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Improvements for the numerical instability of lattice Boltzmann method

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Heathrow, London

Lattice Boltzmann Method

$$\frac{\partial f(\vec{r},\vec{\xi},t)}{\partial t}+\vec{\xi}\cdot\frac{\partial f(\vec{r},\vec{\xi},t)}{\partial \vec{r}}+\vec{a}\cdot\frac{\partial f(\vec{r},\vec{\xi},t)}{\partial \vec{\xi}}=\iint(\mathrm{F}_1\mathrm{F}_2-f_1 f_2)d_D^2\left|\vec{g}\right|\cos\theta d\Omega d\vec{\xi}_1 \qquad (1)$$

Continuous Boltzmann equation

$$\frac{\partial f(\vec{r},\vec{\xi},t)}{\partial t}+\vec{\xi}\cdot\frac{\partial f(\vec{r},\vec{\xi},t)}{\partial \vec{r}}+\vec{a}\cdot\frac{\partial f(\vec{r},\vec{\xi},t)}{\partial \vec{\xi}}=\Omega_f=\frac{1}{\tau}\left[f^{eq}(\vec{r},\vec{\xi})-f(\vec{r},\vec{\xi},t)\right] \qquad (2)$$

Continuous BGK Boltzmann equation

Bhatnagar, Gross and Krook approximation

Lattice Boltzmann Method

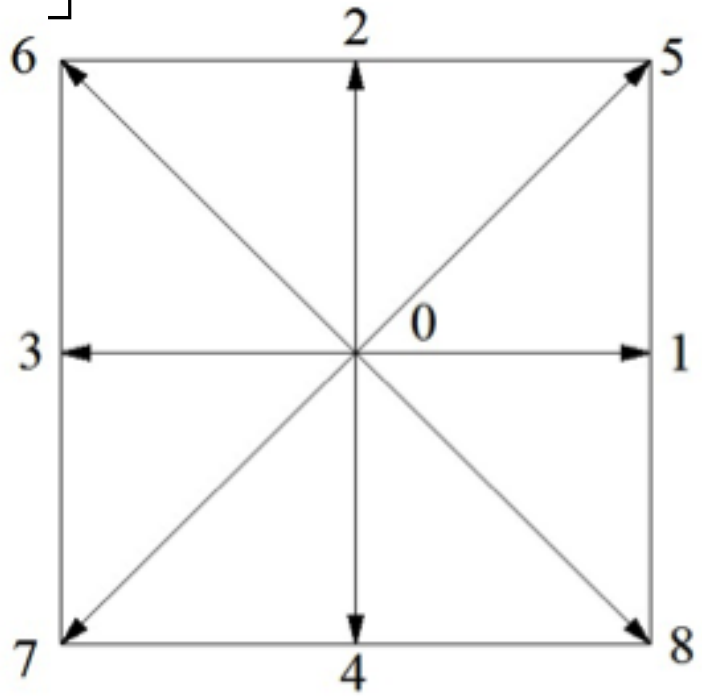
$$f_\alpha(\vec{r}+\vec{e}_\alpha\Delta t, t+\Delta t) - f_\alpha(\vec{r},t) = \Omega_f^\alpha = \frac{1}{\tau}\left[f_\alpha^{eq}(\vec{r},\vec{\xi}) - f_\alpha(\vec{r},\vec{\xi},t)\right] \qquad (3)$$

Lattice Boltzmann equation

$$f_\alpha^{eq} = \omega_\alpha \rho\left[1+\frac{\vec{e}_\alpha\cdot\vec{u}}{c_s^{\,2}}+\frac{(\vec{e}_\alpha\cdot\vec{u})^2}{2c_s^{\,4}}-\frac{u^2}{2c_s^{\,2}}\right] \qquad \alpha = 0,1,\ldots,8$$

$$\vec{e} = c\begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

$$c_s = \frac{c}{\sqrt{3}} \qquad \omega_\alpha = \begin{cases} 4/9 & \vec{e}_\alpha^{\,2} = 0 \\ 1/9 & \vec{e}_\alpha^{\,2} = c^2 \\ 1/36 & \vec{e}_\alpha^{\,2} = 2c^2 \end{cases}$$



LBGK D2Q9 model

Numerical instability

$$\mathrm{Re} = UL/\nu \qquad (4)$$

$$\nu = (2\tau - 1)\Delta x^2 / (6\Delta t) \qquad (5)$$

$$\tau = 3UL/(\mathrm{Re}\Delta x) + 0.5 \qquad (6)$$

[1]S. Succi, “Lattice Boltzmann Equation for Fluid Dynamics and Beyond,” Oxford: Clarendon Press. (2001)

[2]Y.L. He, Y. Wang, and Q. Li, “Lattice Boltzmann Method: Theory and Applications,” Science Press, Beijing. (2008)

[3]R. Du, B.C. Shi, and X.W. Chen, “Multi- relaxation-time lattice Boltzmann model for incompressible flow,” Physics Letters A. 359, 564-572 (2006).

