG & 0 \\ -iG & \chi & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$m_\pm = m_x \pm i m_y, \quad h_\pm = h_x \pm h_y$$

$$m_\pm = \chi_\pm h_\pm, \quad \chi_\pm = \chi \pm G,$$

$$\chi_+ = \chi + G, \quad \chi_- = \chi - G$$

$$\chi = \frac{\chi_0 \omega_0^2}{\omega_0^2 - 2i\omega\Gamma - \Gamma^2 - \omega^2}, \quad G = \frac{\chi_0 \omega}{\omega_0^2 - 2i\omega\Gamma - \Gamma^2 - \omega^2}.$$

Now we are searhcing for the parameters, *T* temperature, *B* magnetic field, *r* radius of the ferromagnetic nanoparticles, *x* concentration of Cd in $\mathrm{Hg_{1-x}Cd_xTe}$ - compound, f_1, f_2, f_3 the volume filling factors, to get

$$\mathrm{Re}\,[\mu(\omega)] < 0, \quad \mathrm{Re}\,[\varepsilon(\omega)] < 0, \quad \mathrm{Im}\,[\mu(\omega)] > 0, \quad \mathrm{Im}\,[\varepsilon(\omega)] > 0.$$

Effective dielectric permittivity: Briggemann theory:

$$f_1 \frac{\varepsilon_1 - \varepsilon}{\varepsilon_1 + 2\varepsilon} + f_2 \frac{\varepsilon_2 - \varepsilon}{\varepsilon_2 + 2\varepsilon} + f_3 \frac{\varepsilon_3 - \varepsilon}{\varepsilon_3 + 2\varepsilon} = 0$$

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ are the dielectric permittivities of the components

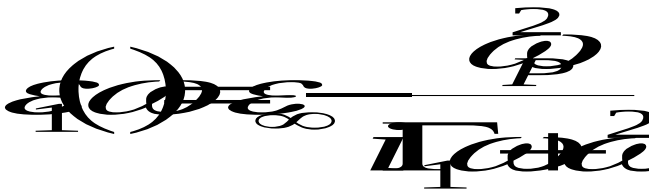
ε is the dielectric permittivity of the composite

f_1, f_2, f_3 the percentage of the components in a mixture

Dielectric function of particular components

1. Silver

A silver is a metal, its dielectric function we calculate from Drude model:



