





A Generalized Method for Calculating Phase Matching Conditions in Biaxial Crystals

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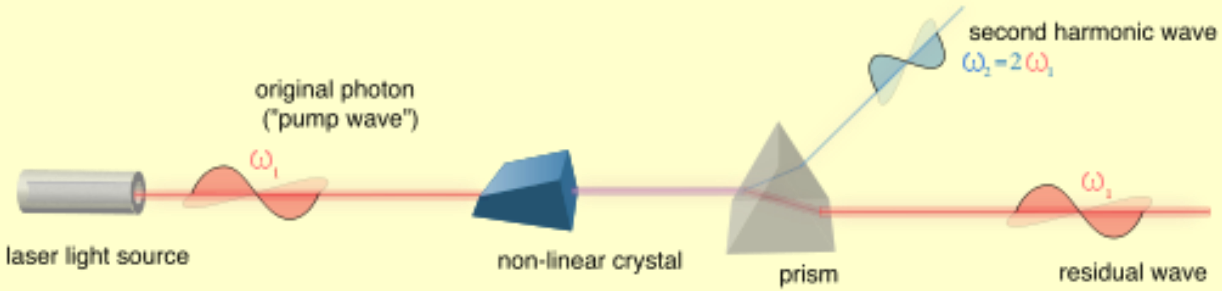
1 Background

2 Our work

3 Outlook

Second harmonic generation was first demonstrated by *Peter Franken, A. E. Hill, C. W. Peters, and G. Weinreich at theUniversity of Michigan, Ann Arbor,* in 1961.

Phys Rev Lett. ***7****(4), 118-119 (1961)*













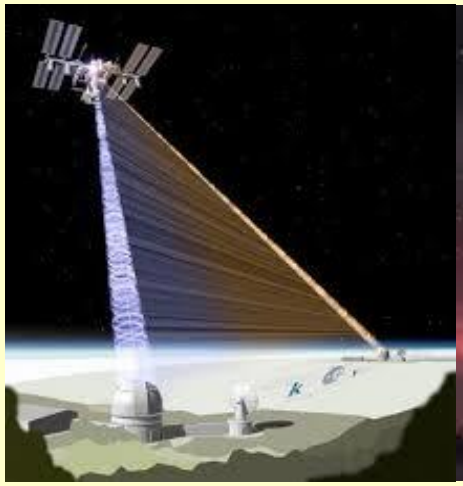


1

Background

My doctor degree thesis focus on “the ultrafast entangled photons generation and detection”.

Crystals’ selection is the key problem for spontaneous parametric down conversion (SPDC).















1.High nonlinearity (coefficient)
2.Vesatile phase matching
3.High damage threshold
4. Inertness to moisture (nonhygroscopic)

$$n_p(\omega_p) = \frac{n_i\left(\frac{\omega_p}{2}\right) + n_s\left(\frac{\omega_p}{2}\right)}{2}$$

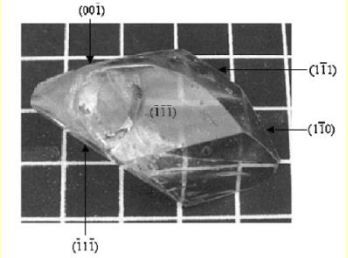


Fig. 1. Photograph of BIBO crystal (bottom view).







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Yao's method solving quadratic Fresnel equations

$$\sin^2\theta\cos^2\varphi/(n_\lambda^{-2}-n_{x,\lambda}^{-2})+\sin^2\theta\sin^2\varphi/(n_\lambda^{-2}-n_{y,\lambda}^{-2})+\cos^2\theta/(n_\lambda^{-2}-n_{z,\lambda}^{-2})=0,$$

$$\sin^2\theta\cos^2\varphi/(n_{2\lambda}^{-2}-n_{x,2\lambda}^{-2})+\sin^2\theta\sin^2\varphi/(n_{2\lambda}^{-2}-n_{y,2\lambda}^{-2})+\cos^2\theta/(n_{2\lambda}^{-2}-n_{z,2\lambda}^{-2})=0,$$

$$n_p(\omega_p)=\frac{n_i\left(\frac{\omega_p}{2}\right)+n_s\left(\frac{\omega_p}{2}\right)}{2}$$

J. Appl. Phys. ***55****, 65 (1984)*
J. Opt. Soc. Am. B ***9****, 891 (1992)*

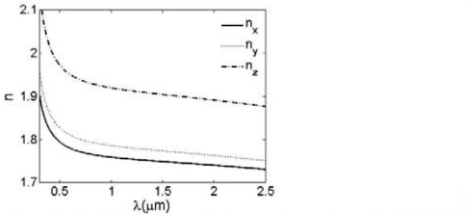


Figure3. Dispersion relations of the refractive indices and calculated curves at room temperature (295 K).

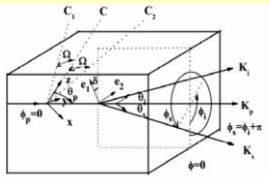


Figure 2.The schematic diagram of the SPDC process in the crystal with the pump light in $x-z$ plane. C_1, C_2 is the two axes of crystal and C is the z axis orientation. θ_j and ϕ_j $(j=p,s,i)$ is the angle of pump, signal and idler. Ω is the angle between C_1, C_2 and C, which is the auxiliary angle of the polarization.







