UNITE DE CATALYSE ET DE CHIMIE DU SOLIDE *- UMR CNRS 8181

Predicting the unpredictable with an isomorphic single-control parameter structure of the Laser-Lorenz equations

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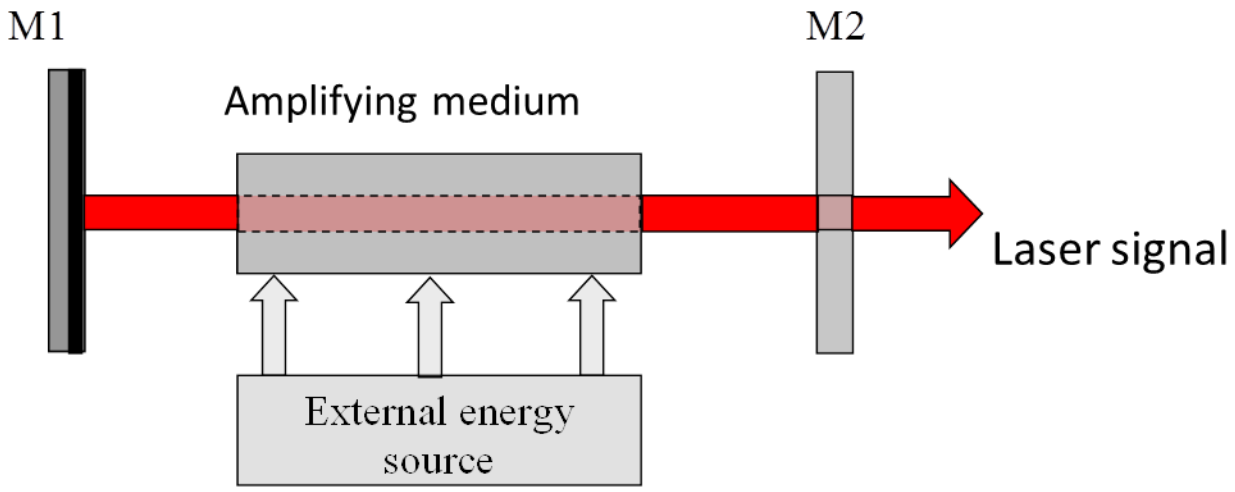
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**Équipe Couches Minces & Nanomatériaux*

UNIVERSITÉ D'ARTOIS

Basic Laser Scheme



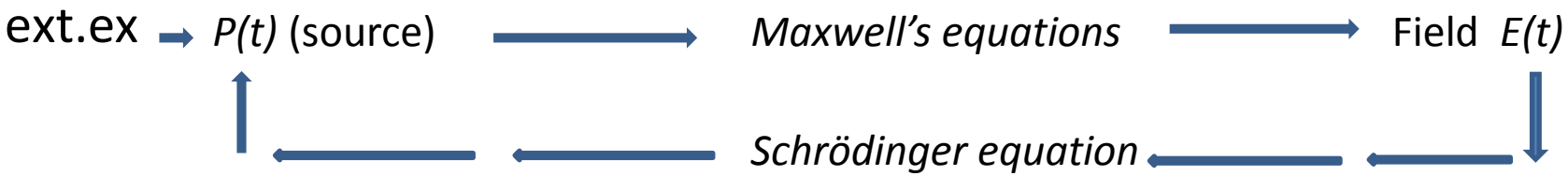
Light-matter interactions

Electric field-amplitude $E(t)$ ⟶ *Cavity decay-rate* k

Amplifying medium $\begin{cases} Population\ inversion\ D(t) \\ Medium\ Polarisation\ P(t) \end{cases}$ Relax. rates $\begin{cases} \gamma_{\parallel} \\ \gamma_{\perp} \end{cases}$

