Masses of black hole merger for GW150914

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Experimental test of General Relativity

- Time delay experiment of the solar system, using planets as reflectors.
- I. I. Shapiro et al (1979): Accuracy 1 in 10^3
- B.Bertotti et.al. (2003): Nature 1 in 10^5

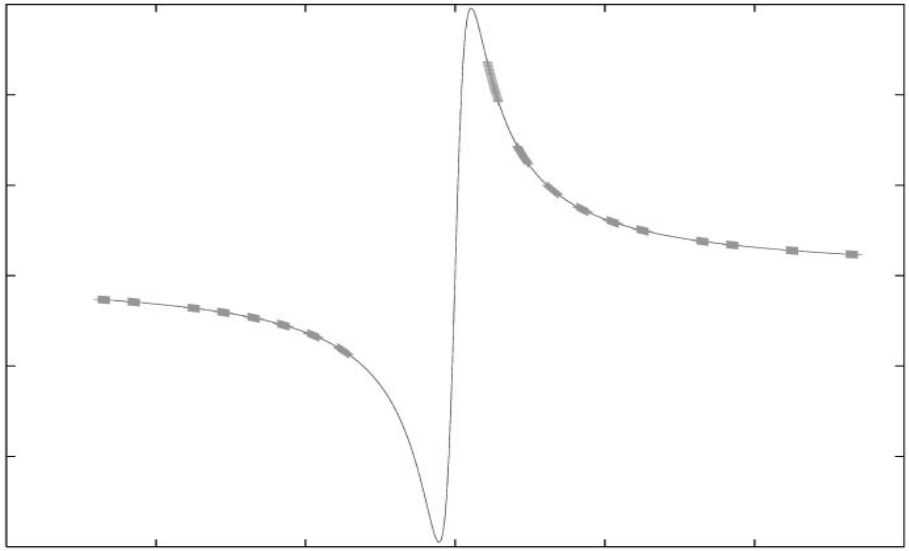
  using the Cassini satellite as reflector





Experimental test of General Relativity

- B.Bertotti et. al. Nature 425, 274 (2003): Using Cassini satellite (Saturn probe) as a reflector. The accuracy of 1 in 10^5
- $\Delta t=2(1+\gamma)GM_{\odot}/c^3\ln(4r_1r_2/b^2)$
- $\gamma=1+(2.1\pm2.3)*10^{-5}$



Prediction of General Relativity

- $ds^2=e^{\nu(r)}dt^2-e^{\lambda(r)}dr^2-e^{\mu(r)}r^2d\Omega$
- For the Schwarzschild metric, $e^{\mu(r)}=1$,

  $\Delta t=2(r_s)/c[\ln(2r/b)+(1/2)((r-b)/(r+b))^{1/2}]$,

  where $r_s=2GM_\odot/c^2$ is the Schwarzschild radius, $M_\odot$ is the solar mass and b is the impact parameter.

  (See S. Weinberg's Gravitation pp202 or R. M. Wald, General Relativity, pp146)

Prediction of General Relativity 2

- For the physical metric, $e^{\nu(r)}=e^{\mu(r)}$, (speed of light on the spherical direction is c)
- $\Delta t=2(r_s)/c[\ln 2r/b]$
- (See Y. Tomozawa, J Modern Physics 6, 335 (2015))
- The physical metric is the right metric that gives the correct formula for time delay experiment (but not the Schwarzschild metric).
- Notice that the spherical direction is perpendicular to the radial direction, which is the direction of gravity.