[4]Z.H. Chai, B.C. Shi, and L. Zheng, “Simulating high Reynolds number flow in two-dimensional lid-driven cavity by multi-relaxation-time lattice Boltzmann method,” Chinese Physics. 15, 8. (2006)

[5]J.H. Lu, H.F. Han, B.C. Shi, and Z.L. Guo, “Immersed boundary lattice Boltzmann model based on multiple relaxation times,” Physical Review E. 85, 016711 (2012).

[6]A. Fakhari, and T. Lee, “Multiple-relaxation-time lattice Boltzmann method for immiscible fluids at high Reynolds numbers,” Physical Review E. 87, 023304. (2013)

[25]L.S. Lin, H.W. Chang, and C.A. Lin, “Multi relaxation time lattice Boltzmann simulations of transition in deep 2D lid driven cavity using GPU,” Computers & Fluids. 80, 381-387 (2013).

[26]S. Hou, J. Sterling, S. Chen, and G. D. Doolen, “A Lattice Boltzmann Subgrid Model for High Reynolds Number Flows, Pattern Formation and Lattice Gas Automata,” A. T. Lawniczak, and R. Kapral, eds., American Mathematical Society, Providence, RI, pp. 151-166. 6: 149. (1996)

[27]C.M. Teixeira, “Incorporating turbulence models into the lattice Boltzmann method,” International Journal of Modern Physics C. 9, 8, 1159-1175 (1998).

[28]H.D. Yu, S.S. Girimaji, and L.S. Luo, “DNS and LES of decaying isotropic turbulence with and without frame rotation using lattice Boltzmann method,” Journal of Computational Physics. 209, 2, 599-616 (2005).

Modified LBM

1) The entropic lattice Boltzmann Method
2) The fractional step lattice Boltzmann method
3) The turbulence models lattice Boltzmann method
4) The multiple-relaxation-time LBM (MRT-LBM)
5) The large-eddy-simulation LBM (LES-LBM)

Multiple relaxation time-LBM

$$\begin{cases} f_\alpha(\vec{r}+e_\alpha\Delta t, t+\Delta t) - f_\alpha(\vec{r},t) = 1/\tau\,(f_\alpha^{eq}(\vec{r},\vec{\xi}) - f_\alpha(\vec{r},\vec{\xi},t)) & LBGK-Single \\ \vec{f}_\alpha(\vec{r}+e_\alpha\Delta t, t+\Delta t) - \vec{f}_\alpha(\vec{r},t) = \vec{\Lambda}_{\alpha\beta}[\vec{f}_\beta^{eq}(\vec{r},\vec{\xi}) - \vec{f}_\beta(\vec{r},\vec{\xi},t)] & LBM-MRT \end{cases} \quad (7)$$

Singular Relaxation time term

$$\tau = 3UL/(\mathrm{Re}\Delta x) + 0.5 \quad (6)$$

Multiple-relaxation-time term

$$\vec{\Lambda}_{\alpha\beta} = \begin{pmatrix} 1/\tau_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/\tau_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/\tau_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\tau_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\tau_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/\tau_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/\tau_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/\tau_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/\tau_9 \end{pmatrix}$$

Large-eddy-simulation LBM

Original LBM

$$\mathrm{Re} = UL/\nu \qquad (4)$$

$$\nu = (2\tau - 1)\Delta x^2 / (6\Delta t) \qquad (5)$$

$$\tau = 3UL/(\mathrm{Re}\Delta x) + 0.5 \qquad (6)$$

LES LBM

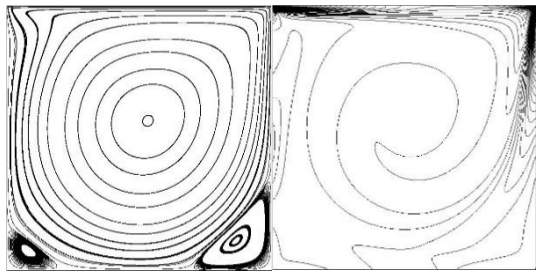
$$\nu_{total} = \nu_0 + \nu_t \qquad (8)$$

$$\nu_0 = UL/\mathrm{Re} \qquad (9)$$

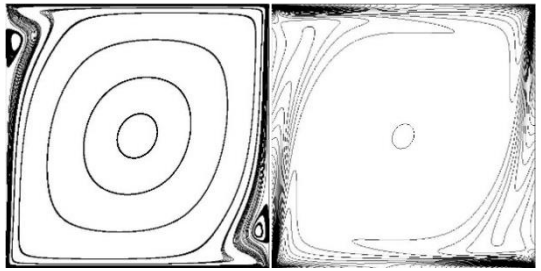
$$\nu_t = C\Delta_f^2 \left|\bar{S}_{ij}\right| \qquad (10)$$

$$\tau_{total} = 3(\nu_0 + \nu_t) + 0.5 \qquad (11)$$

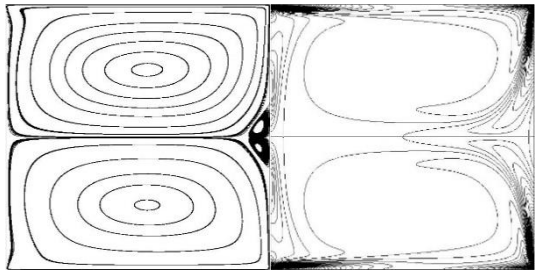
Numerical results (low *Re*)