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Background

$\theta=152.00^\circ$, $\phi=90^\circ$ and $d_{eff}=3.48$ pm/V

Proc.of SPIE Vol.8333,833313-1-10 (2012)

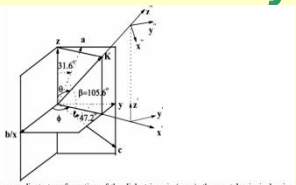


Figure 1.The schematic showing the coordinate transformation of the dielectric axis (xyz), the crystal principal axis (abc) and the lab frame ($x'y'z'$). θ is the angle between the z axis and the wave vector $\vec{k}$. ϕ is the angle between the x axis and the wave vector $\vec{k}$ project to the x - y plane. β is the angle between the crystal principal axis a and c .

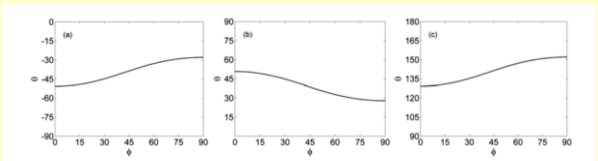


Figure 4.The variation of θ dependent on ϕ . (a)~(c) correspond to the case of ($\theta=-90^\circ\sim0^\circ$, $\phi=0^\circ\sim90^\circ$), ($\theta=0^\circ\sim90^\circ$, $\phi=0^\circ\sim90^\circ$), and ($\theta=90^\circ\sim180^\circ$, $\phi=0^\circ\sim90^\circ$), respectively.

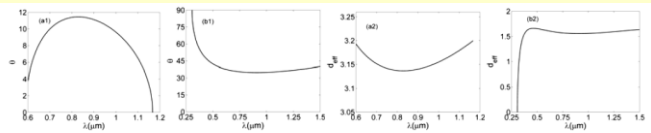


Figure 7. The variation relations of θ (1) and the absolute d_{eff} (2) in the x - z plane as a function of fundamental wavelength.

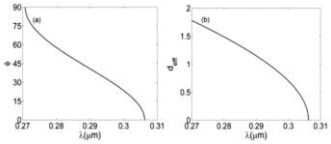


Figure 5.The variation relations of θ (a) and the absolute d_{eff} (b) in the x - y plane as a function of fundamental wavelength.

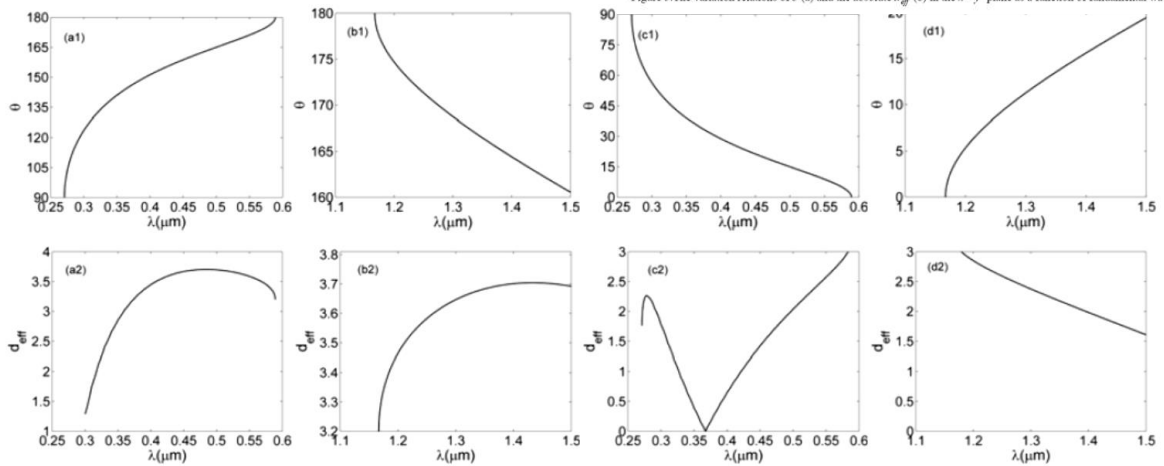


Figure 6. The variation relations of θ and the absolute d_{eff} in the y -z plane as a function of fundamental wavelength.











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Background

Type II, ω= 800 nm, GVD=0

Optik,124,6627-6630 (2013)

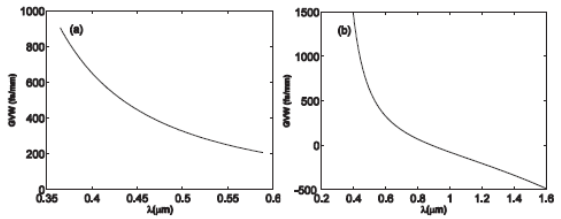


Fig. 1. The GVW as a function of fundamental wavelength: (a) the type I phase matching and (b) the type II phase matching.

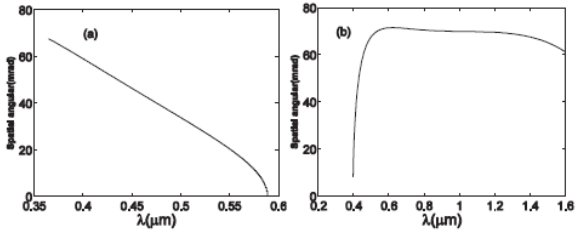


Fig. 3. The spatial walk-off angles as a function of fundamental wavelength: (a) the type I and (b) the type II.