Physical Metric

- $\omega = e^{\nu(r)} = e^{\mu(r)}$
- Schwarzschild metric $\rightarrow e^{\nu(r')} = 1 + r_s/r'$,

  $r' = r\omega^{1/2} \rightarrow \quad 1 - r_s/r\omega^{1/2} = \omega \rightarrow$

  $r_s/r = \omega^{1/2}(1-\omega)$
- $e^{\lambda(r)} = (d/dr(r\omega^{1/2}))^2/(1 - r_s/r\omega^{1/2})$

  $= (2\omega/(3\omega - 1))^2$

Physical metric, the internal solution

- $(1/A)(1+Dr_s/r') \rightarrow (1/A)(1+Dr_s/r\omega^{1/2})=\omega \rightarrow$

  $Dr_s/r=\omega^{1/2}(A\omega-1)$

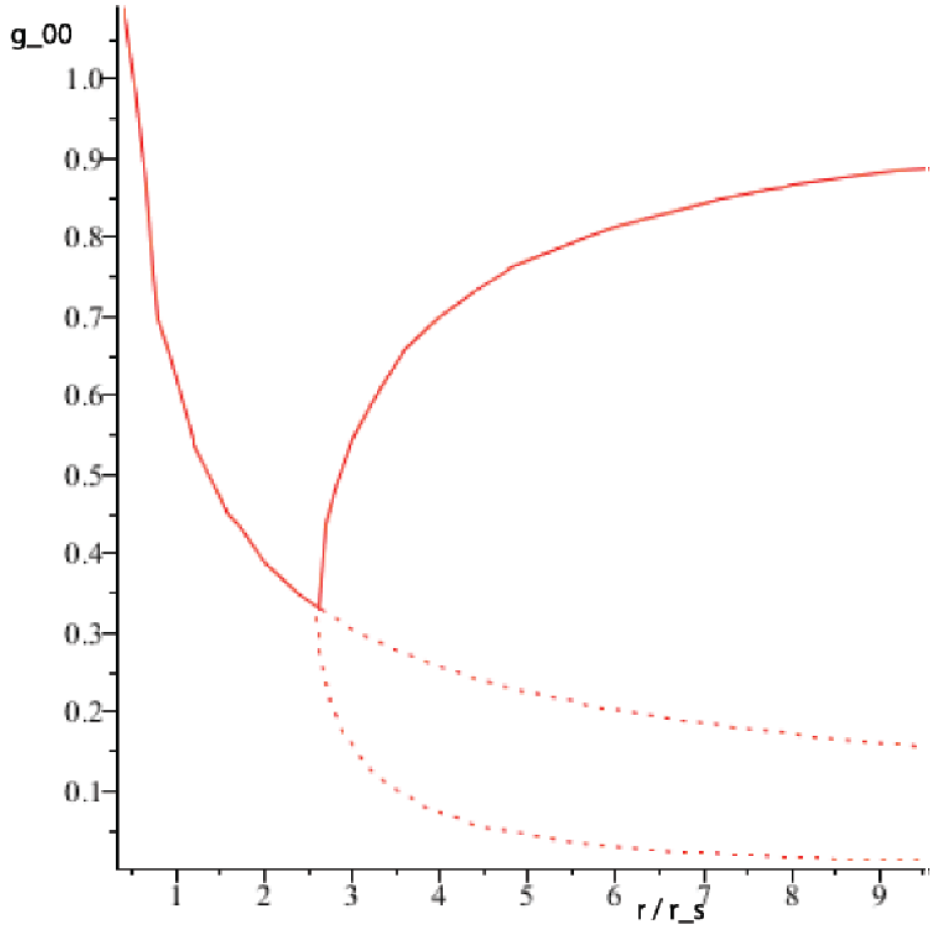
- Continuity of ω to the outside solution→

  $A=2D+3$ (We choose $A>3$, $D>0$)

  Then all the metric functions are positive definite.

  $e^{\lambda(r)}=A(2\omega/(3A\omega-1))^2$

Physical metric (ω=g_00)



Physical metric for internal solution with constant density

- Constant mass distribution, ρ.
- ω=1/(B+8πGρr^2/3)
- e^λ(r)=Bω^2
- B=2.615 (From continuity of ω at R)
- The both internal solutions are repulsive,

  with constant density or point source.
- p = - ρ This is consistent with cosmology data.

