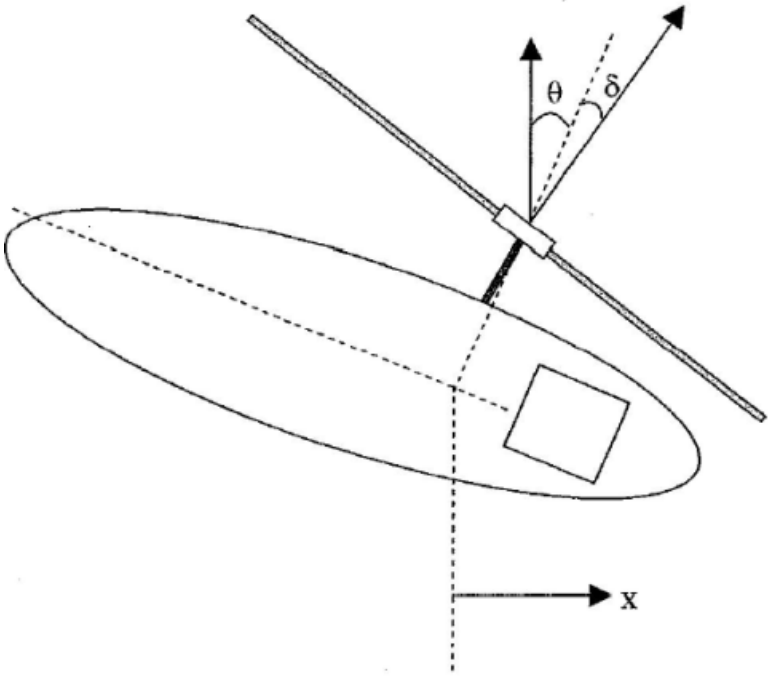
2nd World Congress on
Automation and Robotics
June 13-15, 2016 Philadelphia, USA

Predictive Controller Design using ANOVA

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National University of Computer and Emerging Sciences (NU-FAST), Karachi, Pakistan

Conceptual Model of Helicopter



$$\ddot{\theta} = -\tau_1 \dot{\theta} - \alpha_1 \dot{x} + \vartheta_1 \delta$$
$$\ddot{x} = g\theta - \alpha_2 \dot{\theta} - \tau_2 \dot{x} + g\delta$$

$$\tau_1 = 0.415 \quad \alpha_1 = 0.0111 \quad \vartheta_1 = 6.27$$
$$\tau_2 = 0.019 \quad \alpha_2 = 0.0111 \quad g = 9.81$$

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

System states

- State equations

$$\dot{x}_1 = \dot{\theta} = x_2$$
$$\dot{x}_2 = \ddot{\theta} = -0.415x_2 - 0.0111\,x_4 + 6.27\,\delta$$
$$\dot{x}_3 = \dot{x} = x_4$$
$$\dot{x}_4 = \ddot{x} = 9.81\,x_1 - 1.43\,x_2 - 0.0198x_4 + 9.81\delta$$

- State model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.415 & 0 & -0.0111\\ 0 & 0 & 0 & 1\\ 9.81 & -1.43 & 0 & -0.0198\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 6.27\\ 0\\ 9.81\end{bmatrix}\delta + v(k)$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + w(k)$$

System States

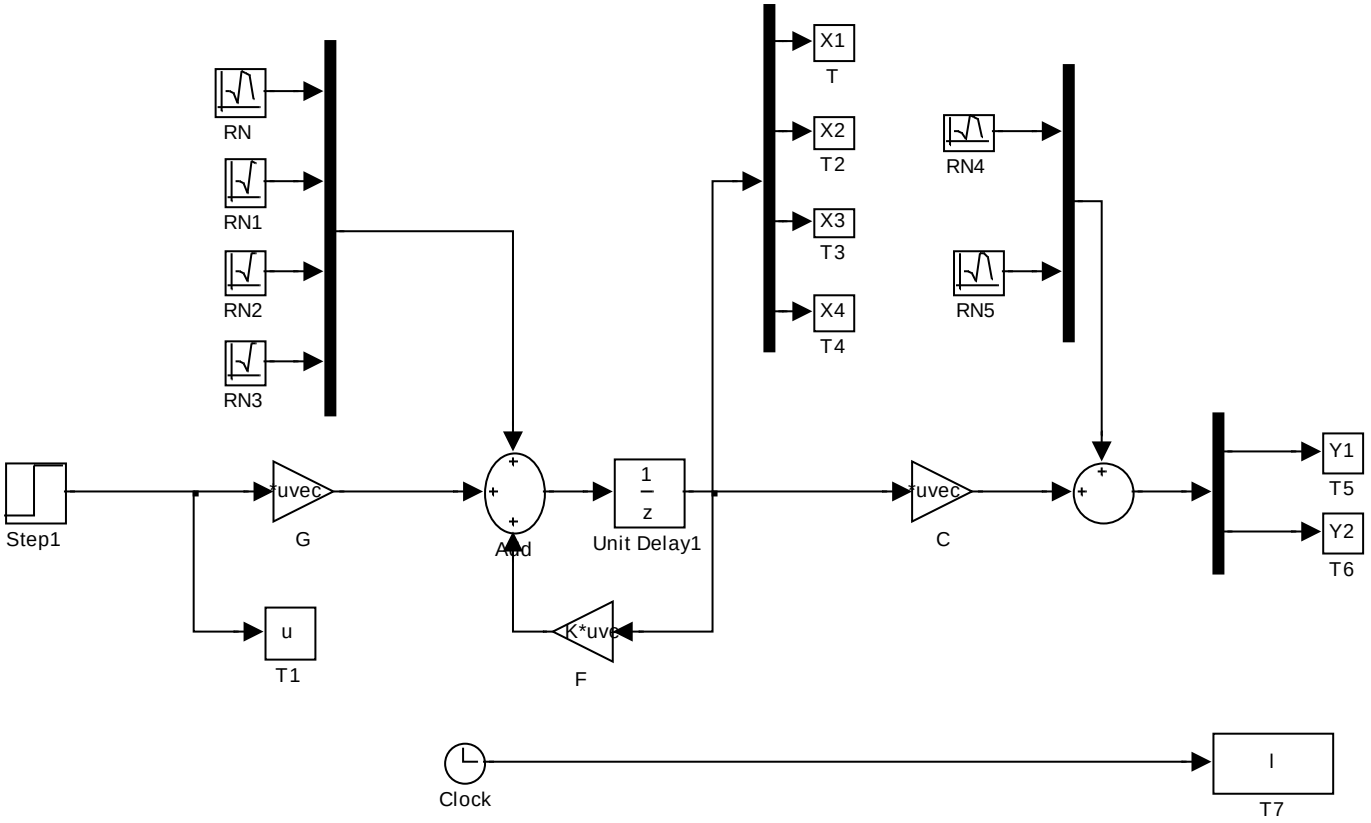
- Continuous to discrete model: A,B,C => F,G,C