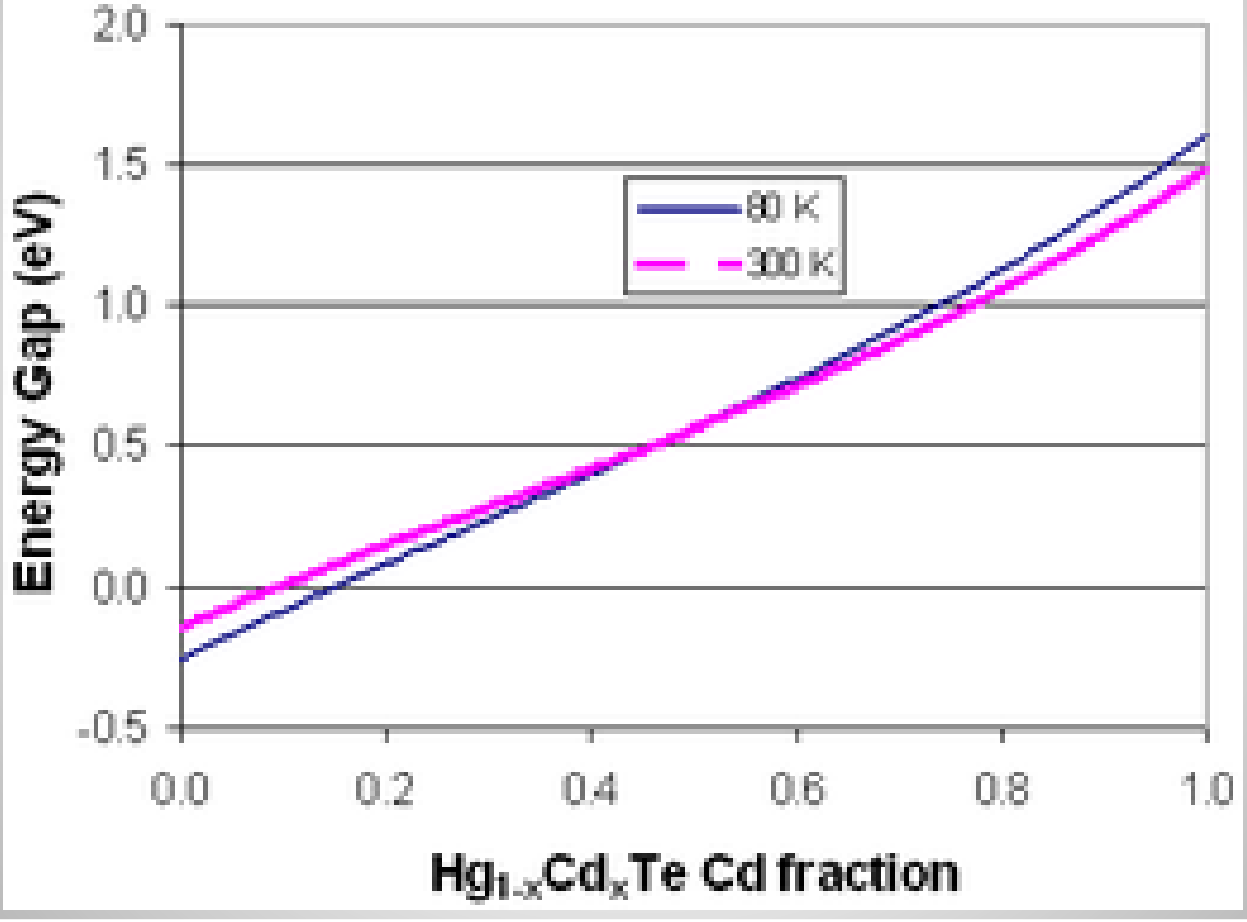
where:

ε_∞ = 5.00 is the permittivity in the limit of very high frequencies,

ω_p ~ 14.00 $\times 10^{15}$ Hz is the plasma frequency,

Γ_1 ~ 0.032 $\times 10^{15}$ Hz is the damping constant.

Energy gap of mercury cadmium telluride vs cadmium concentration



2. Hg(1-x)Cd(x)Te

$$\varepsilon_2(\omega) = \varepsilon_\infty(x) - \frac{1}{\varepsilon_0}\frac{\sigma_0\Gamma_2}{\omega^2+\Gamma_2^2} + \frac{i\sigma_0\Gamma_2^2}{\omega(\omega^2+\Gamma_2^2)}$$

$$\varepsilon_\infty = 15.2 - 15.6x + 8.2x^2$$

$\sigma_0(x,T)$ -is the conductivity,

$\Gamma_2 = 0.1 \times 10^9 \mathrm{Hz}$ - is the damping constant

$$\sigma_0(x,T) = e\, n_e(x,T)\mu_{\mathrm{mob}}(x,T)$$

Conductivity depends on carrier concentratuon and mobility.