The Lorenz-Haken equations

Lamb self-consistent analysis



$$\frac{d}{dt}E(t) = -\kappa\left\{E(t) + 2CP(t)\right\} \qquad (1a)$$

$$\frac{d}{dt}P(t) = -P(t) + E(t)D(t) \qquad (1b)$$

$$\frac{d}{dt}D(t) = -\wp\left\{D(t) + 1 + E(t)P(t)\right\} \qquad (1c)$$

Excitation parameter *2C* quantifies the pumping mechanism with respect to its level at lasing threshold, and

$$\wp = \frac{\gamma_{\parallel}}{\gamma_{\perp}}, \qquad \kappa \rightarrow \frac{\kappa}{\gamma_{\perp}}, \qquad t \rightarrow \gamma_{\perp} t \Rightarrow$$ Normalized quantities

Back to Lorenz notations and main characteristics

$$\partial_t X = -\kappa\{X + \rho Y\} \quad \text{(2a)}$$

$$\partial_t Y = -Y + XZ \quad \text{(2b)}$$

$$\partial_t Z = -\wp\{Z + 1 + XY\} \quad \text{(2c)}$$

Where ρ = 2C

Instability threshold

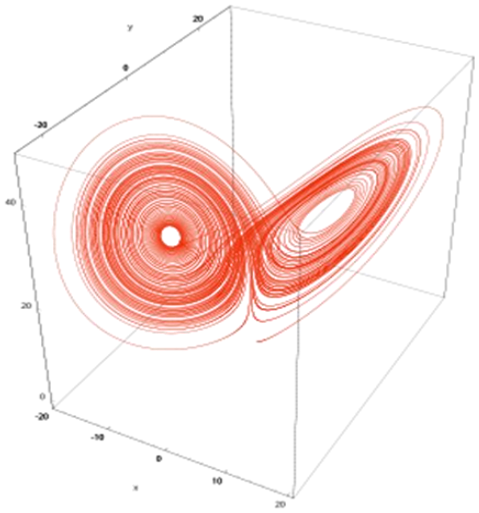
$$\rho_{2th} = \frac{\kappa(\kappa + 3 + \wp)}{\kappa - 1 - \wp}$$

Bad-cavity condition

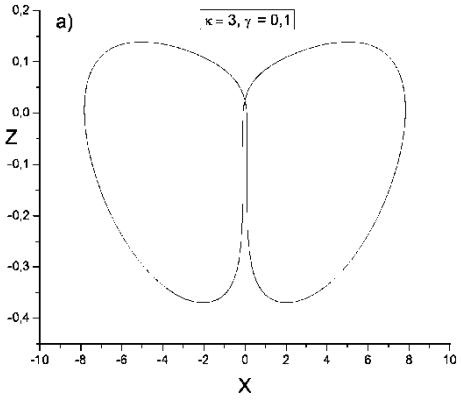
$$\kappa > 1 + \wp$$

$\rho > \rho_{2th}$ → Unstable solutions

Three control parameters (ρ, κ, $\wp$) govern the dynamics of Eqs (2)