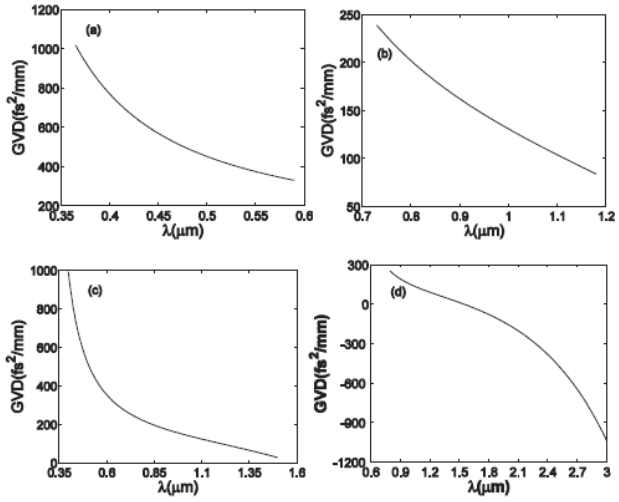


Fig. 2. (a) (c) The fluctuant curve of GVD and fundamental wavelength for type I and type II; (c) (d) the curve of GVD and SPDC pulses for type I and type II.

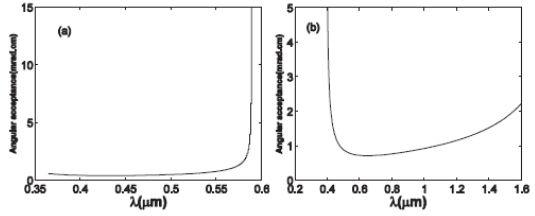


Fig. 4. The FWHM of internal angle acceptance $\Delta\theta L$ for fundamental wavelength: (a) the type I and (b) the type II.

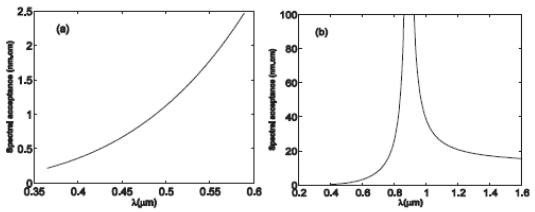


Fig. 5. The FWHM of spectral acceptance as a function of fundamental wavelength: (a) the type I and (b) the type II.











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Back

$\Phi=11^\circ$, Type II, $\omega=800$ nm,

$d_{eff}=2.5445$pm/V

Journal of Nonlinear Optical Physics & Materials, 22(1), 1350010 (2013)

$$d_{\text{eff}}^{\text{fss}} = -(d_{11}\cos^2\phi + 3d_{12}\sin^2\phi)\cos^3\theta\cos\phi\sin\delta\cos^3\delta$$
$$-[d_{11}\sin^2\phi + d_{12}(3\cos^2\phi - 2)]\cos\theta\cos\phi\sin^3\delta$$
$$+2[d_{11}\sin^2\phi - d_{12}(3\sin^2\phi - 1)]\cos\theta\cos\phi\sin\delta\cos^2\delta$$
$$+2[d_{11}\cos^2\phi - d_{12}(3\cos^2\phi - 1)]\cos^2\theta\sin\phi\sin^2\delta\cos\delta$$
$$-[d_{11}\cos^2\phi - d_{12}(3\cos^2\phi - 1)]\cos^2\theta\sin\phi\cos^3\delta$$
$$-(d_{11}\sin^2\phi + 3d_{12}\cos^2\phi)\sin\phi\sin^2\delta\cos\delta$$
$$-d_{13}\sin^2\theta\cos\delta[3\cos\theta\cos\phi\sin\delta\cos\delta + \sin\phi(3\cos^2\delta - 2)]$$
$$+d_{14}[\sin 2\theta\cos 2\phi\cos\delta(3\sin^2\delta - 1)$$
$$+\sin\theta\sin 2\phi\sin\delta(3\cos^2\theta\cos^2\delta + 3\cos^2\delta - 1)], \quad (3.2)$$

$$d_{\text{eff}}^{\text{fss}} = -[d_{11}\cos^2\phi - d_{12}(3\cos^2\phi - 1)]\cos^2\theta\sin\phi$$
$$-d_{13}\sin^2\theta\sin\phi - d_{14}\sin 2\theta\cos 2\phi. \quad (3.3)$$

$$d_{\text{eff}}^{\text{ffs}} = d_{\text{eff}}^{\text{fsf}}$$
$$= +(d_{11}\cos^2\phi + 3d_{12}\sin^2\phi)\cos^3\theta\cos\phi\sin^2\delta\cos\delta$$
$$+[d_{11}\sin^2\phi + d_{12}(3\cos^2\phi - 2)]\cos\theta\cos\phi\cos^3\delta$$
$$-2[d_{11}\sin^2\phi - d_{12}(3\sin^2\phi - 1)]\cos\theta\cos\phi\sin^2\delta\cos\delta$$
$$+2[d_{11}\cos^2\phi - d_{12}(3\cos^2\phi - 1)]\cos^2\theta\sin\phi\sin\delta\cos^2\delta$$
$$-[d_{11}\cos^2\phi - d_{12}(3\cos^2\phi - 1)]\cos^2\theta\sin\phi\sin^3\delta$$
$$-(d_{11}\sin^2\phi + 3d_{12}\cos^2\phi)\sin\phi\sin\delta\cos^2\delta$$
$$+d_{13}\sin^2\theta\sin\delta[3\cos\theta\cos\phi\sin\delta\cos\delta - \sin\phi(3\sin^2\delta - 2)]$$
$$+d_{14}[\sin 2\theta\cos 2\phi\sin\delta(3\cos^2\delta - 1)$$
$$-\sin\theta\sin 2\phi\cos\delta(3\cos^2\theta\sin^2\delta + 3\sin^2\delta - 1)]. \quad (3.4)$$

