Applied Physics of Air-Sea-Land Interaction during a Tropical Cyclone

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1. Introduction

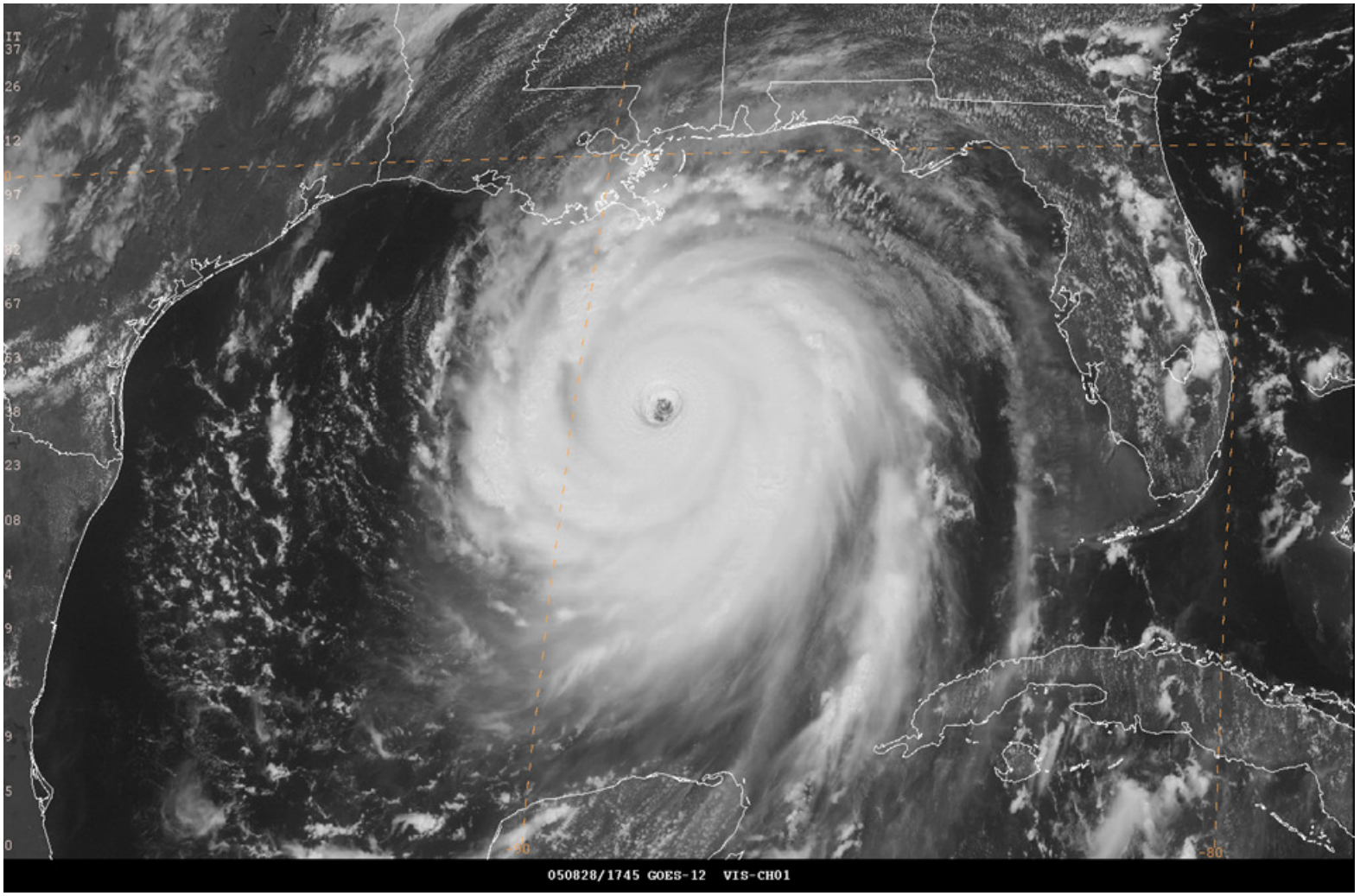









Photo by S. Douglass

2. Estimating Hurricane Winds

From the cyclostrophic equation when the centrifugal force is balanced by the pressure gradient force, we have

$$U_a^2/ \Upsilon = (1/\varrho)\, \Delta P/\Delta \Upsilon = (1/\varrho)\, (P_n - P_0)/\, (\Upsilon - 0) \quad (1)$$