$$\begin{aligned} n_e(x,T) &= ((5.585 - 3.82x + (1.753 \times 10^{-3})T \\ &\quad -(1.364 \times 10^{-3})xT) \times 10^{14}) E_g^{0.75} T^{1.5} \exp[-E_g/(2k_B T)] \end{aligned}$$

$$E_g(x,T) = -0.3 + 1.93x - 0.81x^2 + 0.832x^3 + 5.35 \times 10^{-4}(1-2x)T.$$

Concentration depends on temperature and energy gap

Dielectric permittivity of the ferromagnetic nanopartilces

- Typical values of the dielectric permittivity of manganese based ferromagnetics :

$$\mathrm{Re}[\varepsilon_3(\omega)] \sim 10$$

$$\mathrm{Im}[\varepsilon_3(\omega)] > 0$$

Computer simulations

We are searching for the corresponding values of the next parameters

T – temperature

B - magnetic field

r – radius of the ferromagnetic nanoparticles

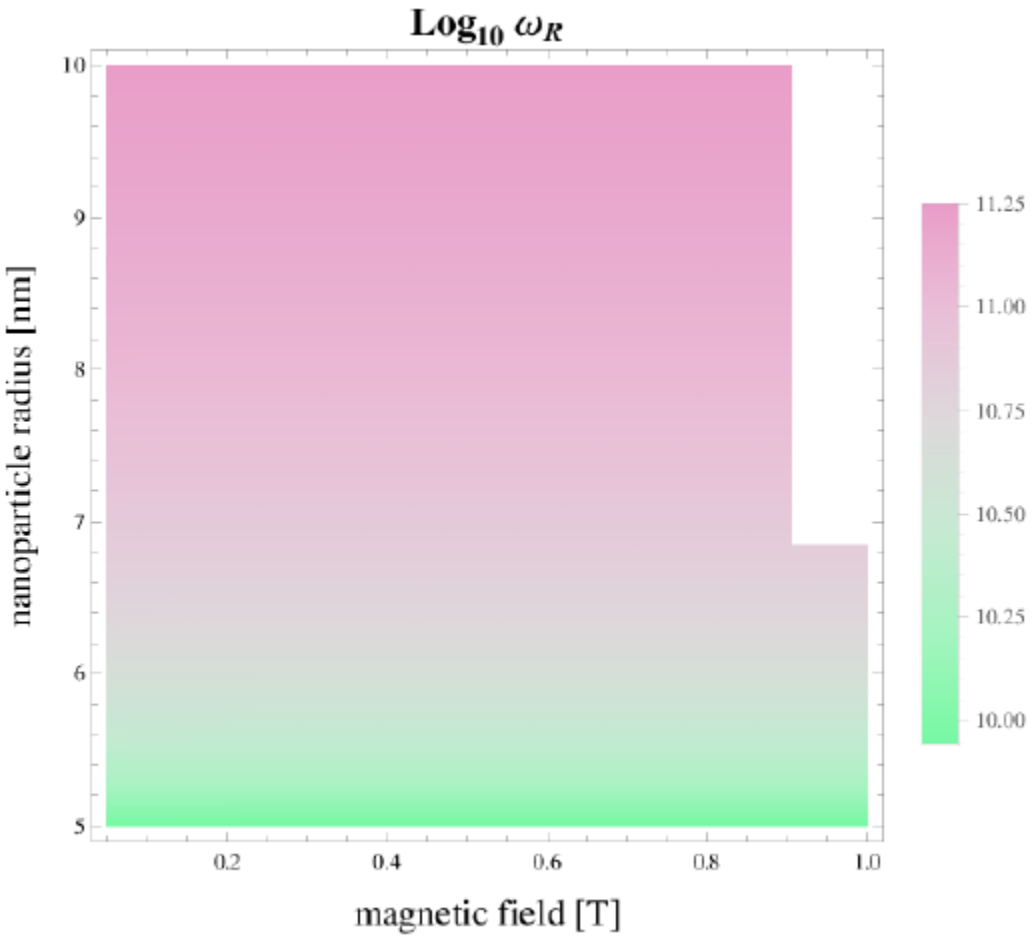
$|\mathbf{m}|$ - magnetic moment of the nanoparticle