Extended horizon =size of BH

- The external solution ends at

  r=R=($\sqrt{3}$ 3/2) r_s = 2.60 r_s

  with the value of ω = 1/3.
- R = 2.60 r_s (r_s is the Schwarzschild radius)
- R is the size of the compact object, black holes and neutron stars. It is called the extended horizon. Neutron star of 1.4 M_ʘ --> R=11km.
- The gravitational red shift on the surface of compact objects is 1+z = $\sqrt{3}$

The size of BH at the center of the Milky Way

- EHT (Event Horizon Telescope) reported that the diameter size of BH at the center of the Milky Way is of the order of 6 SR (Schwarzschild Radius). (Nature 350, 1242 (2015))
- The physical metric predicts 5.2 SR. (2.60 * 2 =5.20) (The Schwarzschild metric predicts 2 SR.)

Repulsive force inside the extended horizon

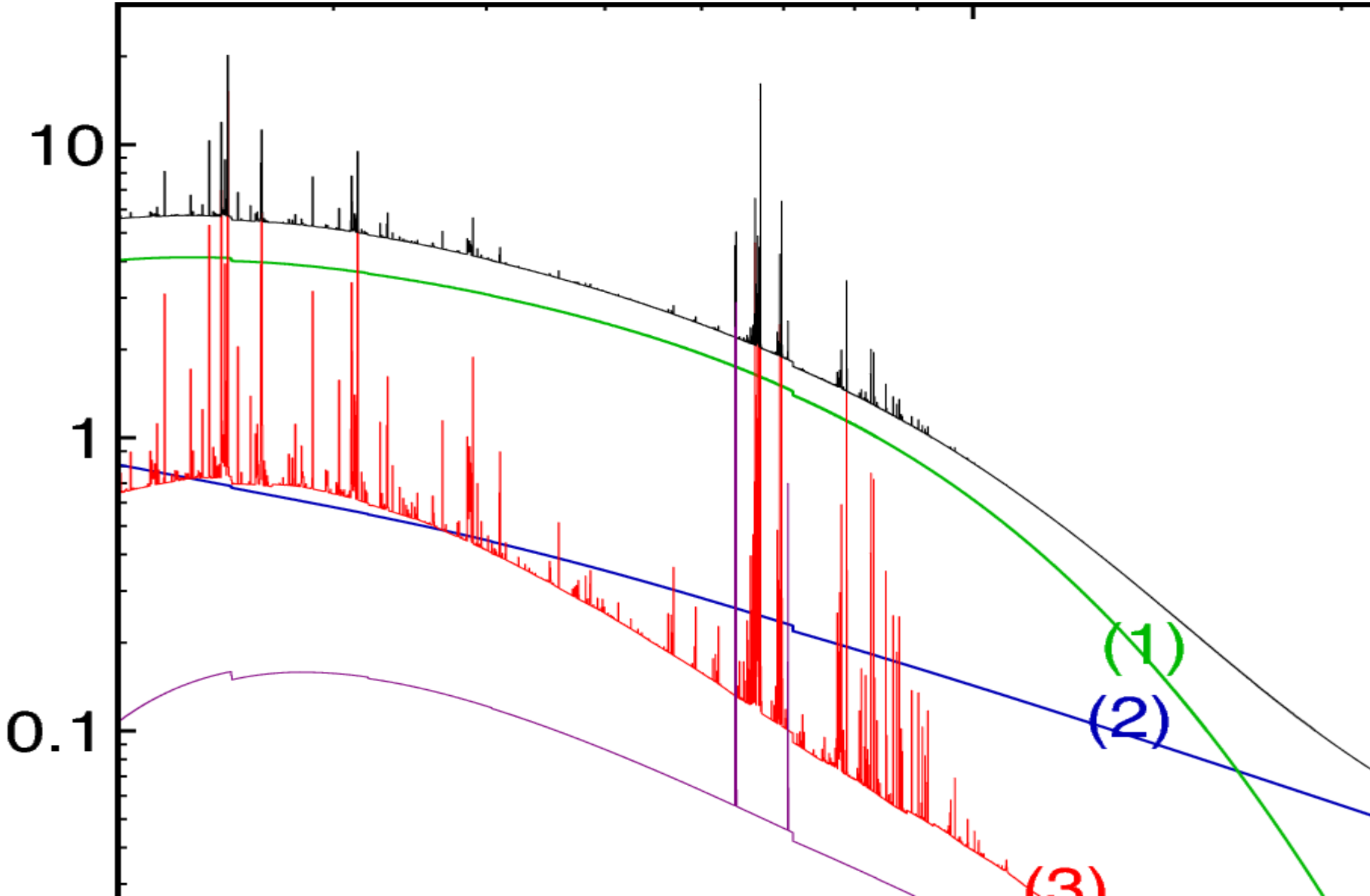
- Black holes can emit particles to outside the extended horizon
- Supernova explosion can be explained
- Cosmic rays can be produced from black holes (AGN, Active Galactic Nuclei, massive black holes). This was proposed by the author. (1985)
- Dark energy can be related with this repulsive force.