$$F = \begin{bmatrix} 1.0000 & 0.0980 & 0 & -0.0001 \\ -0.0005 & 0.9594 & 0 & -0.0011 \\ 0.0490 & -0.0054 & 1.0000 & 0.0999 \\ 0.9801 & -0.0916 & 0 & 0.9981 \end{bmatrix} \quad G = \begin{bmatrix} 0.0309 \\ 0.6136 \\ 0.0478 \\ 0.9460 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

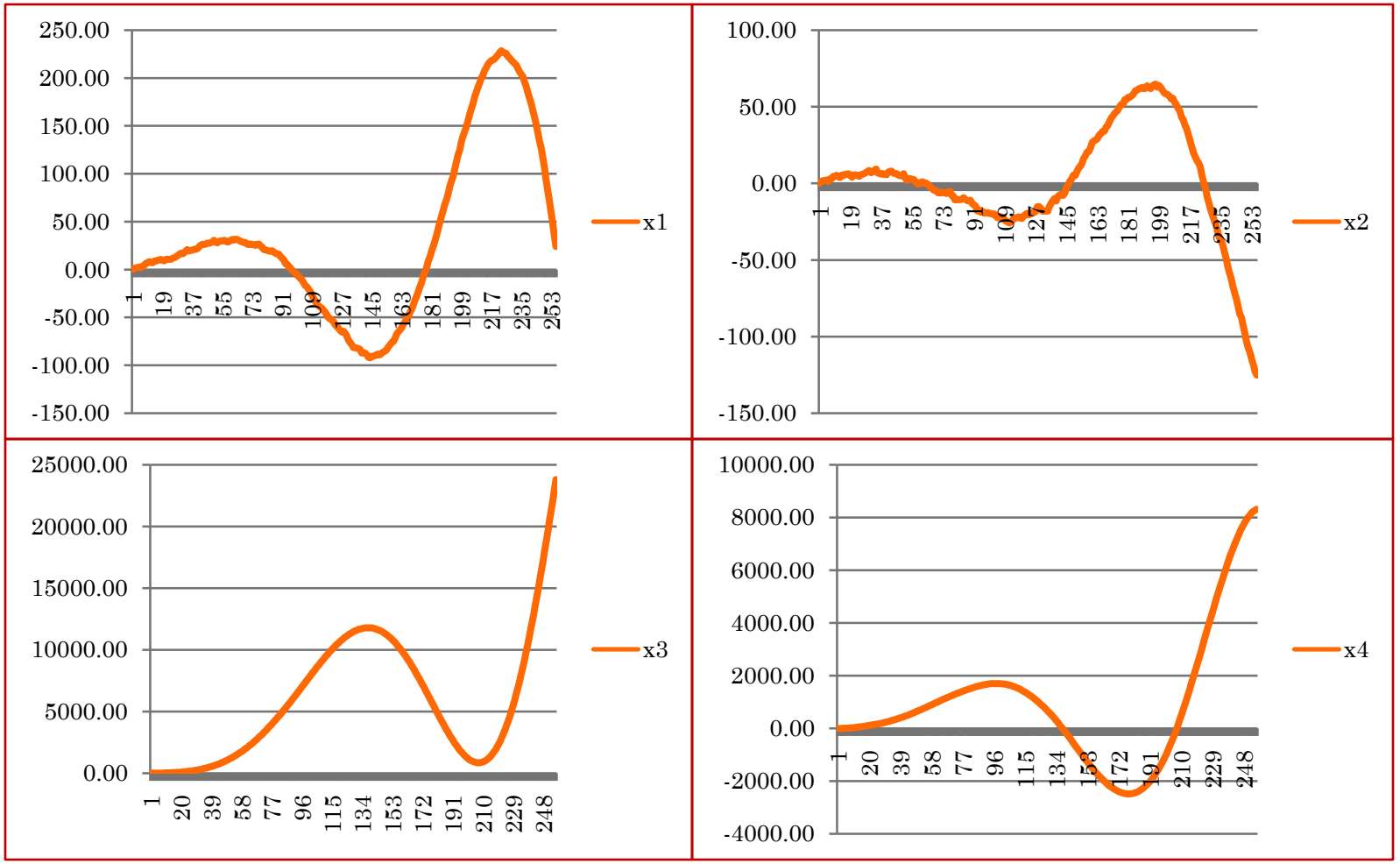
  - Sampling time: 0.1 sec

- System Tests
  - System observable: Mo = observable (F, C)
  - System controllable: Mc = controllable (F, G)
  - System unobservable: length (F) – rank (Mo) > 0
  - Eigen-values of (F) inside unit circle?

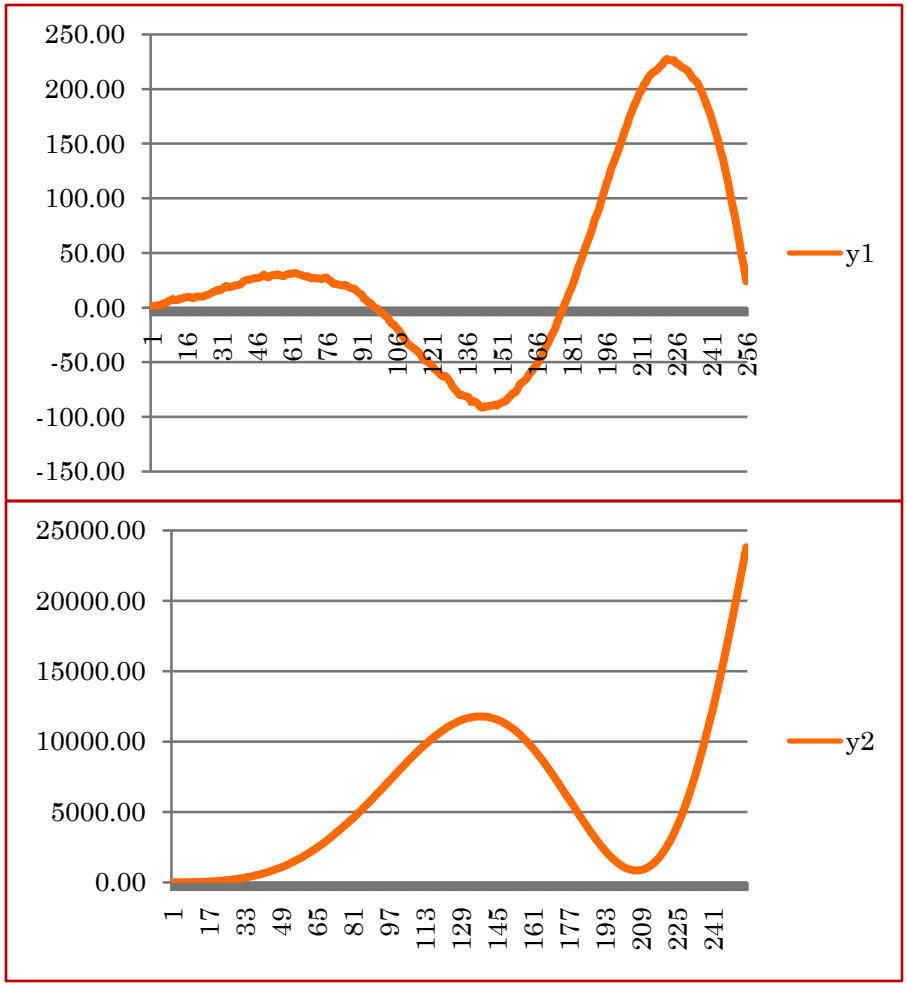
System Model using Simulink



System States



System Output



Kalman Filter Design

- Discrete State Space Model

$$x(k+1) = F\,x(k) + G\,u(k) + v(k)$$
$$y(k) = C\,x(k) + w(k)$$

- State Observer: $\hat{x}(k) = \bar{x}(k) + K\{y(k) - C\,\bar{x}(k)\}$

- State Predictor: $\bar{x}(k) = F\hat{x}(k-1) + G\,u(k-1)$

KALMAN FILTER DESIGN

- System parameters

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R1 = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix} \quad R2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