x – cadmium concentration in the $Hg_{1-x}Cd_xTe$ – compound

f_1, f_2 , f_3 concentrations of the components in a mixture

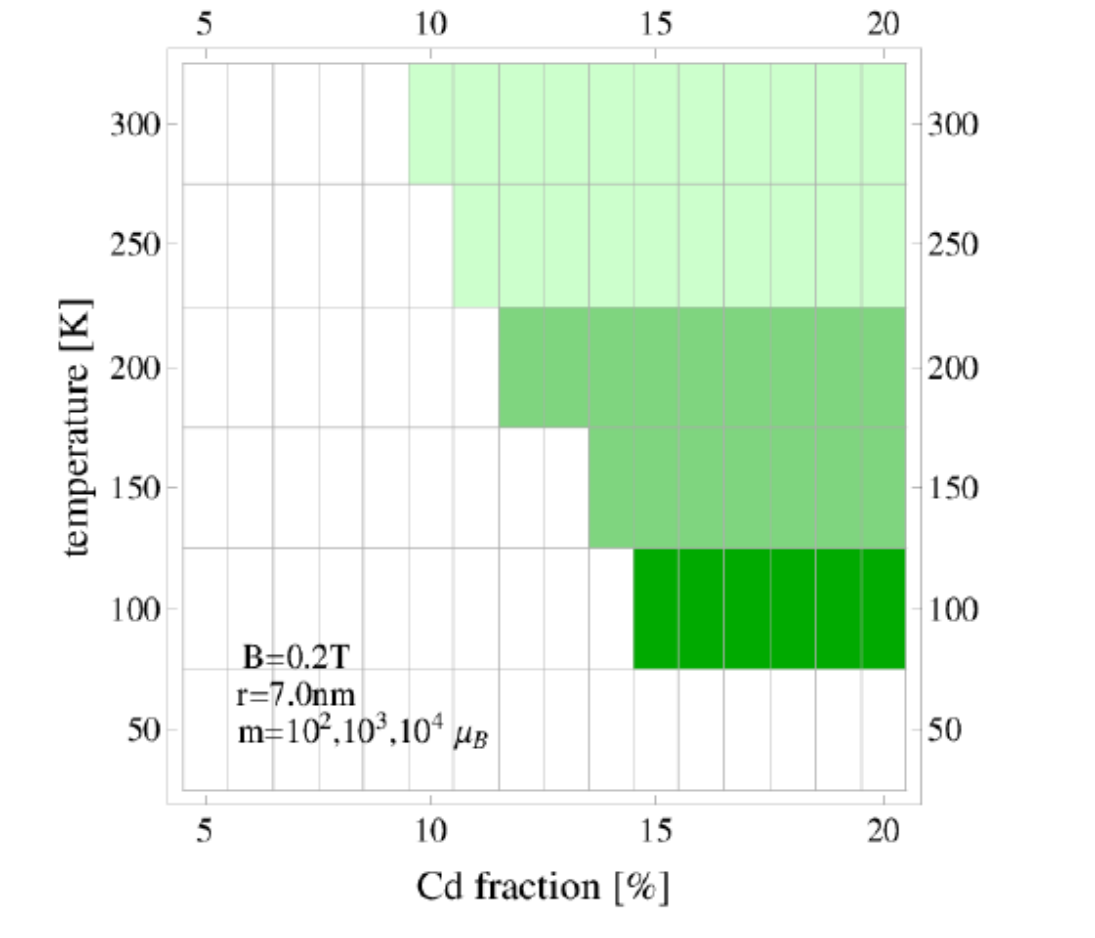
with the additional constrains:

$$\mathrm{Re}[\mu(\omega)] < 0,\ \mathrm{Re}[\varepsilon(\omega)] < 0,\ \mathrm{Im}[\mu(\omega)] > 0,\ \mathrm{Im}[\varepsilon(\omega)] > 0$$