$= 0$, the Eq. (3.4) turns into

$$d_{\text{eff}}^{\text{ffs}} = d_{\text{eff}}^{\text{fsf}}$$
$$= [d_{11}\sin^2\phi + d_{12}(3\cos^2\phi - 2)]\cos\theta\cos\phi + d_{14}\sin\theta\sin 2\phi. \quad (3.5)$$

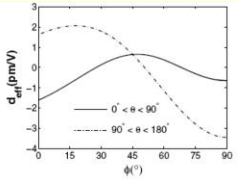


Fig. 5. The variation relation of d_{eff} along with the azimuthal angle ϕ for the type I at 400 nm.

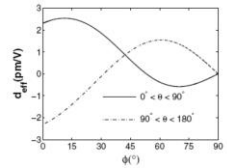


Fig. 6. The variation relation of d_{eff} along with the azimuthal angle ϕ for type II at 800 nm.

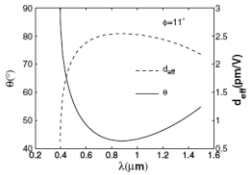


Fig. 7. The phase matching angles θ and absolute effective nonlinear coefficient d_{eff} varying with fundamental wavelength in $\phi = 11^\circ$ plane for the type II SPDC.







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Our work

the longitudinal detuning $n_p(\omega_p) = \frac{n_i\left(\frac{\omega_p}{2}\right) + n_s\left(\frac{\omega_p}{2}\right)}{2}$

$$\Delta_{z''} = \vec{k}_p(\theta,\phi_0) - \vec{k}_1(\theta,\phi_0) - \vec{k}_2(\theta,\phi_0) - \frac{\left(\vec{k}_{1\perp}(\theta,\phi_0) - \vec{k}_{2\perp}(\theta,\phi_0)\right)^2}{2k_p(\theta,\phi_0)}$$

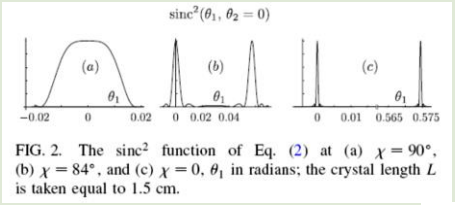
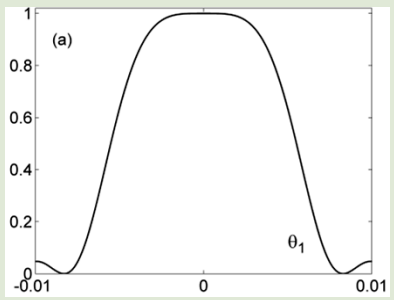
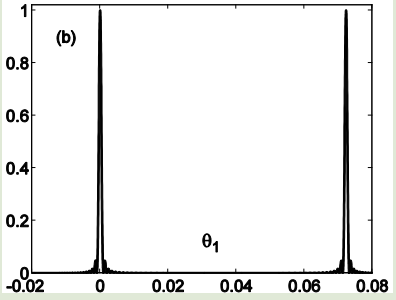
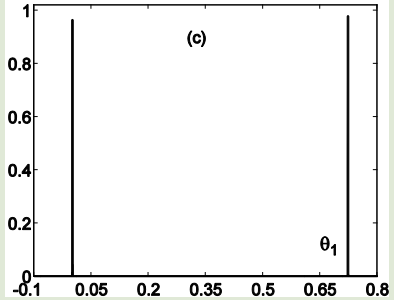


FIG. 2. The sinc² function of Eq. (2) at (a) $\chi = 90°$, (b) $\chi = 84°$, and (c) $\chi = 0$, θ_1 in radians; the crystal length L is taken equal to 1.5 cm.

Phys. Rev. Lett. ***99***, 063901 (2007)
Phys. Rev. A ***77***, 032336 (2008)







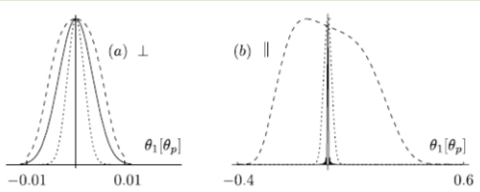
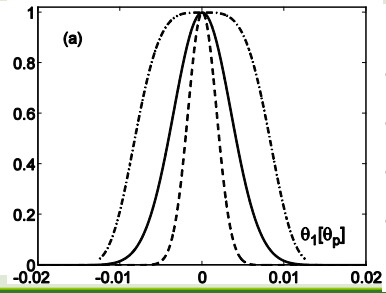
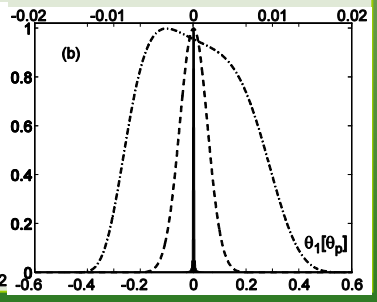


FIG. 3. Coincidence (solid lines) and single-particle (dashed lines) angular distributions for (a) $\chi = \pi/2$ and (b) $\chi = 0$; all curves are normalized by one at their maxima.