Ex. $\kappa = 10$, $\wp = 8/3$, $\rho = 28$ Lorenz attractor

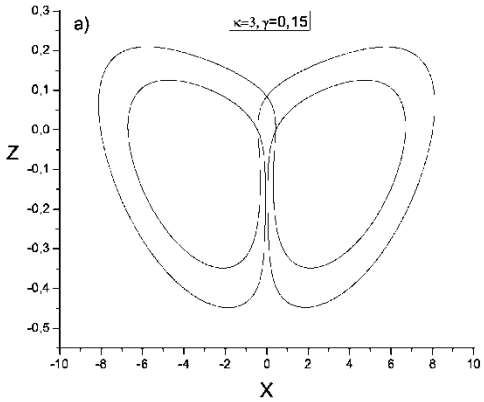


$\kappa = 3$, $\wp = 0{,}1$ $\rho = 10$

Period 1 limit cycle

$\kappa = 3$, $\wp = 0{,}15$, $\rho = 11$

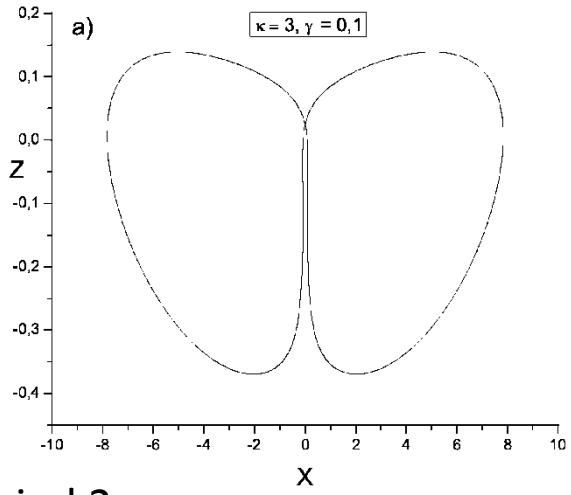
Period-doubling

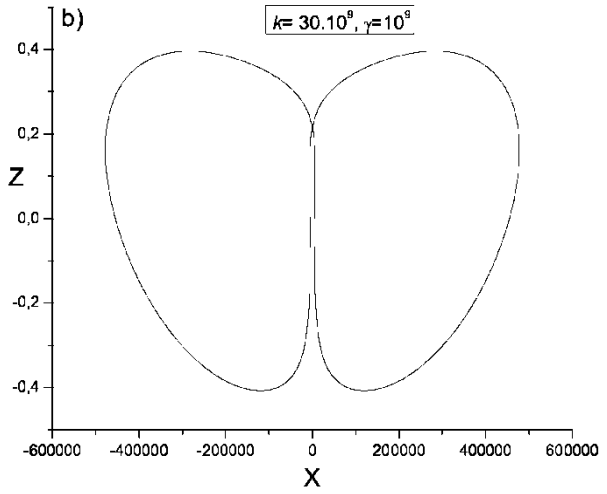


Self-organized solution-recurrence

Period 1

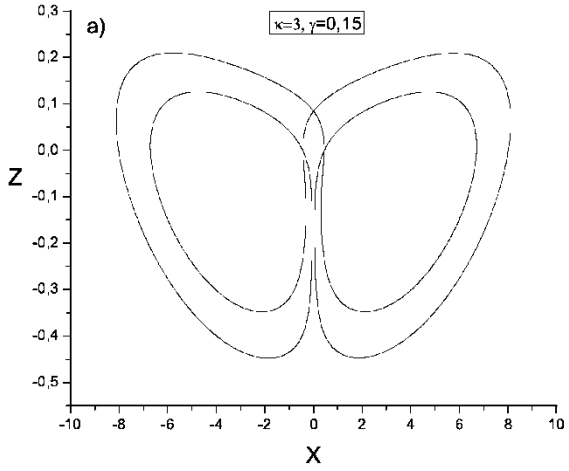
$$\frac{\kappa}{\wp} = 30$$

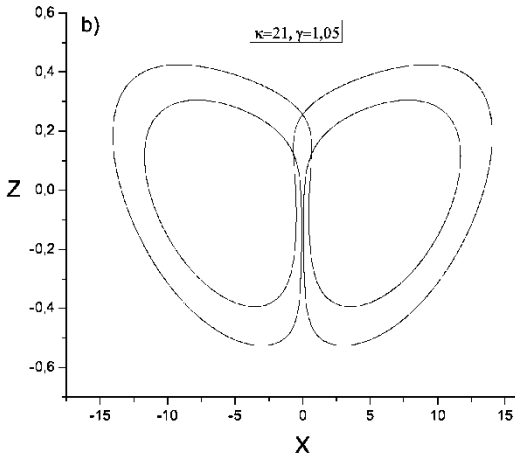


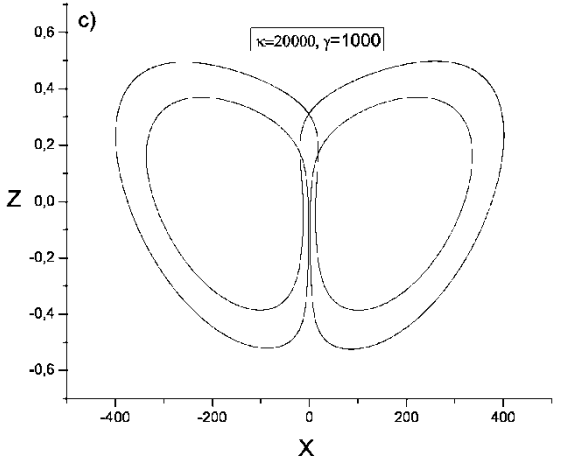


Period 2

$$\frac{\kappa}{\wp} = 20$$