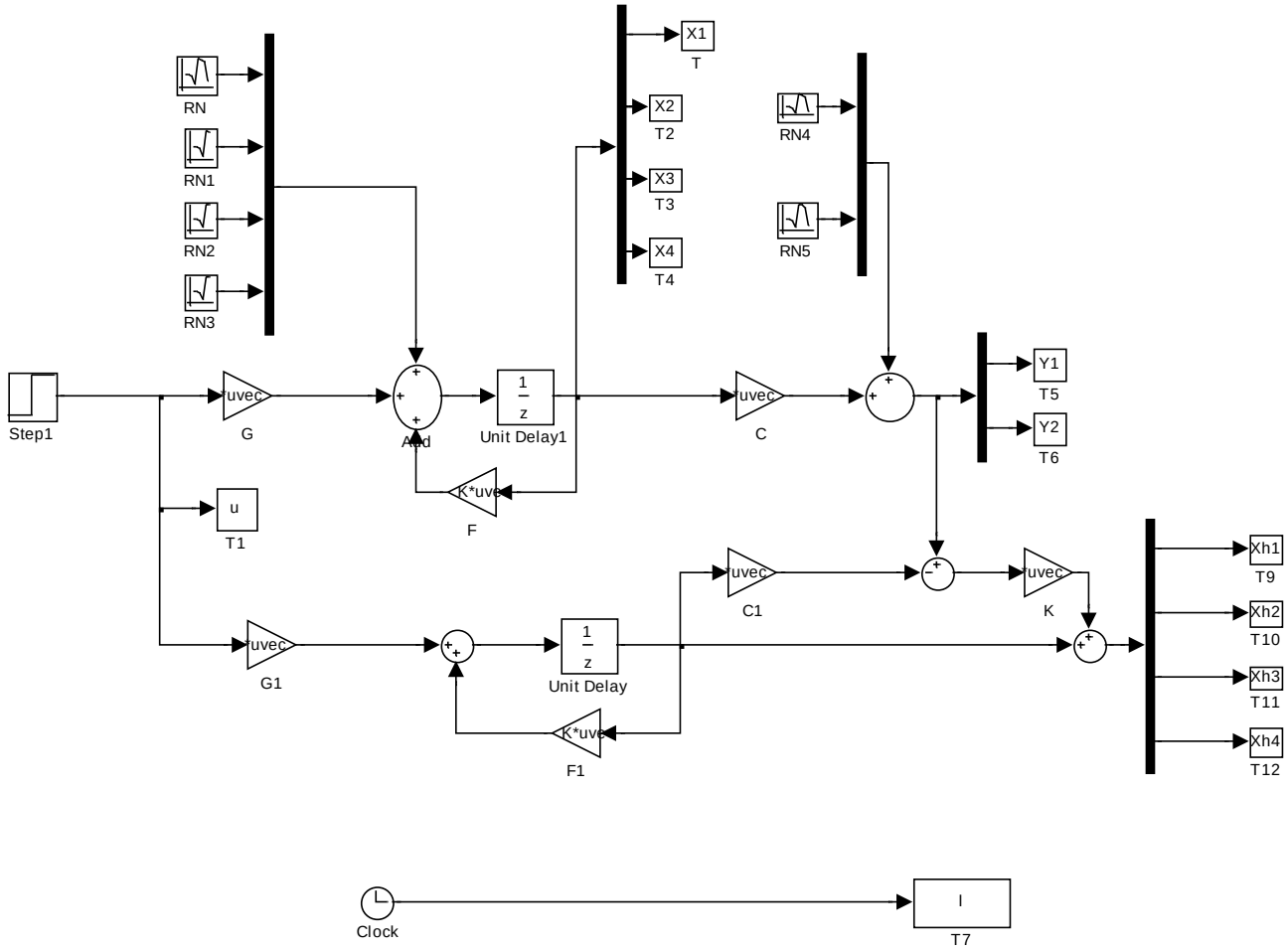
- Kalman gain

$$[K, P] = DLQE\,(F, H, C, R1, R2)$$

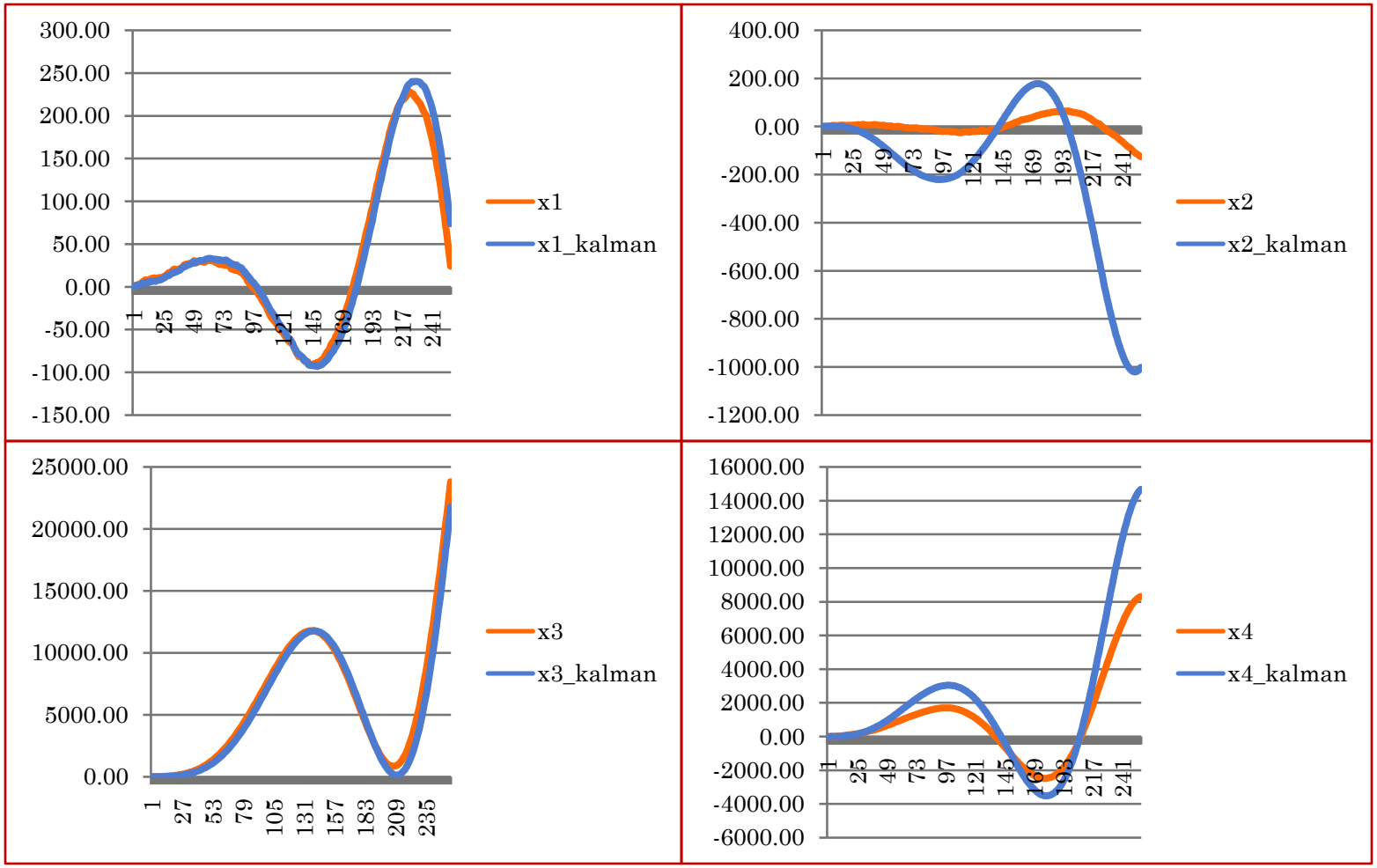
- State Equation using Kalman Prediction

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \\ \hat{x}_3(k+1) \\ \hat{x}_4(k+1) \end{bmatrix} = \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{bmatrix} + \begin{bmatrix} 0.6405 & 0.0006 \\ 0.3695 & -0.1370 \\ 0.0006 & 0.6683 \\ 0.1669 & 0.8703 \end{bmatrix} \left\{ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{bmatrix} \right\}$$

System Model with Kalman Estimation



System State with Kalman Prediction



ANOVA MODEL

- Model

$$\hat{x}(k) = \hat{\mu}_k + \hat{\alpha}_k + \hat{\beta}_k + \epsilon(k)$$

- Parameter estimation

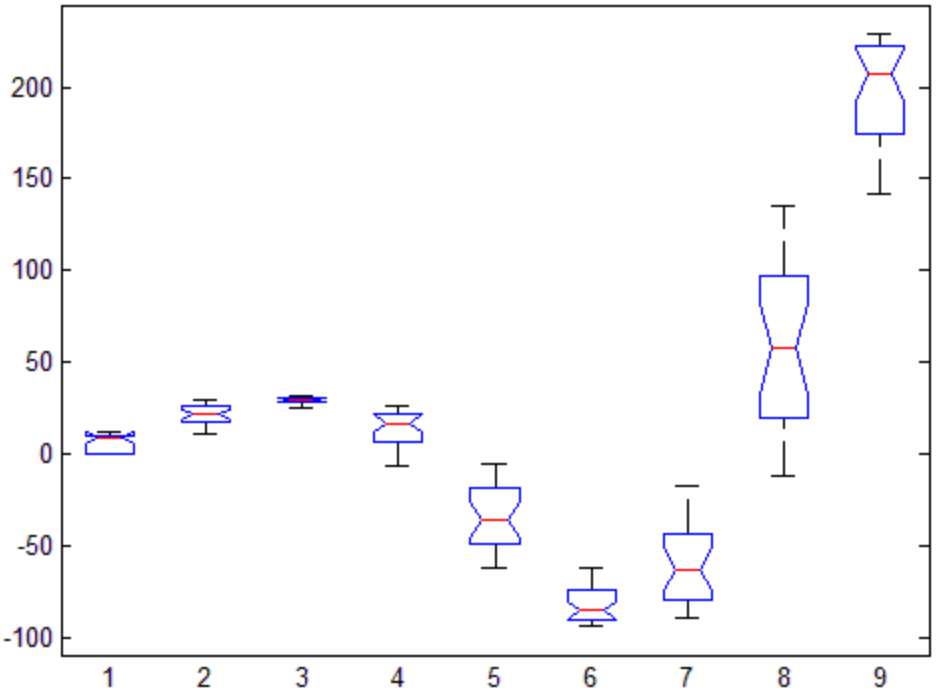
$$\hat{\mu}_k = \frac{1}{8}\sum_{i=(k-8)}^{k-1} x(i) \qquad \hat{\alpha}_k = \frac{1}{7}\sum_{i=(k-7)}^{k-1} [x(i) - x(i-1)]$$