2

Our Work

Fresnel equations change to following form without solving quadratic equations

$$\frac{1}{n_{1,2}^2}=\frac{\cos^2\left[(\theta_1\pm\theta_2)/2\right]}{n_x^2}+\frac{\sin^2\left[(\theta_1\pm\theta_2)/2\right]}{n_z^2}$$

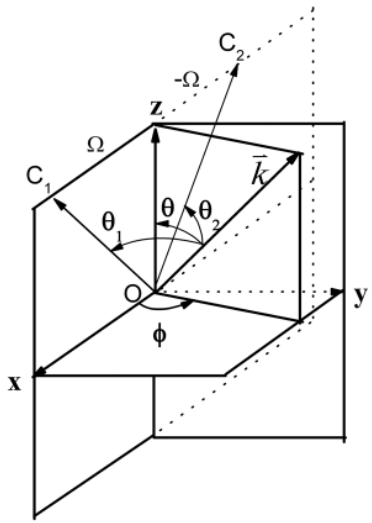


Fig. 1 The schematic diagram of biaxial crystal parameters in coordinate system.

$$\Omega=\arcsin\left(\frac{n_z}{n_y}\sqrt{\frac{n_y^2-n_x^2}{n_z^2-n_x^2}}\right)$$

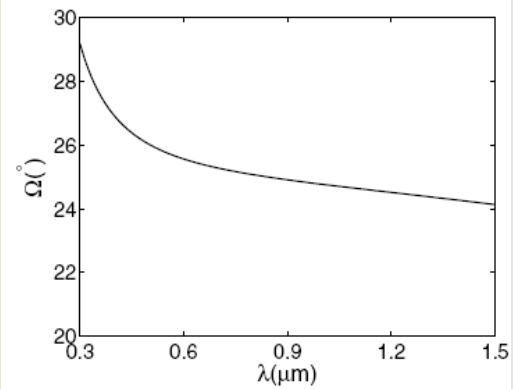


Fig. 2. The dispersion curve of optical axis angle Ω.

Optical Engineering， 53(8),086105 (2014)







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2

Our Work

Japanese mathematician Kodaira Kunihiko

angle is a real number define the rotation of a plane

$$n_p(\omega_p) = \frac{n_i\left(\frac{\omega_p}{2}\right) + n_z\left(\frac{\omega_p}{2}\right)}{2}$$

$$n_1 = \frac{n_x n_z}{\sqrt{n_z^2\cos^2[(\theta_1+\theta_2)/2] + n_x^2\sin^2[(\theta_1+\theta_2)/2]}}$$

$$n_2 = \frac{n_x n_z}{\sqrt{n_z^2\cos^2[(\theta_1-\theta_2)/2] + n_x^2\sin^2[(\theta_1-\theta_2)/2]}}$$

An Introduction to Calculus

(Originally published in Japanese by Iwanami Shoten, Publishers, Tokyo, 2003)

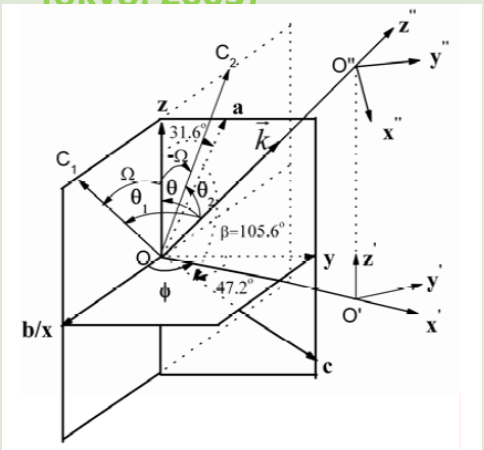


Fig. 1. The schematic diagram of biaxial crystal parameters in different coordinate system: the dielectric axis (xyz), the crystal principal axis (abc) and the lab frame (x"y"z").

$$0^\circ \le \theta < 90^\circ \quad 0^\circ \le \phi \le 90^\circ$$

$$\cos\theta_1 = (\cos\theta / \cos\gamma)\cdot\cos(\Omega-\gamma)$$

$$\cos\theta_2 = (\cos\theta / \cos\gamma)\cdot\cos(\gamma+\Omega)$$

$$\gamma = \arctan(\tan\theta\cdot\cos\phi)$$

$$90^\circ < \theta \le 180^\circ \quad 0^\circ \le \phi \le 90^\circ$$

$$(\pi-\theta)$$

$$\theta = 90^\circ$$

$$\cos\theta_1 = \sin\Omega\cdot\cos\phi$$

$$\cos\theta_2 = \sin\Omega\cdot\sin\phi$$







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Applied Physics B, 120(2), 239-246 (2015)

$$\tan\theta = \frac{\overline{zK}}{\overline{Oz}} \qquad \overline{zK_1} = \overline{zK}\cdot\cos(\pi/2-\phi) \qquad \tan\theta' = \frac{\overline{zK_1}}{\overline{Oz}} = \frac{\overline{zK}\cdot\cos(\pi/2-\phi)}{\overline{Oz}} = \tan\theta\cdot\cos(\pi/2-\phi)$$