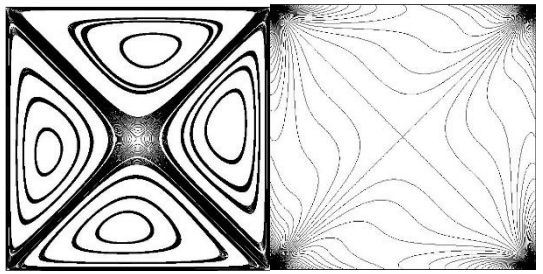
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Re=1000



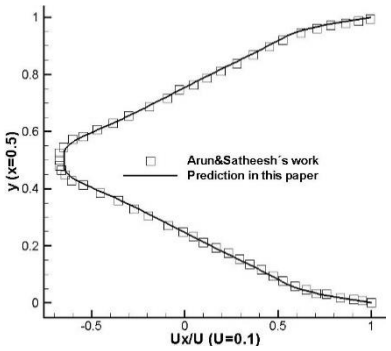
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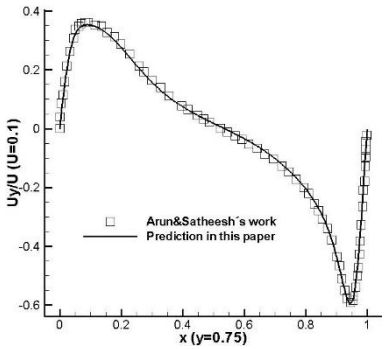


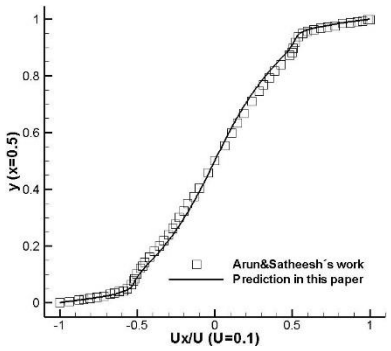
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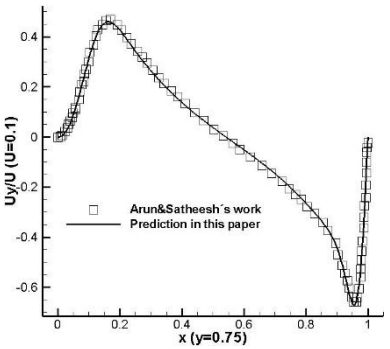
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Left secondary Vortex	X=0.081148	X=0.0815	X=0.0859	X=0.0857
	Y=0.075355	Y=0.076	Y=0.0781	Y=0.0714
Right secondary Vortex	X=0.864711	X=0.865	X=0.8594	X=0.8643
	Y=0.113206	Y=0.1125	Y=0.1094	Y=0.1071
Primary Vortex	X=0.532912	X=0.5325	X=0.5313	X=0.5286
	Y=0.566457	Y=0.566	Y=0.5625	Y=0.5643

Position	This paper	Ref.[]	Ref.[]
Left Primary Vortex	X=0.160056	X=0.161	X=0.160
	Y=0.449111	Y=0.442	Y=0.450
Right Primary Vortex	X=0.839944	X=0.845	X=0.840
	Y=0.550889	Y=0.559	Y=0.550
Top Primary Vortex	X=0.550889	X=0.559	X=0.550
	Y=0.839944	Y=0.845	Y=0.840
Down Primary Vortex	X=0.449111	X=0.442	X=0.450
	Y=0.160056	Y=0.161	Y=0.160

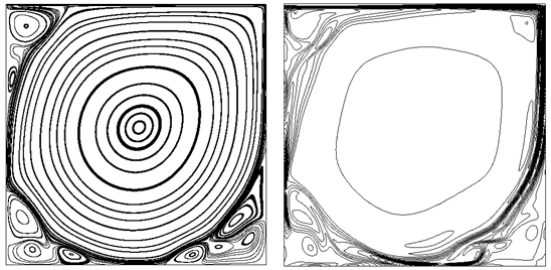




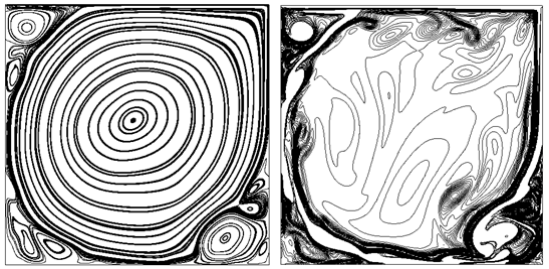




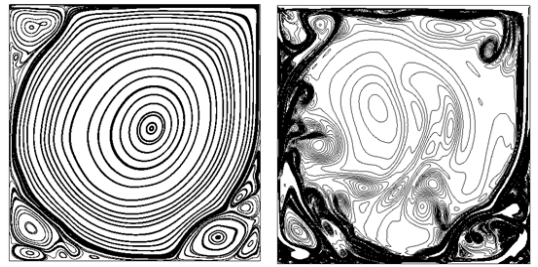
Numerical results (high *Re*)