Because $\varrho = 1.2$ kg m^{-3},

$\Delta P = (1013 - P_0)$ mb, and 1 mb = 100 N m^{-2} = 100 kg m^{-1} s^{-2},

Eq. (1) becomes

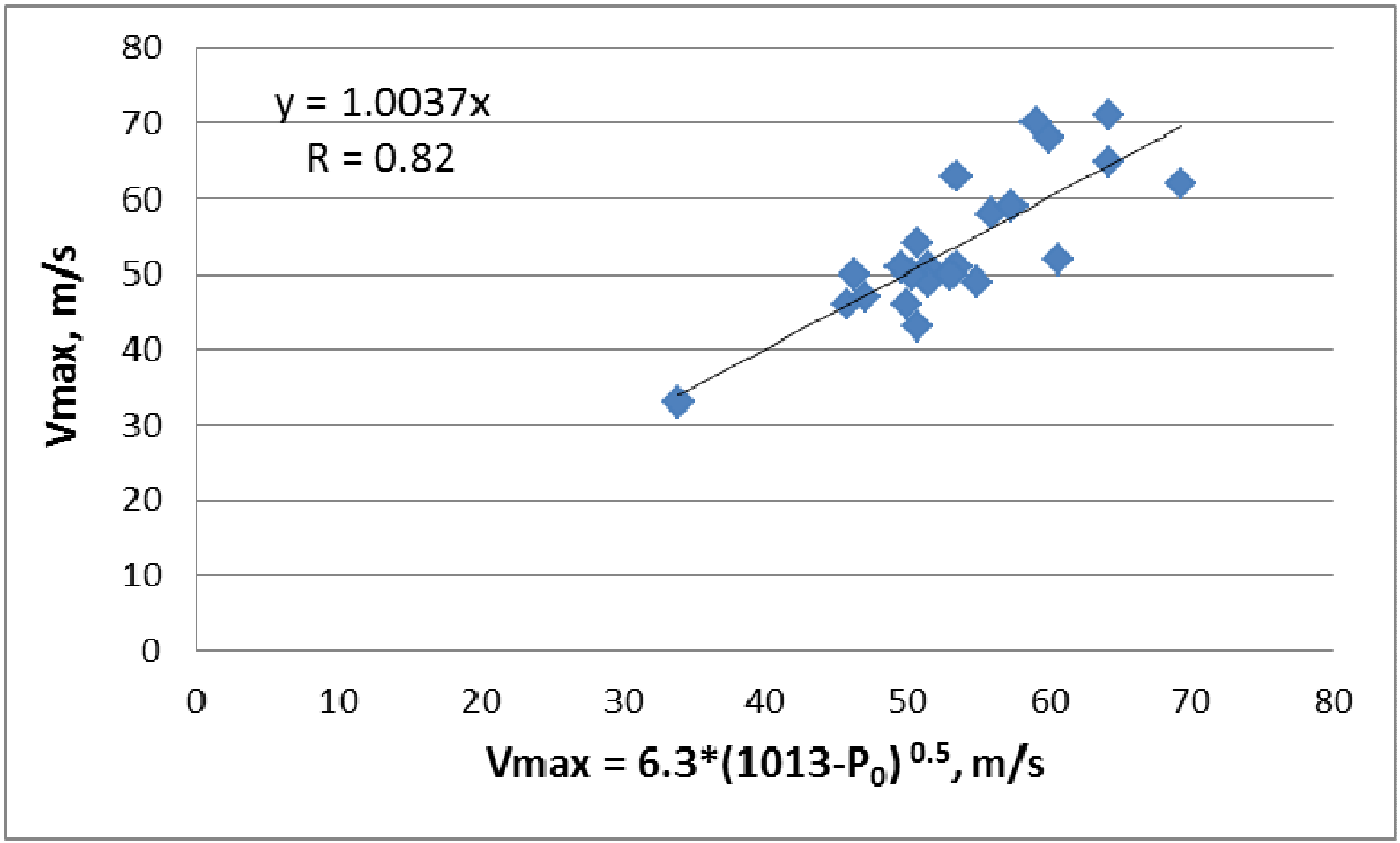
$$U_a = [(100 \text{ kg m}^{-1} \text{ s}^{-2})/ (1.2 \text{ kg m}^{-3})]^{0.5} * (\Delta P)^{0.5}$$