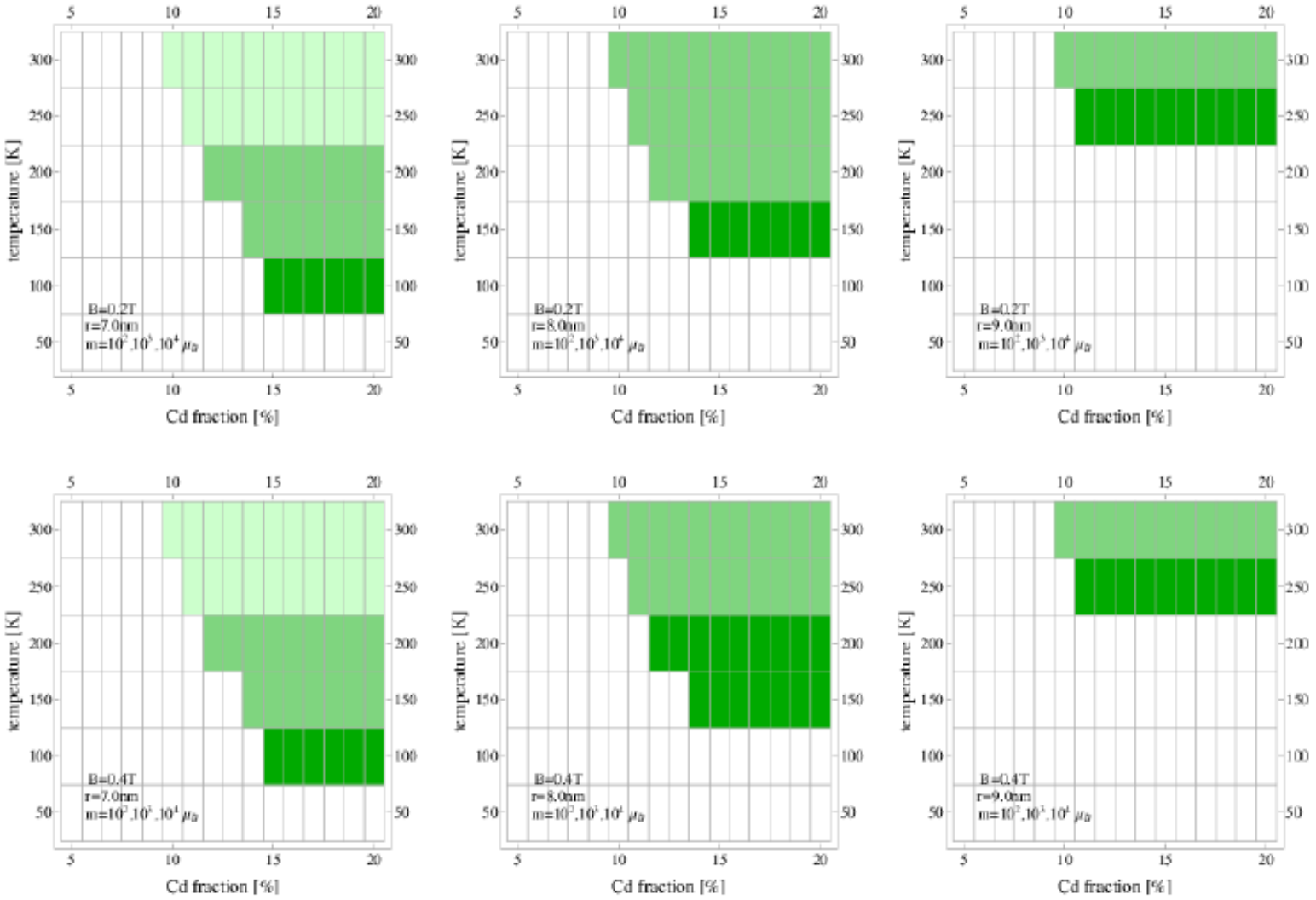


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