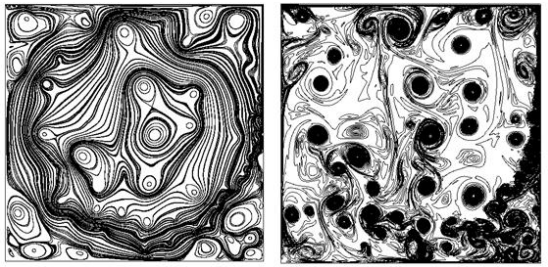
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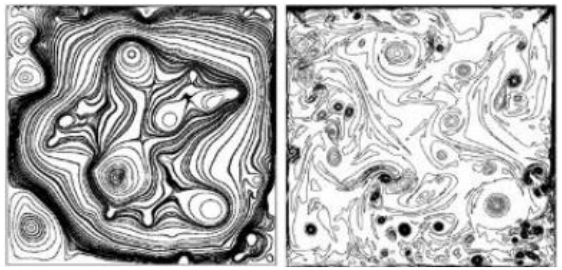
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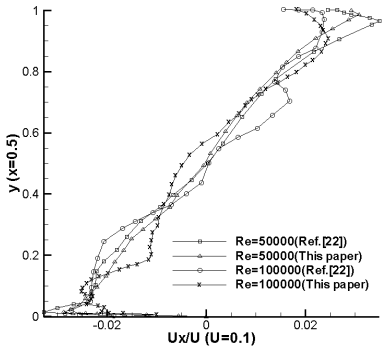
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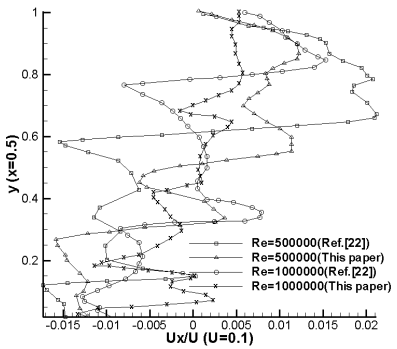


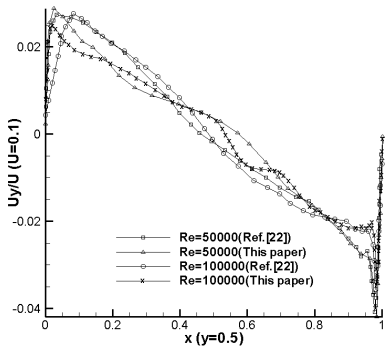
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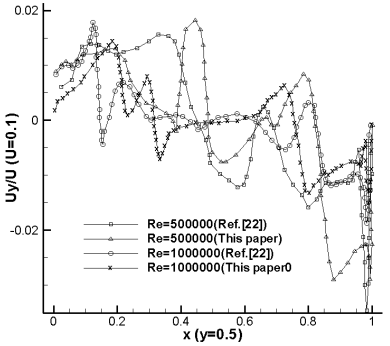


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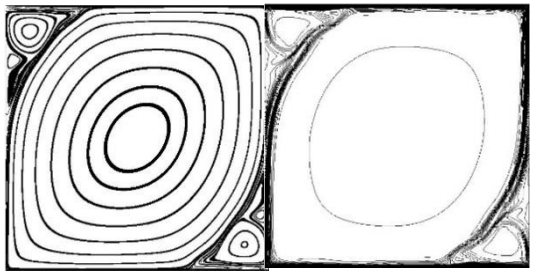




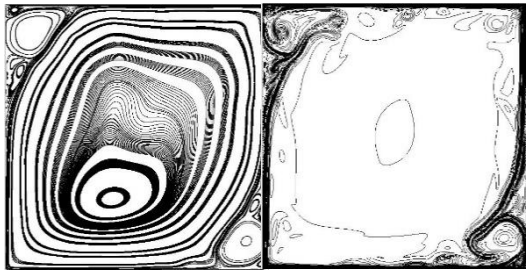




Numerical results (high *Re*)