$$\cos(\angle C_2OK_1) = \cos\theta'\cdot\cos(-\Omega) \qquad \overline{zK_2} = \overline{zK}\cdot\sin(\pi/2-\phi)$$

$$\tan\theta'' = \frac{\overline{zK_2}}{\overline{Oz}} = \frac{\overline{zK}\cdot\sin(\pi/2-\phi)}{\overline{Oz}} = \tan\theta\cdot\sin(\pi/2-\phi) \qquad \cos(\angle C_1OK_1) = \cos\theta'\cdot\cos(\Omega)$$

$$\angle C_1OK_2 = \Omega-\theta'' \qquad \angle C_2OK_2 = \theta''-(-\Omega)$$

$$\theta = 90^\circ \qquad \cos(\angle C_1OO') = \cos(\pi/2-\Omega)\cdot\cos\phi \qquad \cos(\angle C_2OO') = \cos(\pi/2-|-\Omega|)\cdot\cos(\pi-\phi)$$

$$\overline{K_2K} = \overline{zK_1} = \overline{zK}\cdot\cos(\pi/2-\phi) \qquad \overline{K_2O} = \overline{Oz}/\cos\theta'' \qquad \cos\theta = \cos\angle K_2OK\cdot\cos\theta''$$

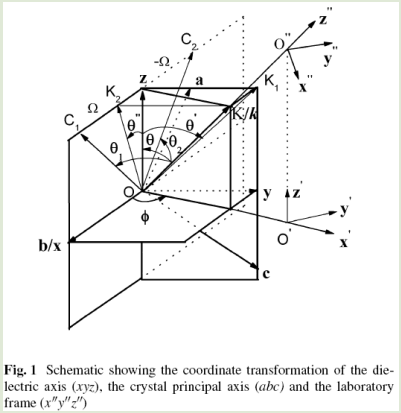


Fig. 1 Schematic showing the coordinate transformation of the dielectric axis (*xyz*), the crystal principal axis (*abc*) and the laboratory frame ($x''y''z''$)

$$\tan\angle K_2OK = \frac{\overline{K_2K}}{\overline{K_2O}} = \frac{\overline{zK}\cdot\cos(\pi/2-\phi)}{\overline{Oz}/\cos\theta''} = \tan\theta\cdot\cos(\pi/2-\phi)\cdot\cos\theta''$$

$$\cos\theta_1 = \cos\angle K_2OK\cdot\cos(\angle C_1OK_2) = \cos\{\arctan[\tan\theta\cdot\cos(\pi/2-\phi)\cdot\cos\theta'']\}\cdot\cos(\Omega-\theta'') = (\cos\theta/\cos\theta'')\cdot\cos(\Omega-\theta'')$$

$$\theta'' = \arctan(\tan\theta\cdot\cos\phi)$$

$$\cos\theta_2 = \cos\angle K_2OK\cdot\cos(\angle C_2OK_2) = \cos\{\arctan[\tan\theta\cdot\cos(\pi/2-\phi)\cdot\cos\theta'']\}\cdot\cos[\theta''-(-\Omega)] = (\cos\theta/\cos\theta'')\cdot\cos(\theta''+\Omega)$$







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Our Work

Applied Physics B, 120(2), 239-246 (2015)

$90^o < \theta \le 180^o \qquad 0^o \le \phi \le 90^o$

$$\theta'' = -\arctan(\tan\theta \cdot \cos\phi)$$

$\theta = 90^o, 0^o \le \phi \le 90^o$

$$\cos\theta_1 = \cos(\angle C_1 OO') = \sin\Omega \cdot \cos\phi$$

$$\cos\theta_2 = \cos(\angle C_2 OO') = -\sin\Omega \cdot \cos\phi$$

$0^o \le \phi \le 90^o \qquad 0^o \le \theta \le 180^o (\theta \ne 90^o)$

$$\theta' = \arctan(\tan\theta \cdot \sin\phi)$$

$$\theta'' = \arctan(\tan\theta \cdot \cos\phi)$$

$\phi = 0^o \qquad \theta' = 0 \qquad \theta'' = \theta$

$$\cos\theta_1 = \cos(\Omega - \theta) \qquad \cos\theta_2 = \cos(\Omega + \theta)$$

$\phi = 90^o \qquad \theta' = 90^o \qquad \theta'' = 0^o$

$$\cos\theta_1 = \cos\theta_2 = \cos\theta \cdot \cos\Omega$$

$\Omega = 0 \qquad \theta_1 = \theta_2 = \theta$

$$n_o = n_x \qquad \frac{1}{n_e^2} = \frac{\cos^2\theta}{n_x^2} + \frac{\sin^2\theta}{n_z^2}$$

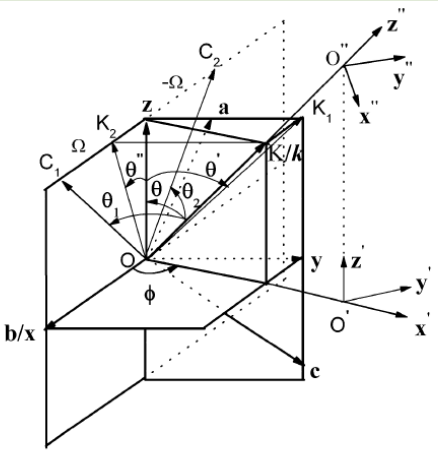


Fig. 1 Schematic showing the coordinate transformation of the dielectric axis (*xyz*), the crystal principal axis (*abc*) and the laboratory frame ($x''y''z''$)