Temperature of compact objects

- dω/dr ~ Temperature
- Outside of extended horizon ≈ high T (infinity at R)
- Schwarzschild metric → Hawking temperature

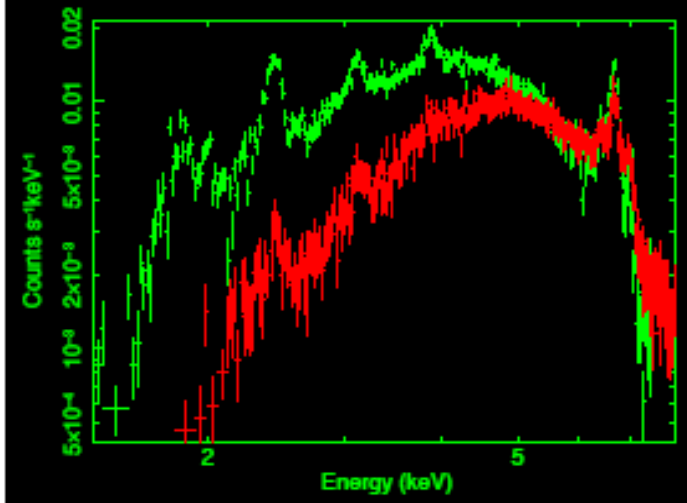
  $T_H = \hbar c^3/8\pi GMk_B$ is very low for a large M
- Negative Temperature inside R (higher than any positive T): Probability = exp(-E/kT) = higher for high E and T < 0

X-ray spectrum



X-ray spectrum from compact objects

Comparison with discrete sources



Discrete sources are on average substantially harder than that of the diffuse emission and may represent magnetic CVs with strong intrinsic abs.

High temperature environment for black holes and neutron stars

- Few keV x-ray ➔ highly ionized atom such as

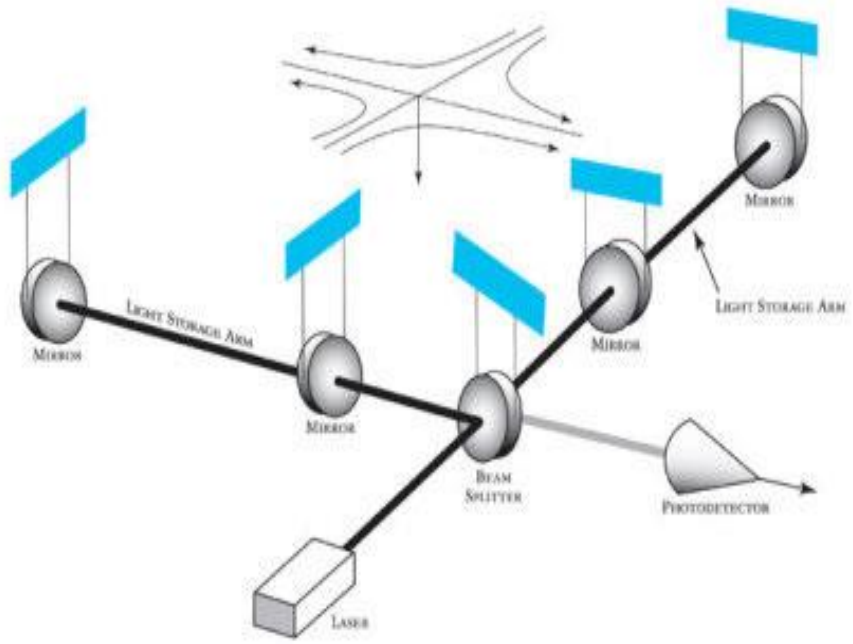
Fe XXVI

- One has to include the effect of gravitational redshift at the extended horizon, R.