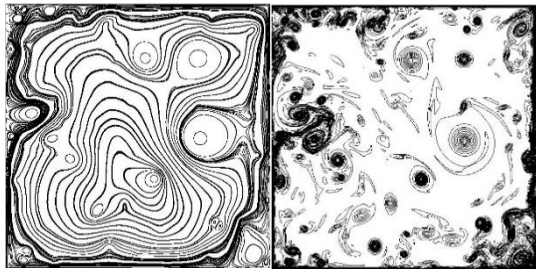
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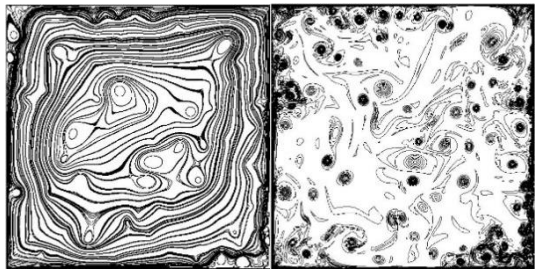
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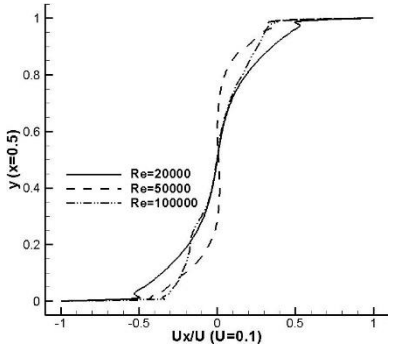
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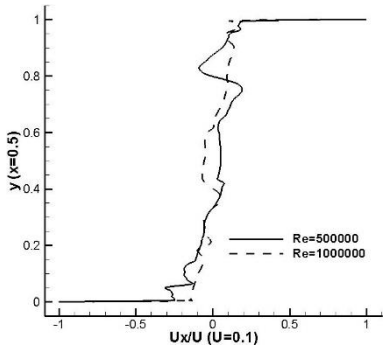


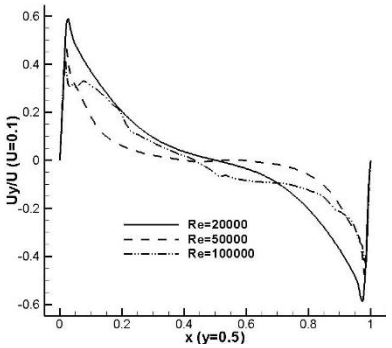
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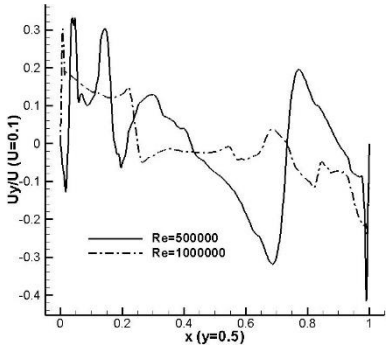


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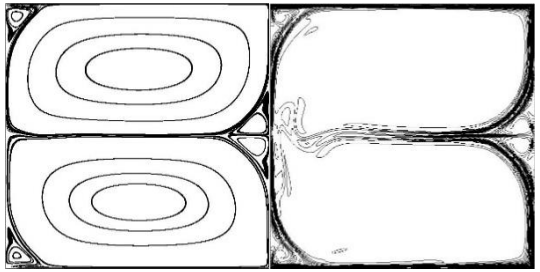




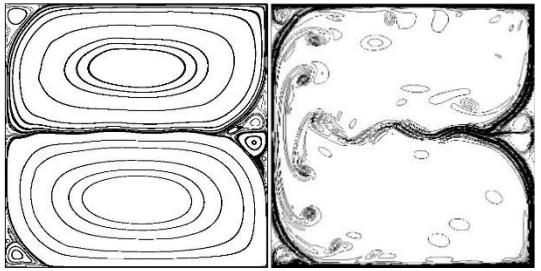




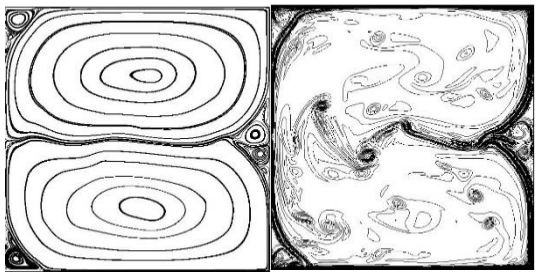
Numerical results (high *Re*)



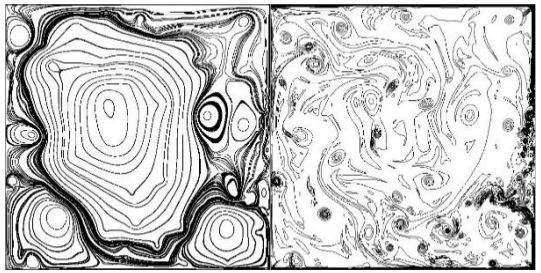
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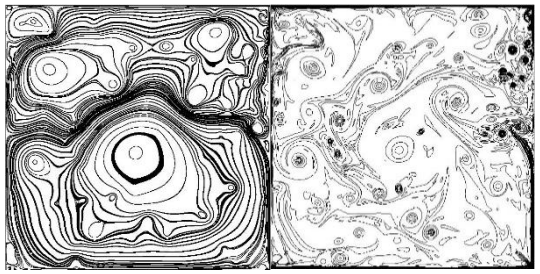
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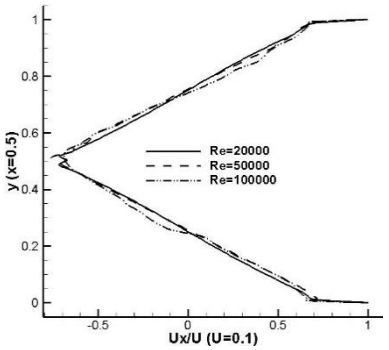
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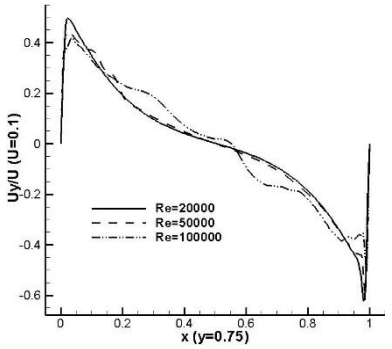