$$\hat{\beta}_k = \frac{1}{2}\{ [x(k-1) - x(k-4) + [x(k-5) - x(k-8)] \}$$

x1 $\hat{x}(k)$ VARIATION

ANOVA Table

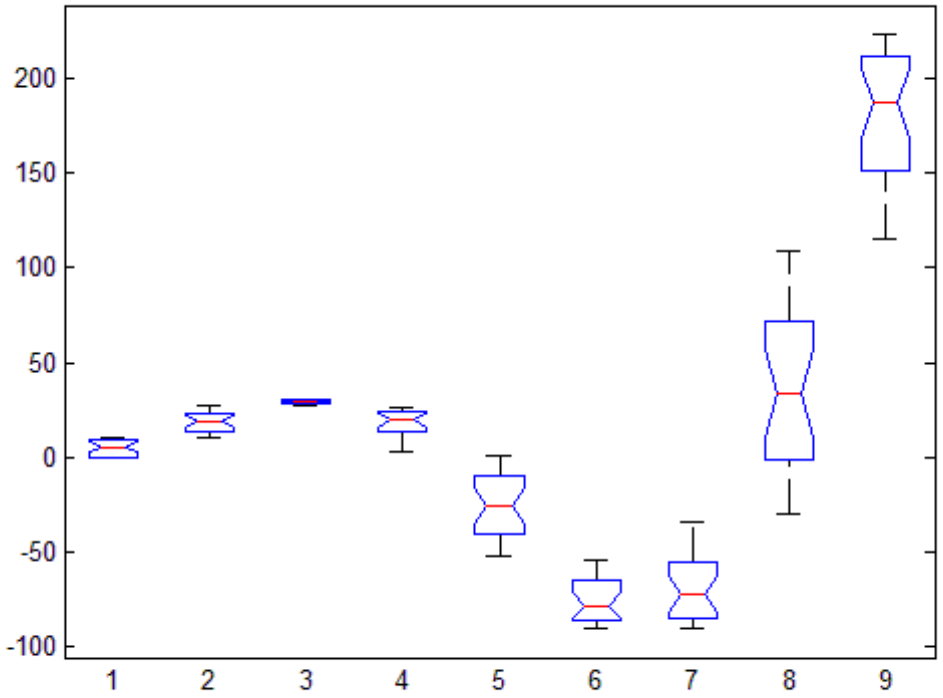
Source	SS	df	MS	F	Prob>F
Columns	1326304.6	8	165788.1	384.17	3.3987e-123
Error	93214.5	216	431.5		
Total	1419519.1	224			



X1 $\hat{\mu}_k$ VARIATION

ANOVA Table

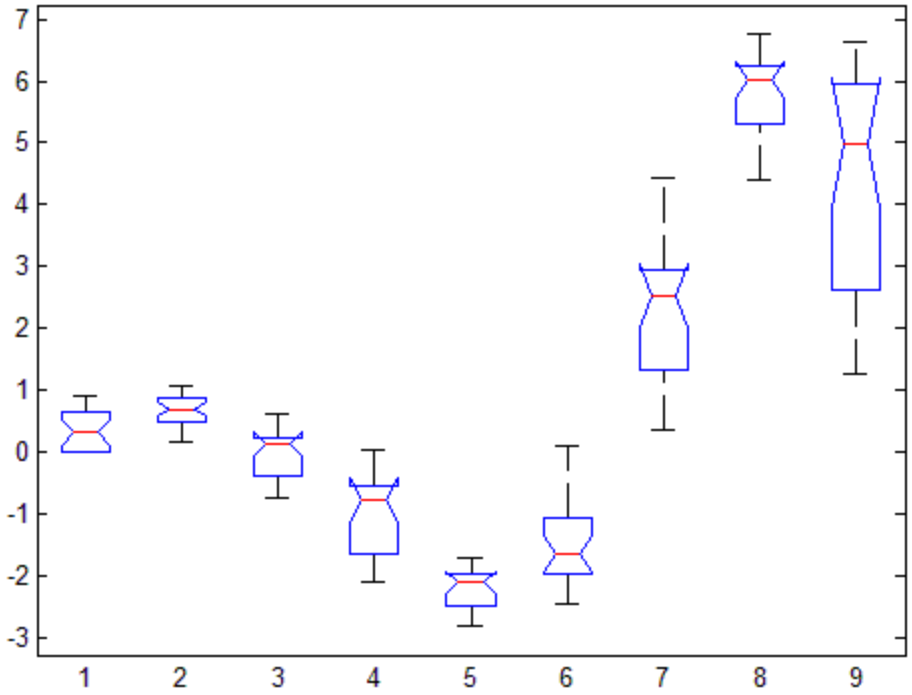
Source	SS	df	MS	F	Prob>F
Columns	1123309.9	8	140413.7	326.37	4.18742e-116
Error	92929.5	216	430.2		
Total	1216239.4	224			