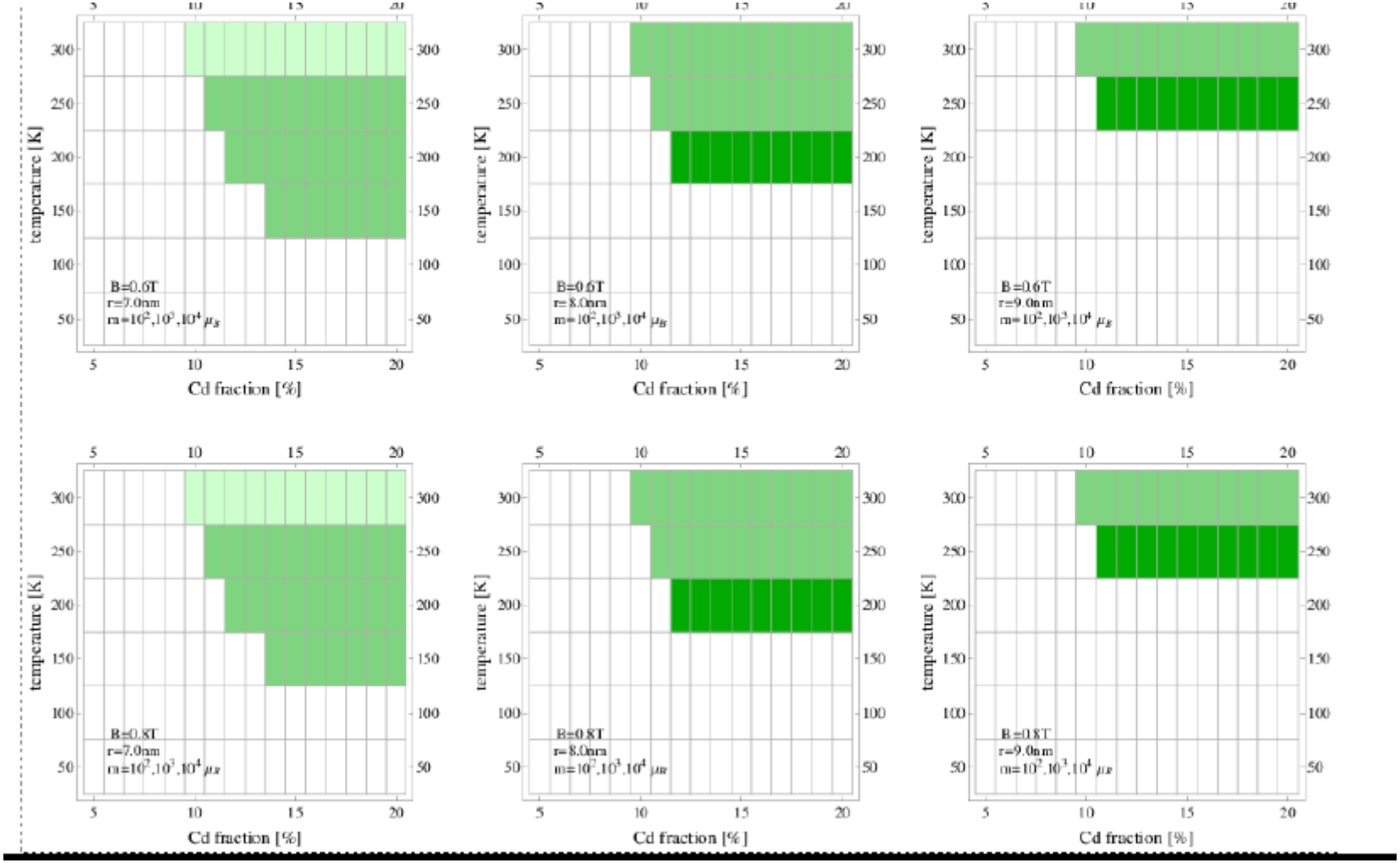
Results of the simulations