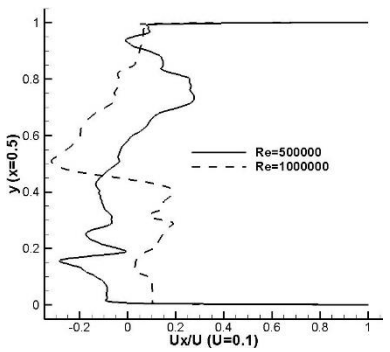
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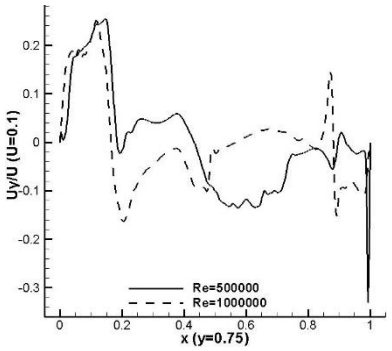


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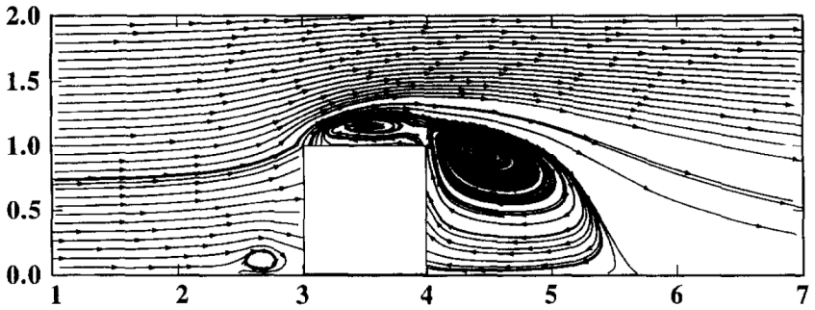




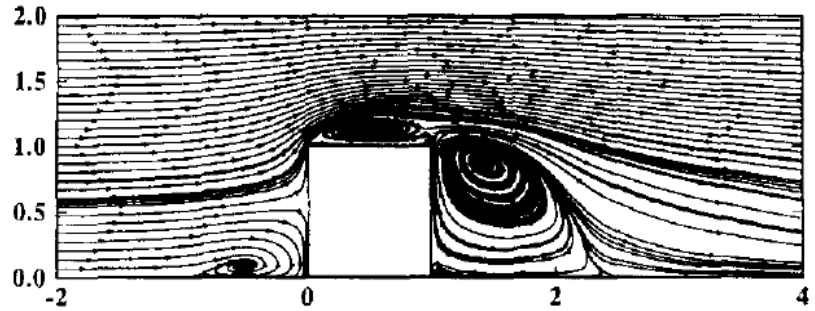




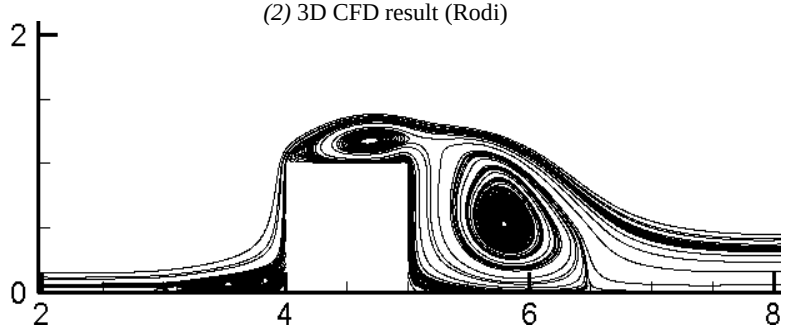
Numerical results (high *Re*)