X1 $\hat{\alpha}_k$ VARIATIONS

ANOVA Table

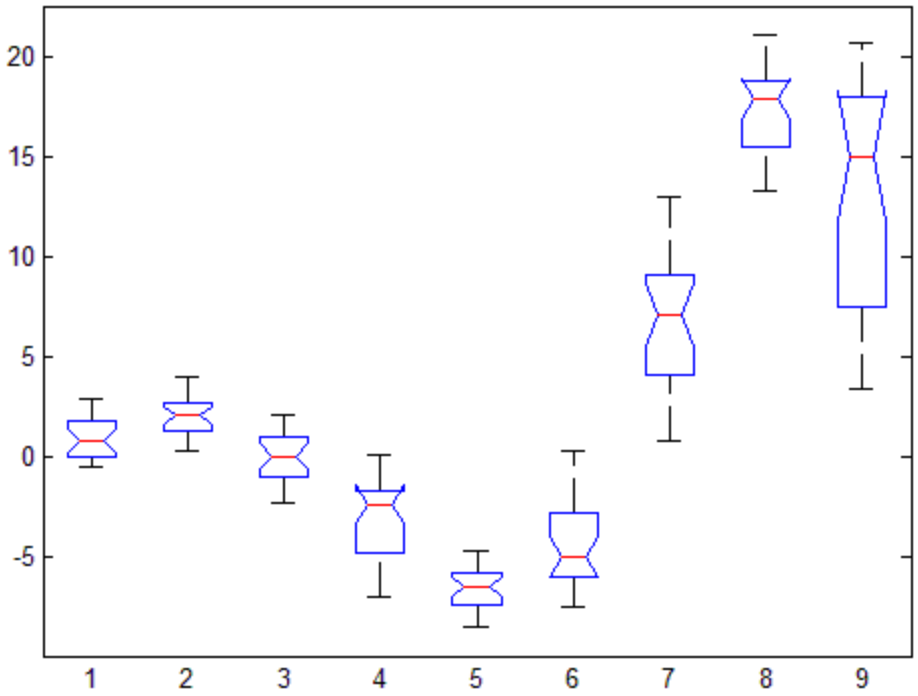
Source	SS	df	MS	F	Prob>F
Columns	1462.96	8	182.869	262.67	8.33451e-107
Error	150.38	216	0.696		
Total	1613.33	224			