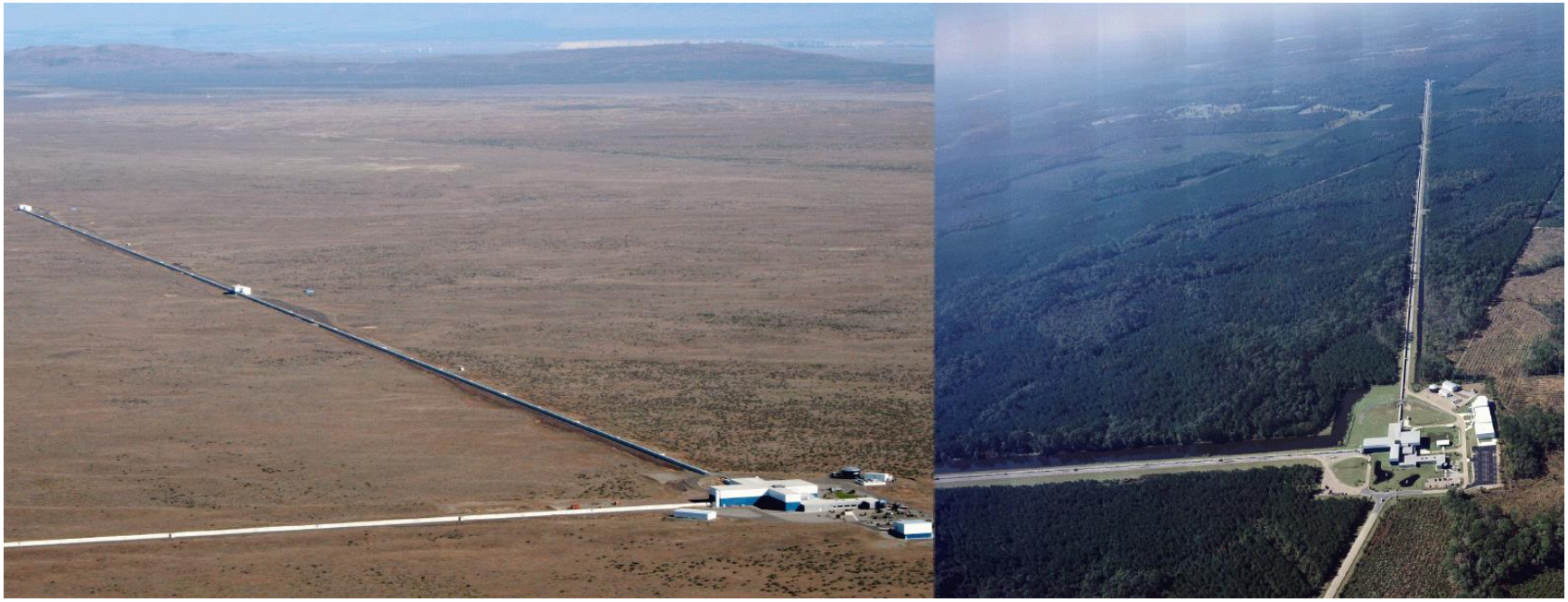
$$1+z = \sqrt{3} = 1.732$$

- The rotation makes a few % change.