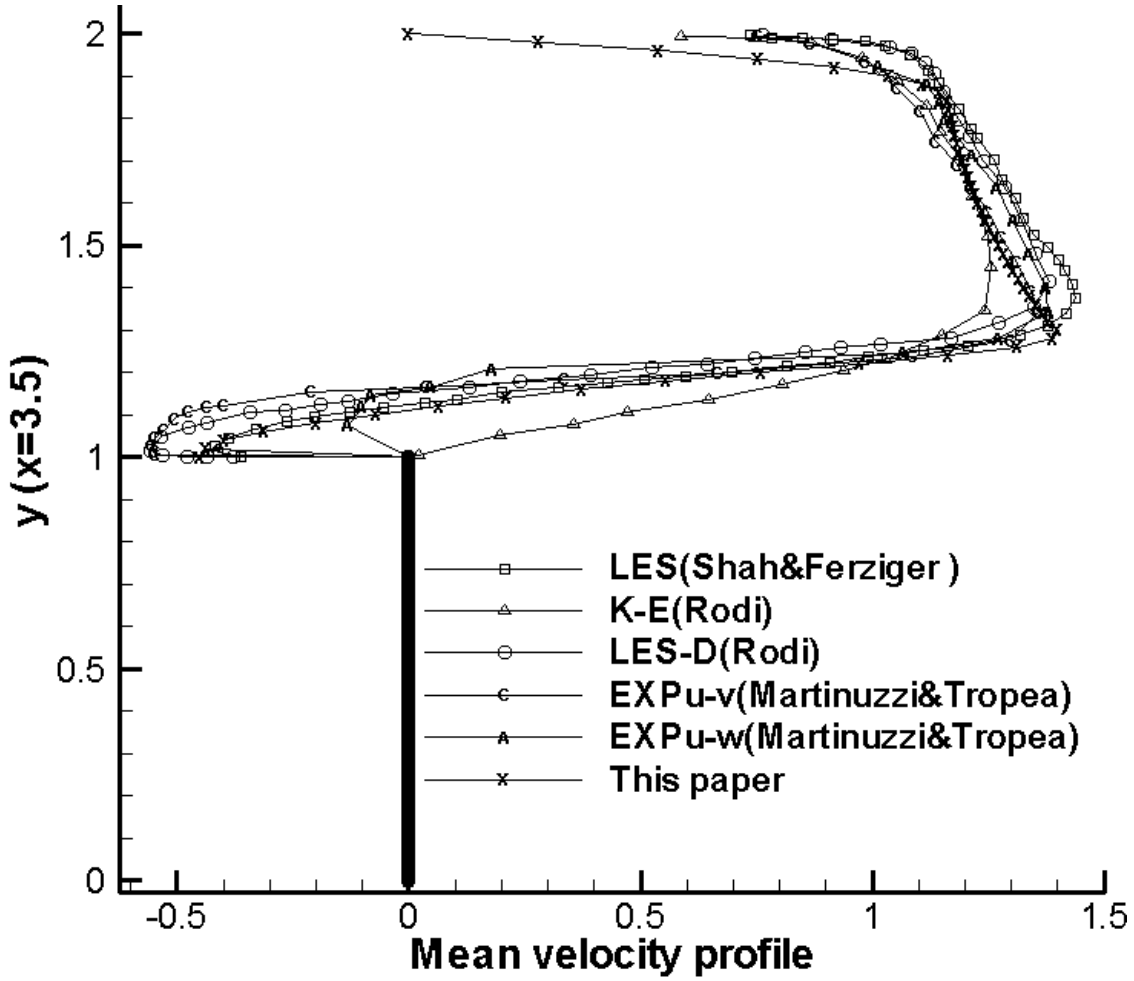
(1) 3Dexperiment (Martinuzzi and Tropea)



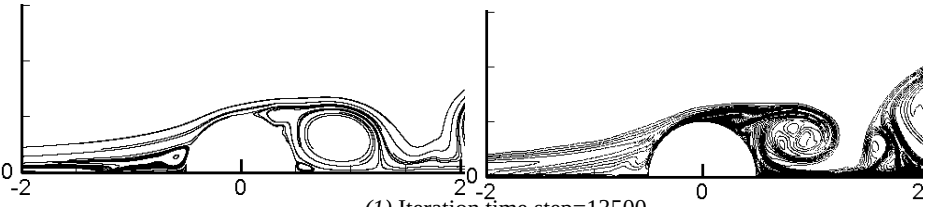
(2) 3D CFD result (Rodi)



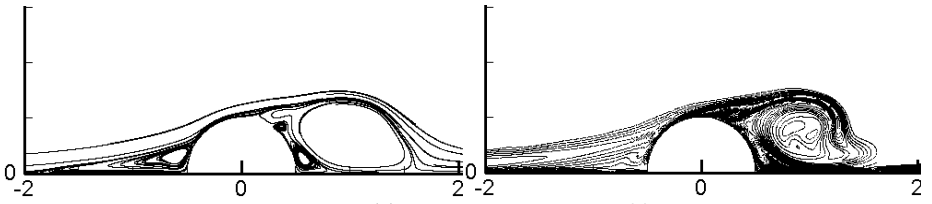
(3) 2D prediction in my application



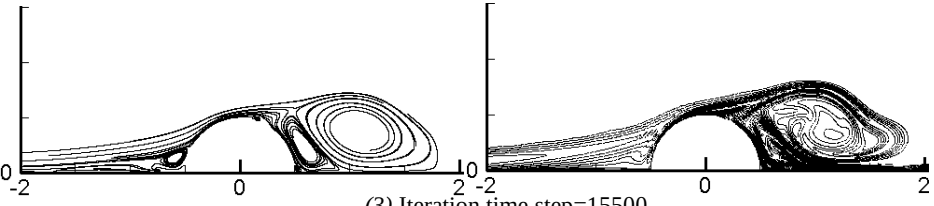
Numerical results (high *Re*)