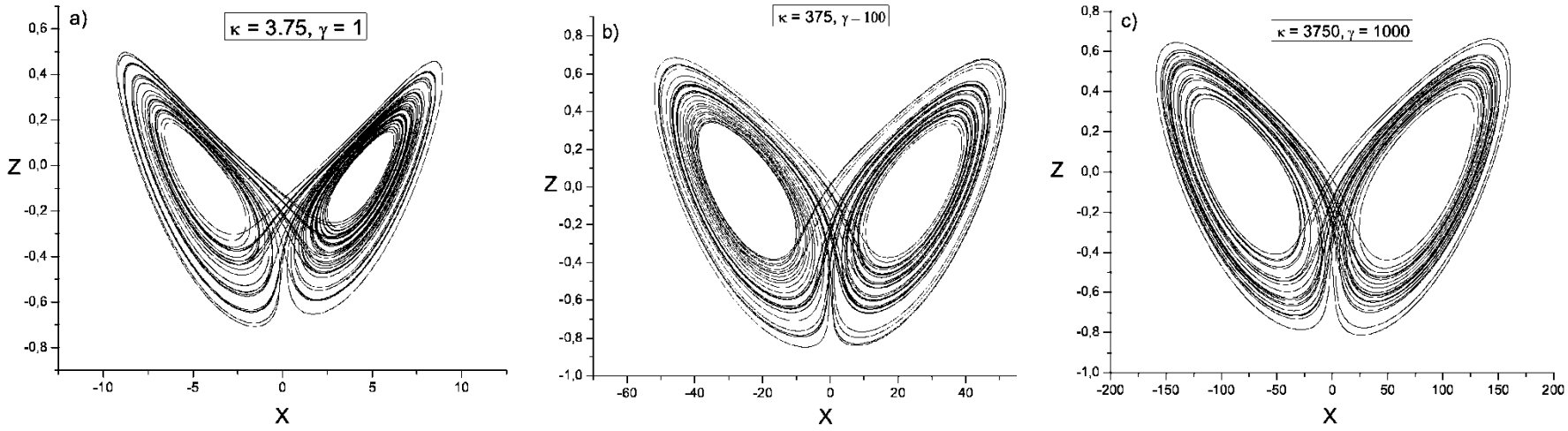






Chaotic attractor

$$\frac{\kappa}{\wp} = 3.75$$



$$\frac{\kappa}{\wp} = \eta$$ defines an isomorphic class of solutions !

Including the particular set $\kappa = \eta$, $\wp = 1$

$(\kappa, \wp, \rho) \longrightarrow (\eta, \rho)$

$$\partial_t X = -\eta\{X + \rho Y\}$$

$$\partial_t Y = -Y + XZ$$

$$\partial_t Z = -\{Z + 1 + XY\}$$

Furthermore, <u>simulations reveal :</u>

$(\eta, \rho) \Longleftrightarrow (\eta', \rho_{2th}(\eta'))$

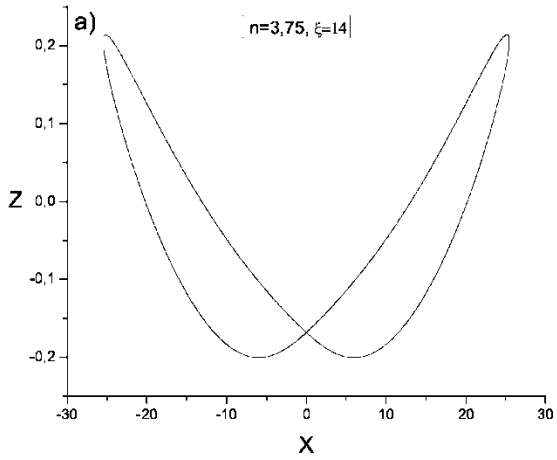
$$\partial_t X = -\eta'\{X + \rho_{2th}(\eta')Y\}$$