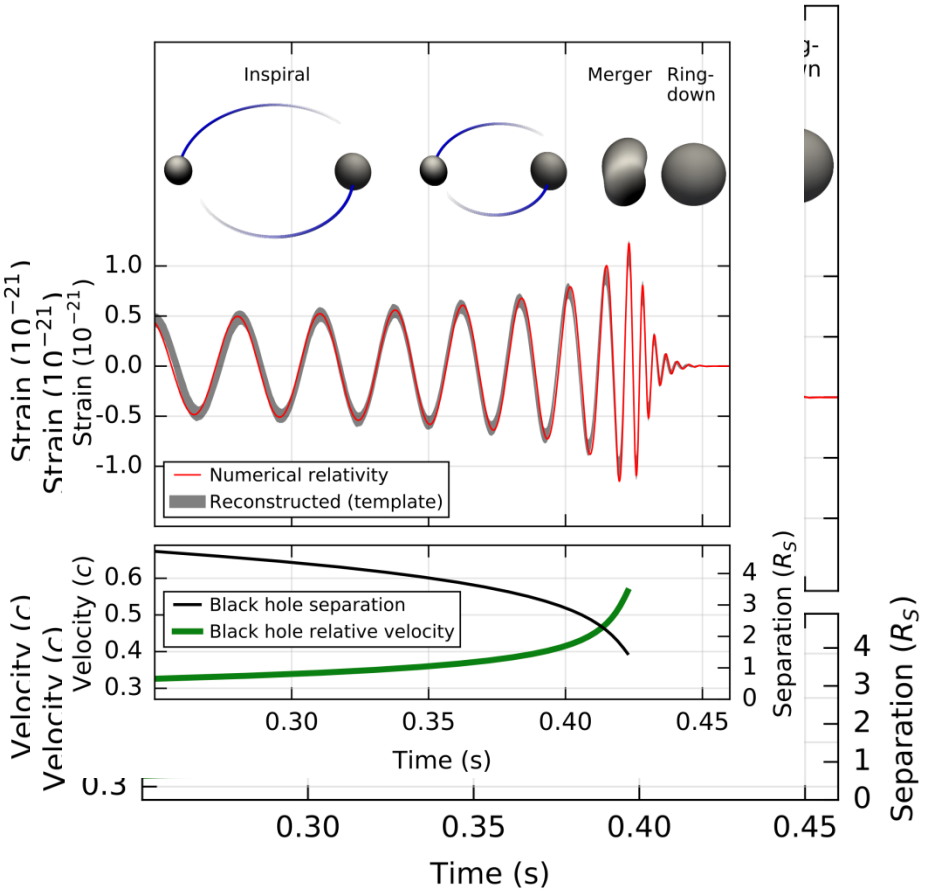
LIGO (gravitational wave detector) x 2
4 km arm length



LIGO



Black hole merger in GW150914



Black hole masses in GW150914

- The numerical simulation indicates the masses of black hole merger to be
- $M_1 = 29\ (+4, -4)\ M_\odot$
- $M_2 = 36\ (+5, -4)\ M_\odot \rightarrow M_f = 62\ (+4, -4) M_\odot$
- These must be changed to (divided by 2.60), since the size of black hole is increased by 2.60
- $M_1 = 11.2\ M_\odot$
- $M_2 = 13.8\ M_\odot \rightarrow M_f = 23.8\ M_\odot$

Observed stellar black hole masses

- Table 1. Confirmed black holes and mass determinations

GRS 1915+105	14 ± 4 $M_\odot$
V404 Cyg	12 ± 2
Cyg X-1	10 ± 3
XTE J1819-254	7.1 ± 0.3
GRO J 1655-40	6.3 ± 0.3
LMC X-3 and 10 others	9.4 – 5.2

Away from the Merger Contact

- The change of energy by GW emission,

  -dE/dt = F (32G/5c^5)*(m1 m2 /(m1+m2))^2 *

  r^4 *ω^6,

where F= (2.60)^2 * (1+z)^2

is the enhancement factor by the size of BH and the gravitational red shift, 1+z = $\sqrt{3}$ =1.732.

- This factor F is good enough to fit the data by smaller masses, m1 = 13.8 and m2 = 11.2 SM.

Conclusion

- Physical metric is the correct metric in GR
- The size of compact objects is 2.60 r_s
- Use the physical metric for GW detection, which makes the estimate of black hole mass to be a factor 2.60 smaller.
- Black holes and neutron stars are high T, with gravitational redshift = $\sqrt{3} - 1 = 0.732$

  (with a few % of error by a rotational effect.)

For the future signatures

- Γ-rays, X-rays
- Cosmic rays (delayed)
- Gamma ray bursts
- Neutrinos (delayed signature)