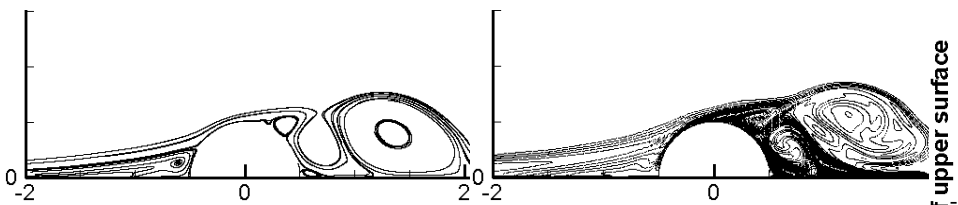
(1) Iteration time step=13500



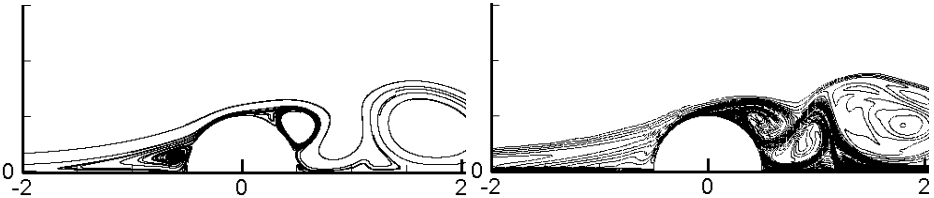
(2) Iteration time step=14500



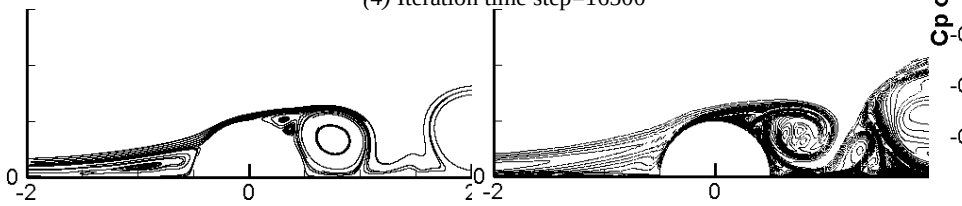
(3) Iteration time step=15500



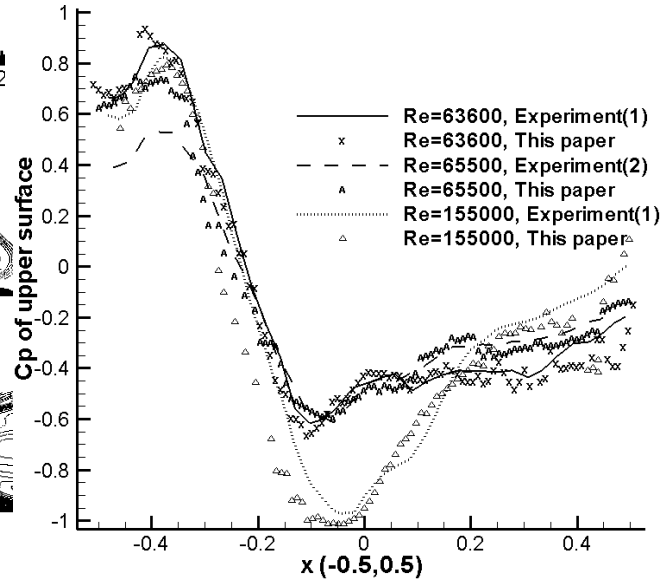
(4) Iteration time step=16500



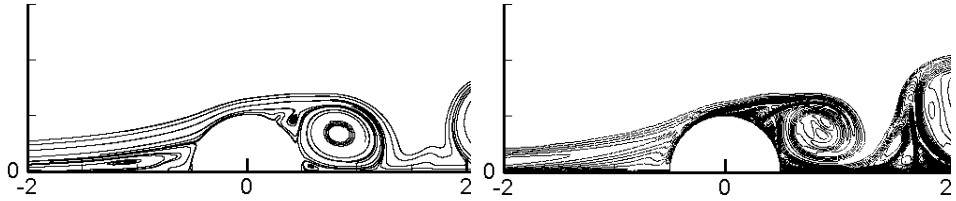
(5) Iteration time step=17500



(7) Iteration time step=18500



$$St = \frac{fL}{U} = 0.178$$



(8) Iteration time step=19000

$$Re = 4000$$

Conclusions and Further Plan

Conclusions

1. The Multiple-relaxation-time LBM model is capable of improving the numerical stability of the flow inside a cavity

2. The Large-eddy-time LBM model works well when dealing with the flow over a hump

Further Plan

1. 3D applications

2. Passive and active flow control


The End