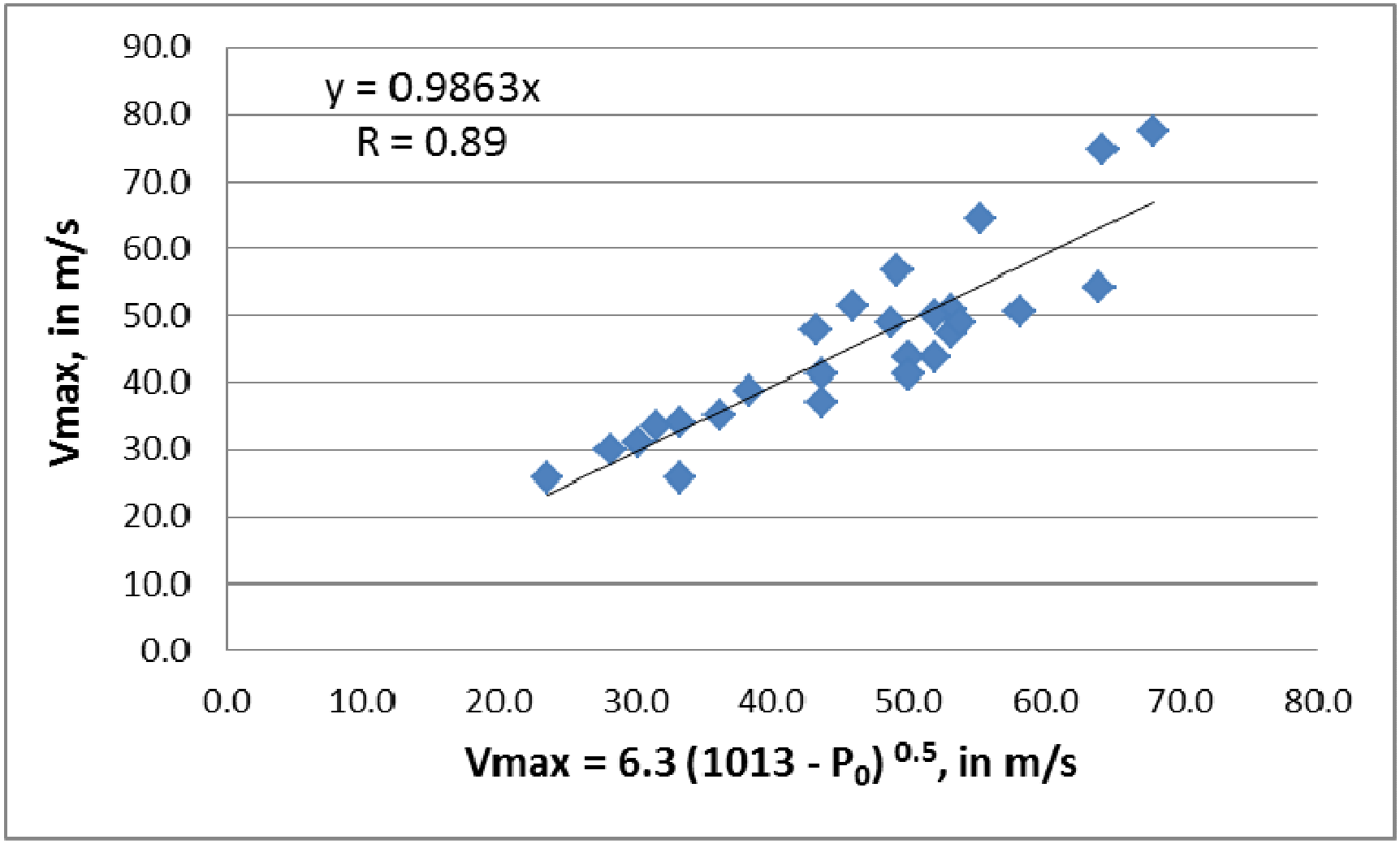
$$U_a = 9 (\Delta P)^{0.5} \qquad (2)$$

According to Powell (1982), $U_{10} = 0.7\ U_a$; therefore,

$$U_{10} = 6.3 (\Delta P)^{0.5} = 6.3 (1013 - P_0)^{0.5} \qquad (3)$$

Where U_{10} is the wind speed at 10 m in m/s and ΔP is in mb.


y = 1.0037x
R = 0.82
Vmax, m/s
0
10
20
30
40
50
60
70
80
Vmax = 6.3*(1013-P0) 0.5, m/s


y = 0.9863x
R = 0.89
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
Vmax, in m/s
Vmax = 6.3 (1013 - P0) 0.5, in m/s