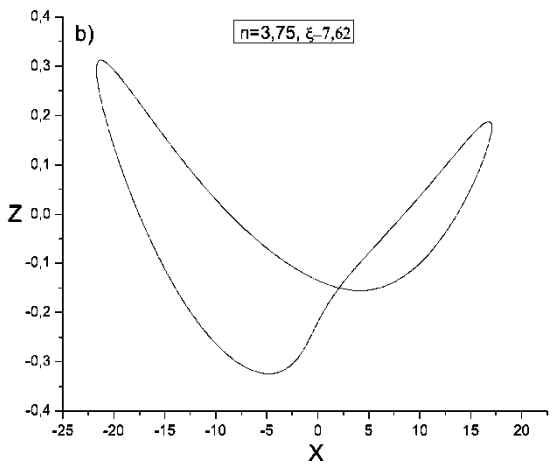
$$\partial_t Y = -Y + XZ$$

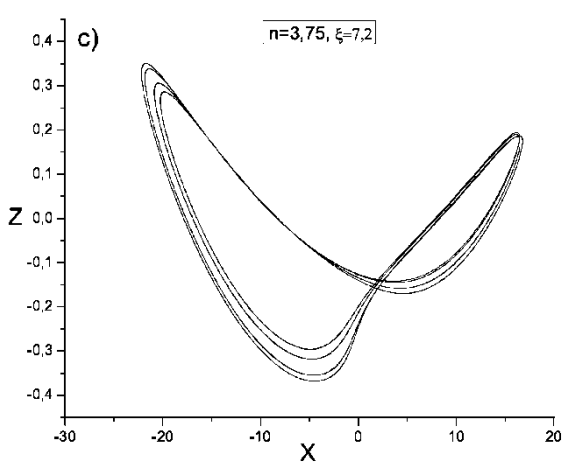
$$\partial_t Z = -\{Z + 1 + XY\}$$

$$\rho = \frac{\eta'(\eta'+4)}{\eta'-2} = \rho_{2th}(\eta')$$

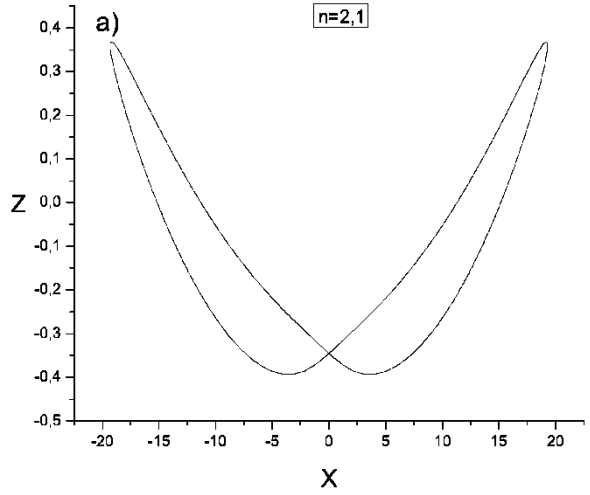
Simulations far above the instability threshold