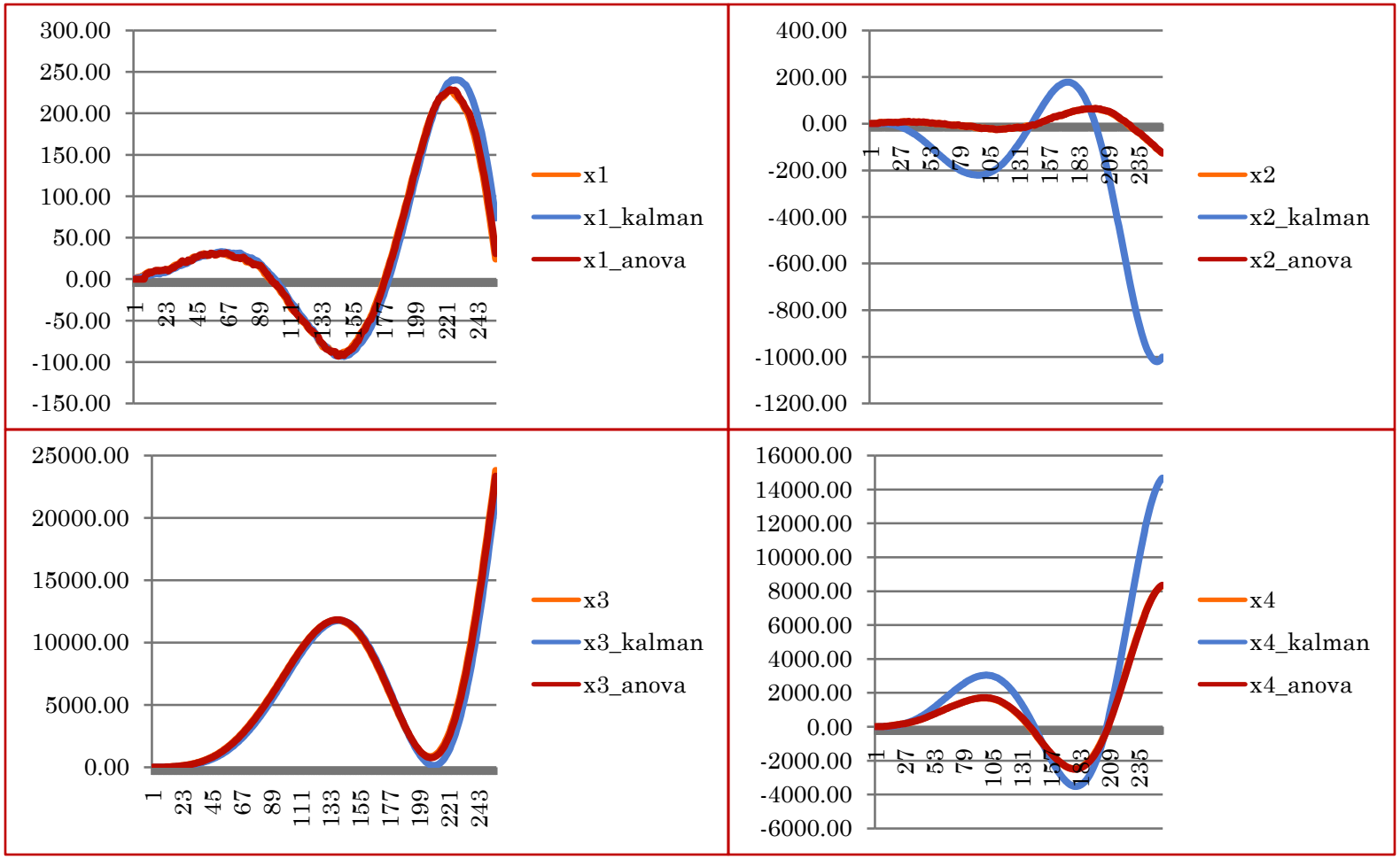
X1 $\hat{\beta}_k$ VARIATION

ANOVA Table

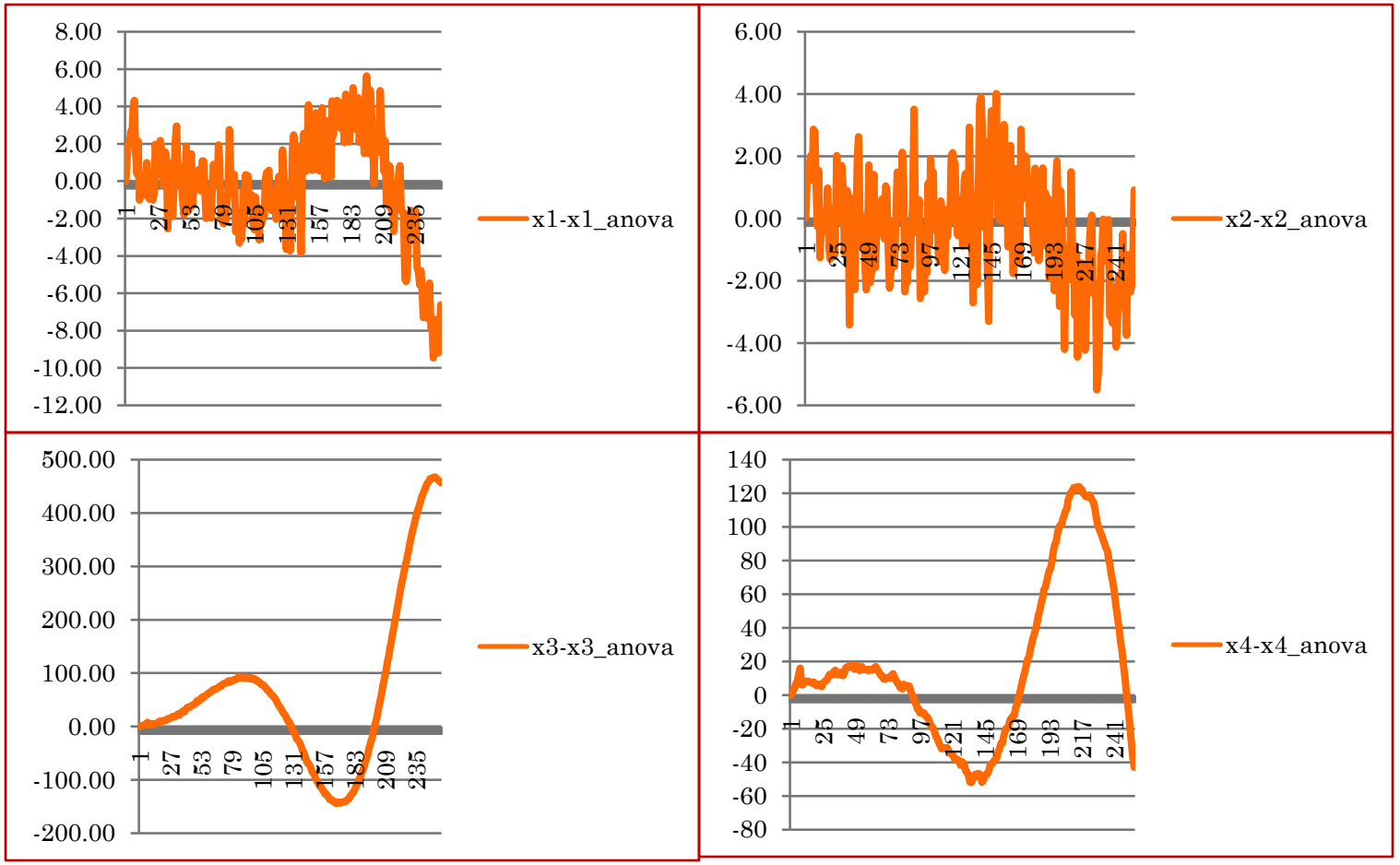
Source	SS	df	MS	F	Prob>F
Columns	13163.8	8	1645.47	254.93	1.53963e-105
Error	1394.2	216	6.45		
Total	14558	224			



System States with ANOVA Prediction



Error using ANOVA Prediction



Conclusions

- Kalman state estimation performs reasonably well when states depend on output.
- ANOVA based estimation performs comparable in all situations.

References

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Thank you

Q &A