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Highlighted by *Advances In Engineering (Canada)* in Nov. 2015（https://advanceseng.com/electrical-engineering/）

the longitudinal detuning

$$n_p(\omega_p) = \frac{n_i\left(\frac{\omega_p}{2}\right) + n_e\left(\frac{\omega_p}{2}\right)}{2}$$

$$\Delta_{z''} = \vec{k}_p(\theta,\phi_0) - \vec{k}_1(\theta,\phi_0) - \vec{k}_2(\theta,\phi_0) - \frac{\left(\vec{k}_{1\perp}(\theta,\phi_0) - \vec{k}_{2\perp}(\theta,\phi_0)\right)^2}{2k_p(\theta,\phi_0)}$$

the linear detuning

$$\begin{aligned}\delta &\equiv \vec{k}_p(\theta,\phi) - \vec{k}_1(\theta,\phi) - \vec{k}_2(\theta,\phi)\\ &= (\omega_p / c)\, n_p(\vec{k}_{p\perp}(\theta,\phi)) - (\omega_1 / c)\, n_1(\vec{k}_{1\perp}(\theta,\phi))\\ &\quad - (\omega_2 / c)\, n_2(\vec{k}_{2\perp}(\theta,\phi))\\ &= (\omega_p / c)\, n'_p(\theta_0,\phi_0) / n_p(\theta_0,\phi_0)\end{aligned}$$

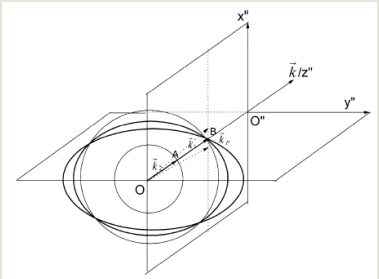


Fig. 7 Umbilicate point for maximal phase matching in biaxial crystal; pump and emission wave vector directions for collinear in the given detection

$$n'_p(\theta_0,\phi_0) = \{[dn_p(\theta,\phi)/d\theta + dn_p(\theta,\phi)/d\phi] - [dn_1(\theta,\phi)/d\theta + dn_1(\theta,\phi)/d\phi]\}\Big|_{\substack{\theta=\theta_0\\ \phi=\phi_0}}$$

$$n'_p(\theta_0,\phi_0) = \left(dn_p(\theta,\phi_0)/d\theta - dn_1(\theta,\phi_0)/d\theta\right)\Big|_{\theta=\theta_0}$$







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Outlook

We present a convenient and accurate theoretical quantification parameter to **estimate the beam width** in nonlinear monoclinic and triclinic crystals. This approach is based on **the gradient of angle-dependent refractive index** and the linear detuning.

$$n_p(\omega_p) = \frac{n_i\left(\frac{\omega_p}{2}\right) + n_e\left(\frac{\omega_p}{2}\right)}{2}$$

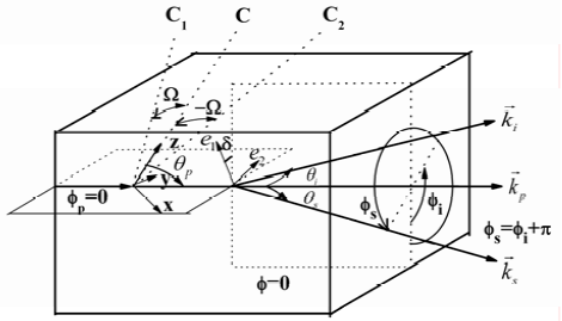


Fig. 2. The schematic diagram of SPDC process in biaxial crystal.

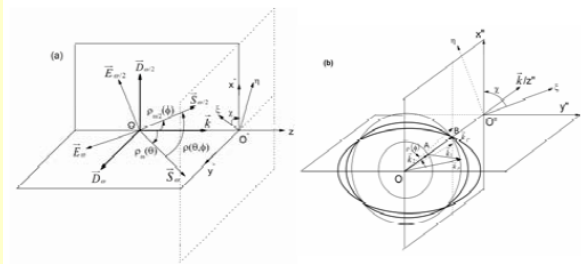


Fig. 2 (a) The spatial walk-off angle relations for pump and emission photons in the given detection coordinate; (b) The scheme of umbilicate point ***B*** for maximal collinear and non-collinear phase matching in biaxial crys-







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Outlook

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first-order Taylor expansion in angular picture of the refractive index

$$n_p(\omega_p)=\frac{n_i\left(\frac{\omega_p}{2}\right)+n_s\left(\frac{\omega_p}{2}\right)}{2}$$

$$\Delta k(\theta,\varphi)\equiv\frac{\omega_i}{c}\Delta n_i(\theta,\varphi)\gamma_i(\theta_0,\varphi_0)+\frac{\omega_s}{c}\Delta n_s(\theta,\varphi)\gamma_s(\theta_0,\varphi_0)$$

$$\Delta n_i(\theta,\varphi)=n_i(\theta,\varphi)-n_p(\theta_0,\varphi_0)\quad \Delta n_s(\theta,\varphi)=n_s(\theta,\varphi)-n_p(\theta_0,\varphi_0)$$

$$\gamma_i(\theta_0,\varphi_0)=\mathrm{n}'_p(\theta_0,\varphi_0)\text{-}\mathrm{n}'_i(\theta_0,\varphi_0)\quad \gamma_s(\theta_0,\varphi_0)=\mathrm{n}'_p(\theta_0,\varphi_0)\text{-}\mathrm{n}'_s(\theta_0,\varphi_0)$$