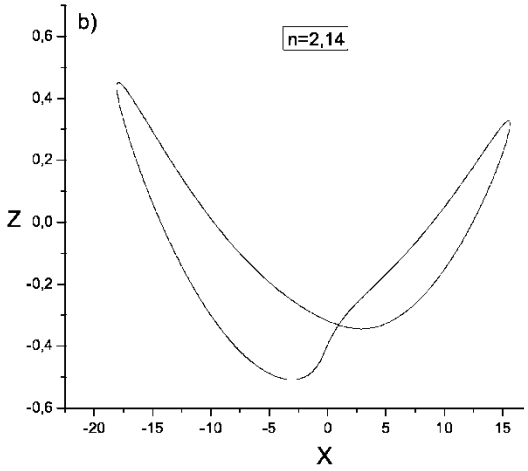


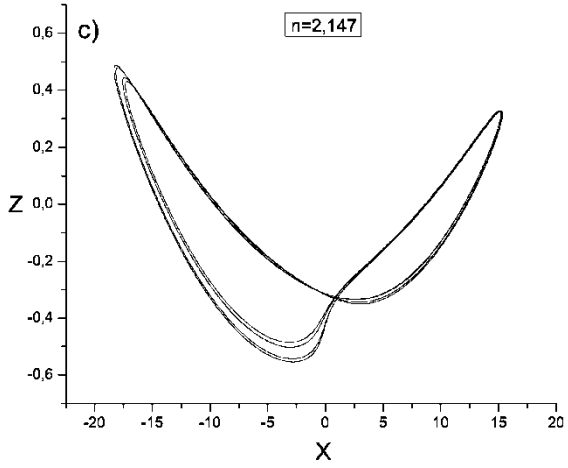




Simulations at the instability threshold, obtained with other η values

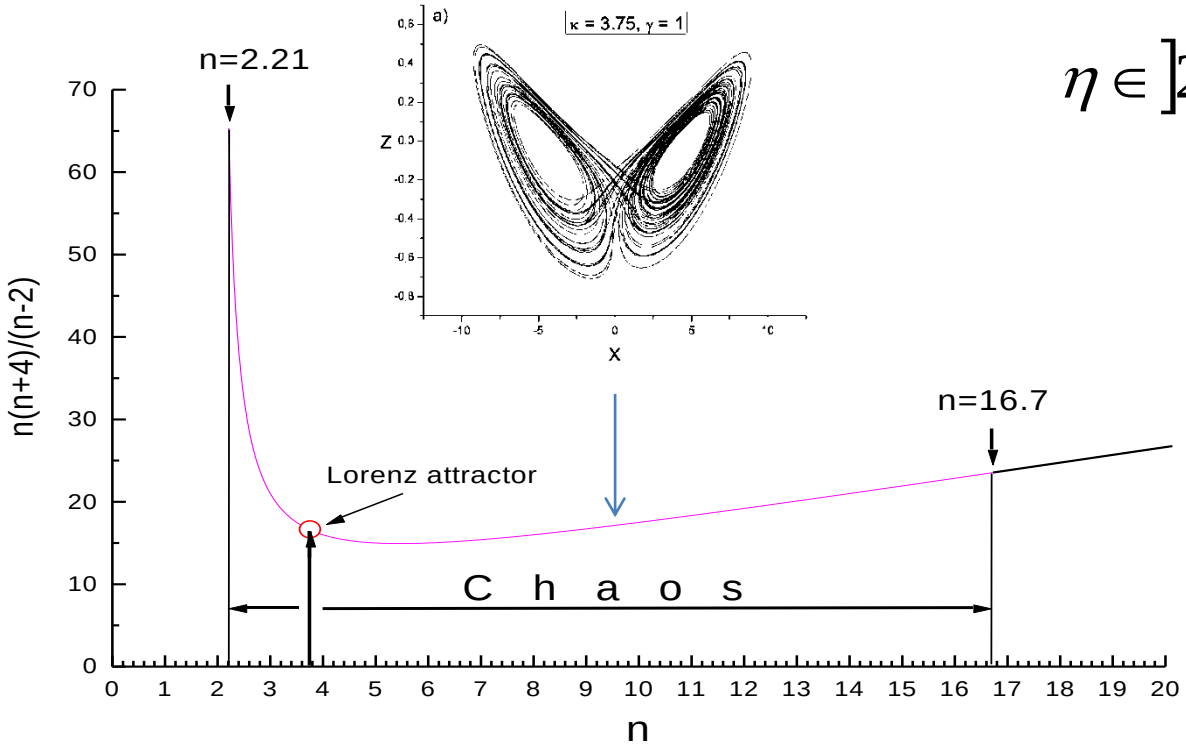






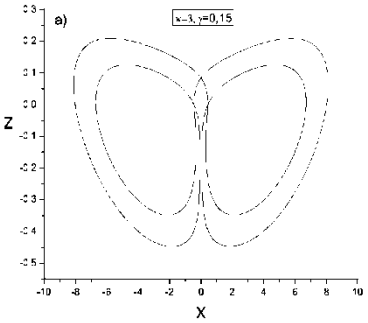
Predicting the unpredictible

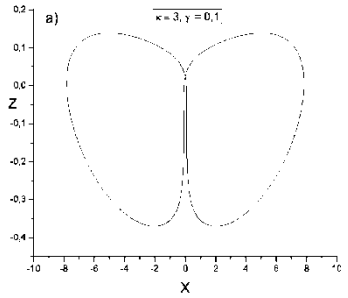