3. Estimating Hurricane Waves

According to the Shore Protection Manual (see USACE, 1984),

$$(g H_s/U_{10}\text{^}2) = 0.0016 (g F/U_{10}\text{^}2) \text{ ^ } (1/2) \quad (4)$$

$$(g T_m/U_{10}) = 0.2857 (g F/U_{10}\text{^}2) \text{ ^ } (1/3) \quad (5)$$

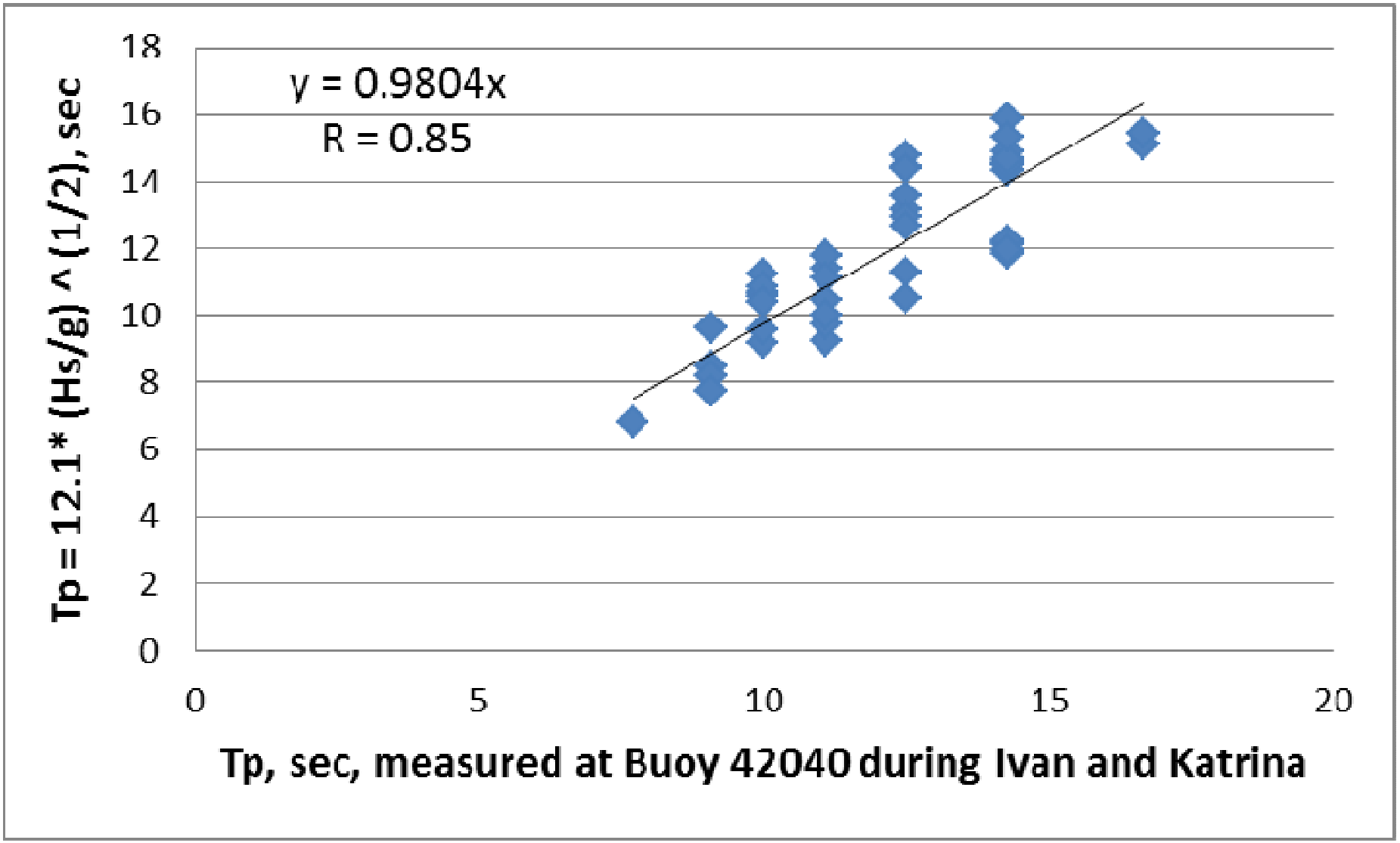
$$T_p = 0.95 T_m \quad (6)$$

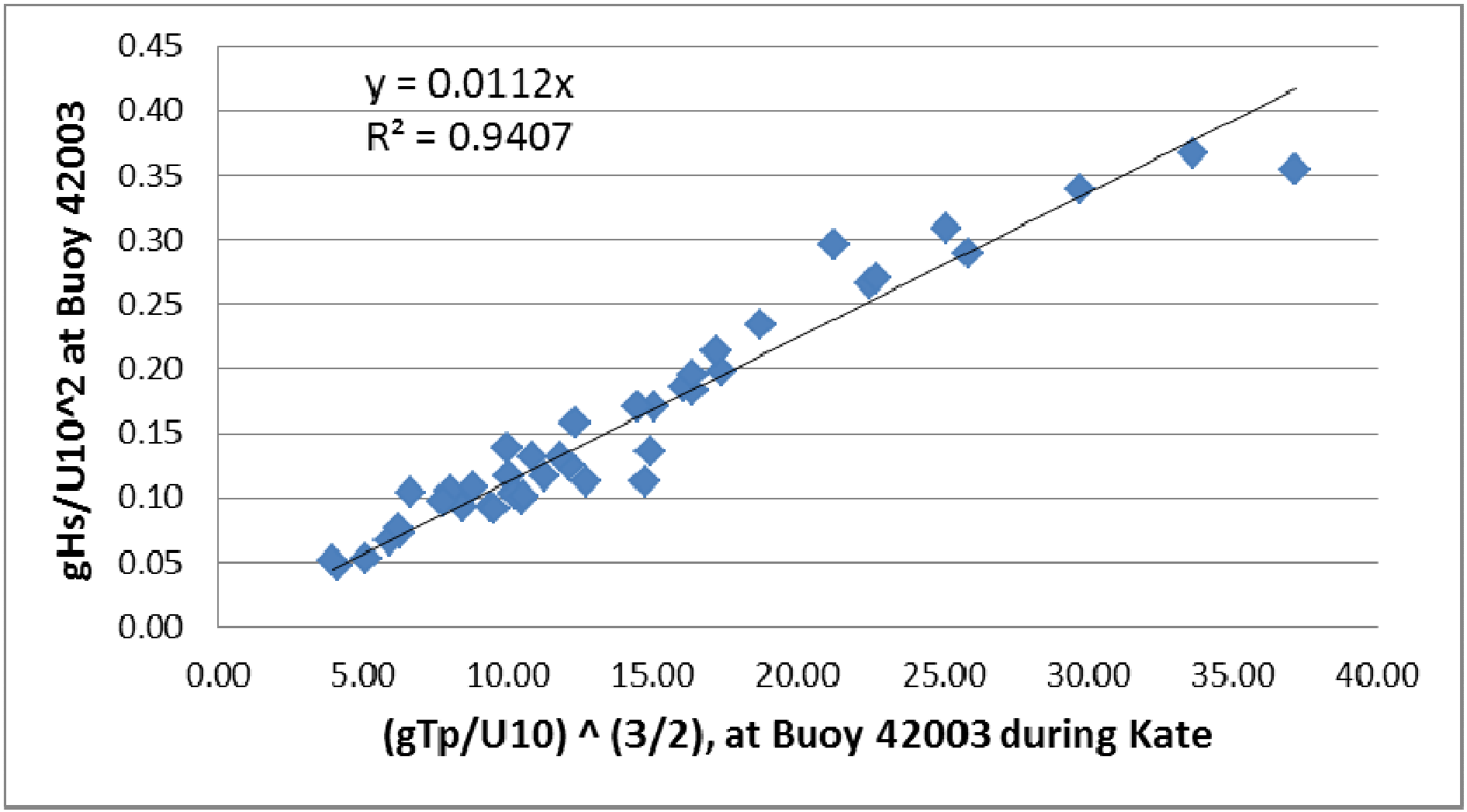
$$T_p = 12.1 (H_s/g) \text{ ^ } (1/2) \quad (7)$$

Now, eliminating the fetch parameter, F, and rearraging Eqs. (4) and (5), we have

$$(g H_s/U_{10}\text{^}2) = 0.0113 (gTp/U_{10}) \text{ ^ } (3/2) \qquad (8)$$

Equation (8) is validated in Fig.9 based on datasets not only in Hsu (2003) but also extending all measurements with the pressure less than 1013mb.


y = 0.9804x
R = 0.85
Tp = 12.1* (Hs/g) ^ (1/2), sec
Tp, sec, measured at Buoy 42040 during Ivan and Katrina
0
2
4
6
8
10
12
14
16
18
0
5
10
15
20


y = 0.0112x
R² = 0.9407
gHs/U10^2 at Buoy 42003
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
(gTp/U10) ^ (3/2), at Buoy 42003 during Kate