Pumped with monochromatic light, the fluorescence spectra of the two photons are identical and have a bandwidth

$$\Omega_f(\theta,\varphi)=\frac{4\pi c}{L\left|\Delta n_i(\theta,\varphi)\gamma_i(\theta_0,\varphi_0)+\Delta n_s(\theta,\varphi)\gamma_s(\theta_0,\varphi_0)\right|}$$

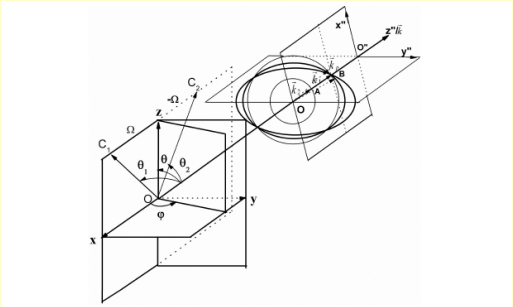


Fig. 1. The schematic diagram showing coordinate transformation of the dielectric axis (xyz), and the lab frame ($x''y''z''$) in biaxial crystals: C_1, C_2, optical axis, Ω, optical axis angle, $\vec{k}_1, \vec{k}_2, \vec{k}_p$, the wave vector of two down-converted photons and pump photon.







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Outlook

Compensation of anisotropy effects in the process of nonlinear three wave mixing.

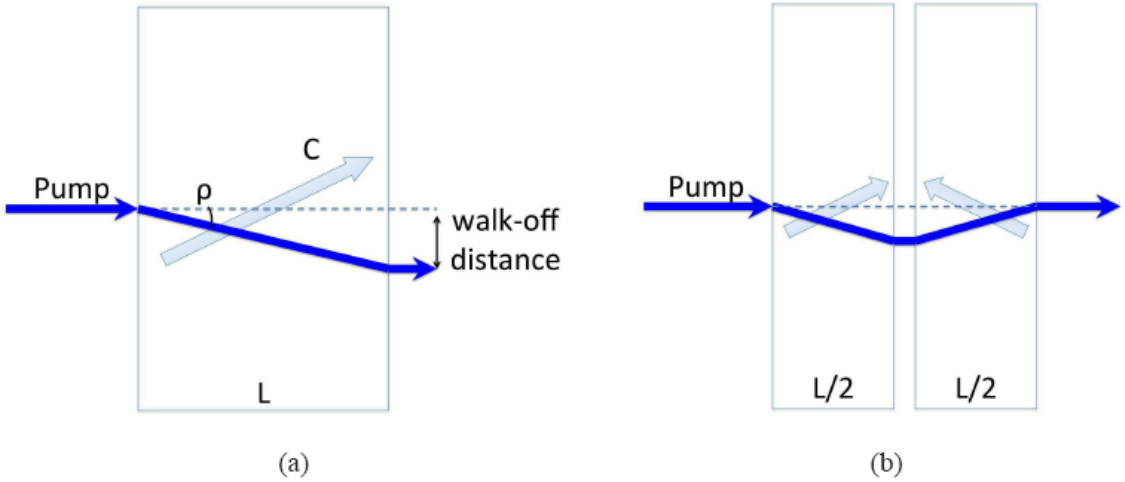


Fig. 1. (a) Transverse walk-off effect: an extraordinarily polarised beam impinging on a crystal normally is tilted while inside. The walk-off angle ρ depends on the angle of the pump wave vector with the optic axis C and the crystal birefringence. (b) Compensation of the anisotropy effect is obtained by using two crystals with the optic axes tilted in opposite directions. In the two crystals, the walk-off, as well as the anisotropy, manifests itself in the opposite directions, so the effects cancel each other.







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Outlook

This method can be extended to triclinic crystal through a more angular projecting process decomposed it into two segments in a plane which are vertical and parallel to each other.

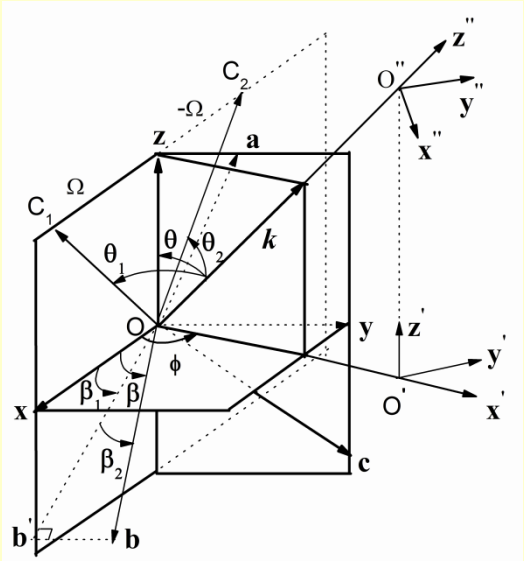


Fig.2. The angle relations and axial projection in triclinic crystal.









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