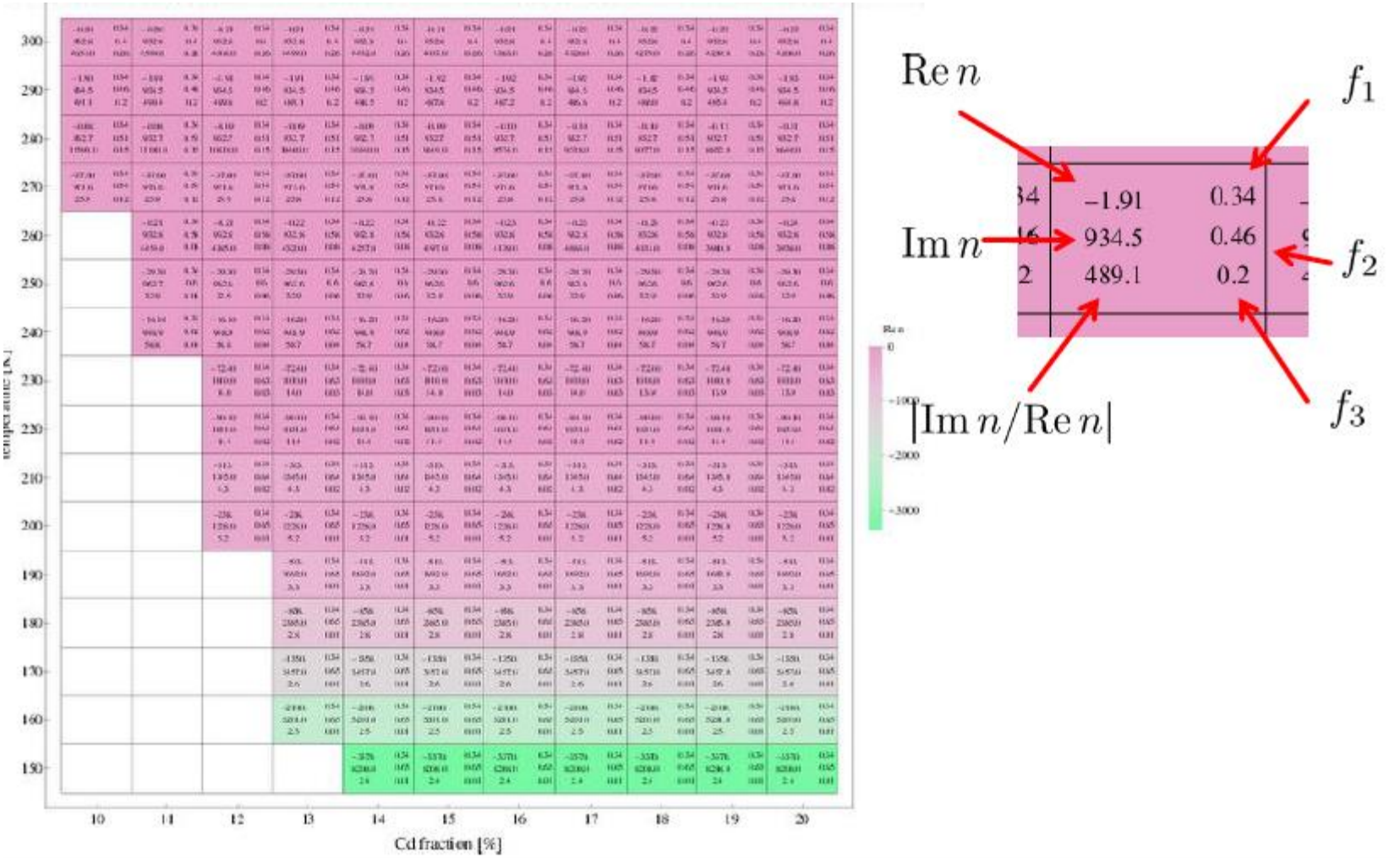
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Thank you for your attention!

Collaboration: Dr Paweł Zięba and Dr Wioletta Paśko, my former PhD students