Dynamic chart 1

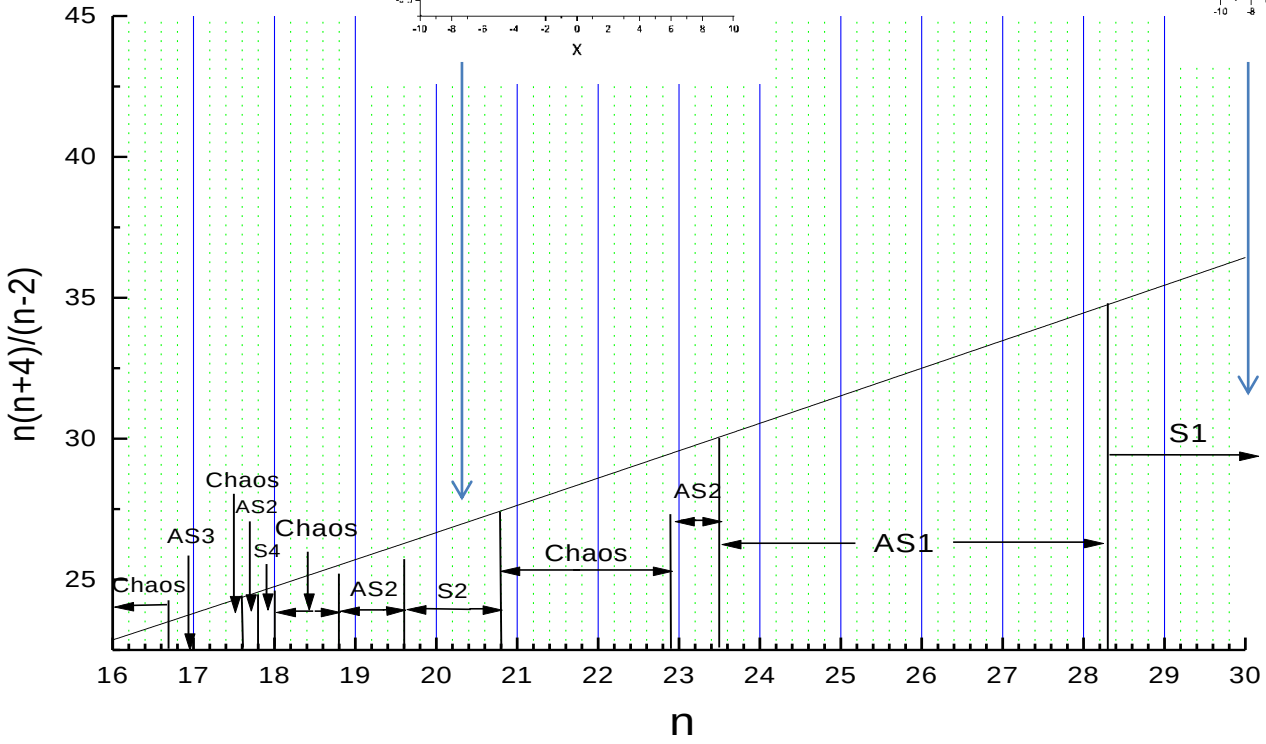


$$\eta \in]2, \infty[$$

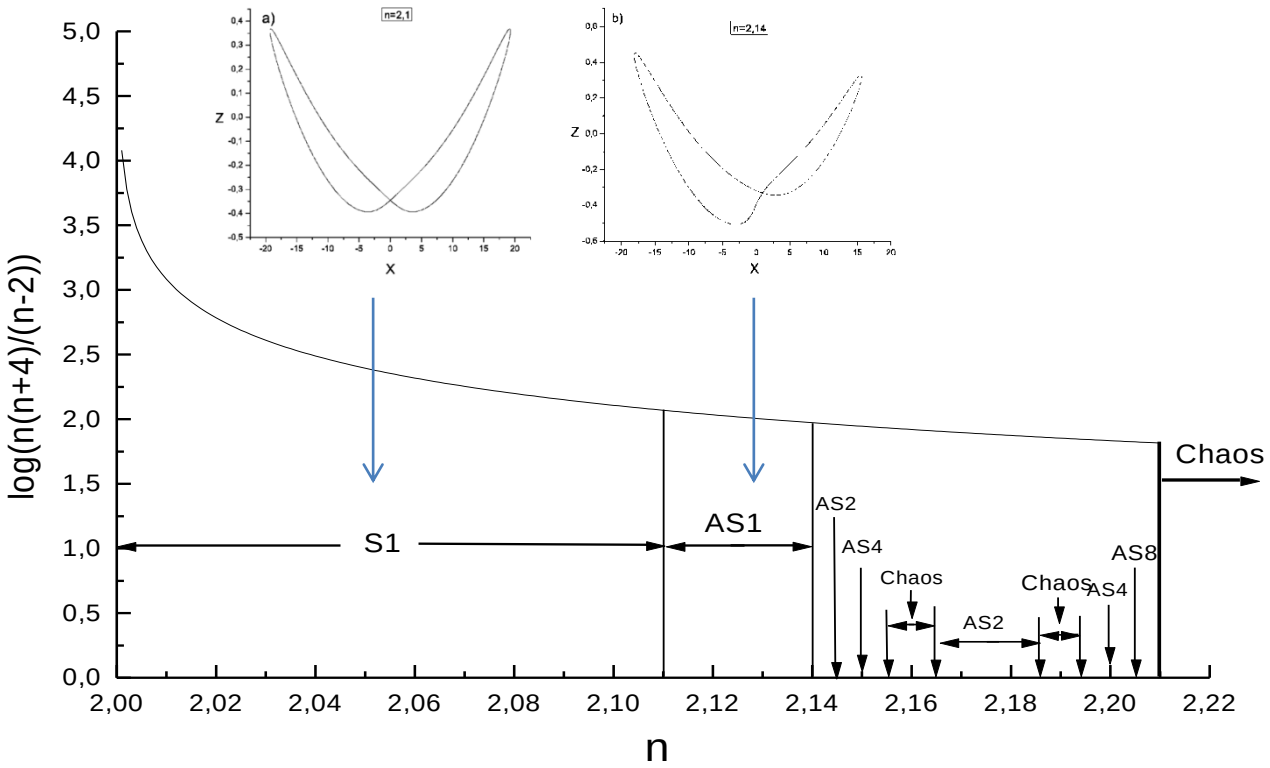
Dynamic chart 2







Dynamic chart 3



Conclusion

Lorenz Equations

$$\partial_t X = -\kappa\{X + \rho Y\}$$

$$\partial_t Y = -Y + XZ$$

$$\partial_t Z = -\wp\{Z + 1 + XY\}$$

3 control parameters

ρ, κ, $\wp$

ISOMORPHISM

$$\partial_t X = -\eta\left\{X + \frac{\eta(\eta+4)}{\eta-2} Y\right\}$$

$$\partial_t Y = -Y + XZ$$

$$\partial_t Z = -\{Z + 1 + XY\}$$

1 single control parameter

η

Predictability in non-linear dynamical systems

Problems *and* *Deterministic solutions*