For operational applications, the coefficient needs to be changed from 0.0113 to 0.0112 so that

$$(g\,H_s/U_{10}\text{^}2) = 0.0112\,(g\,T_p/U_{10})\ \text{^}\ (3/2) \qquad (9)$$

Substituting Eq. (7) into (9), one gets

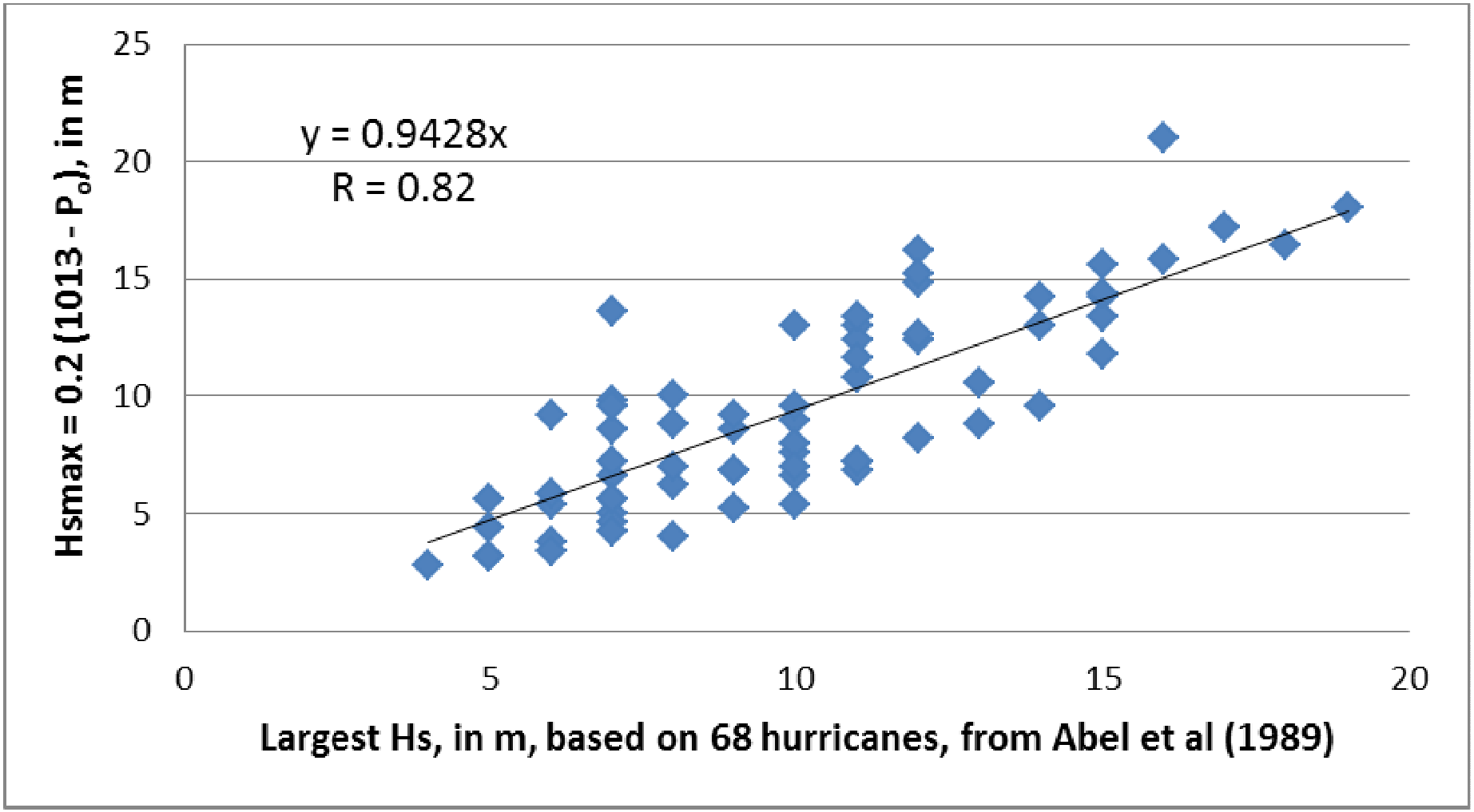
$$Hs = 0.0050\,U_{10}\text{^}2 \qquad (10)$$

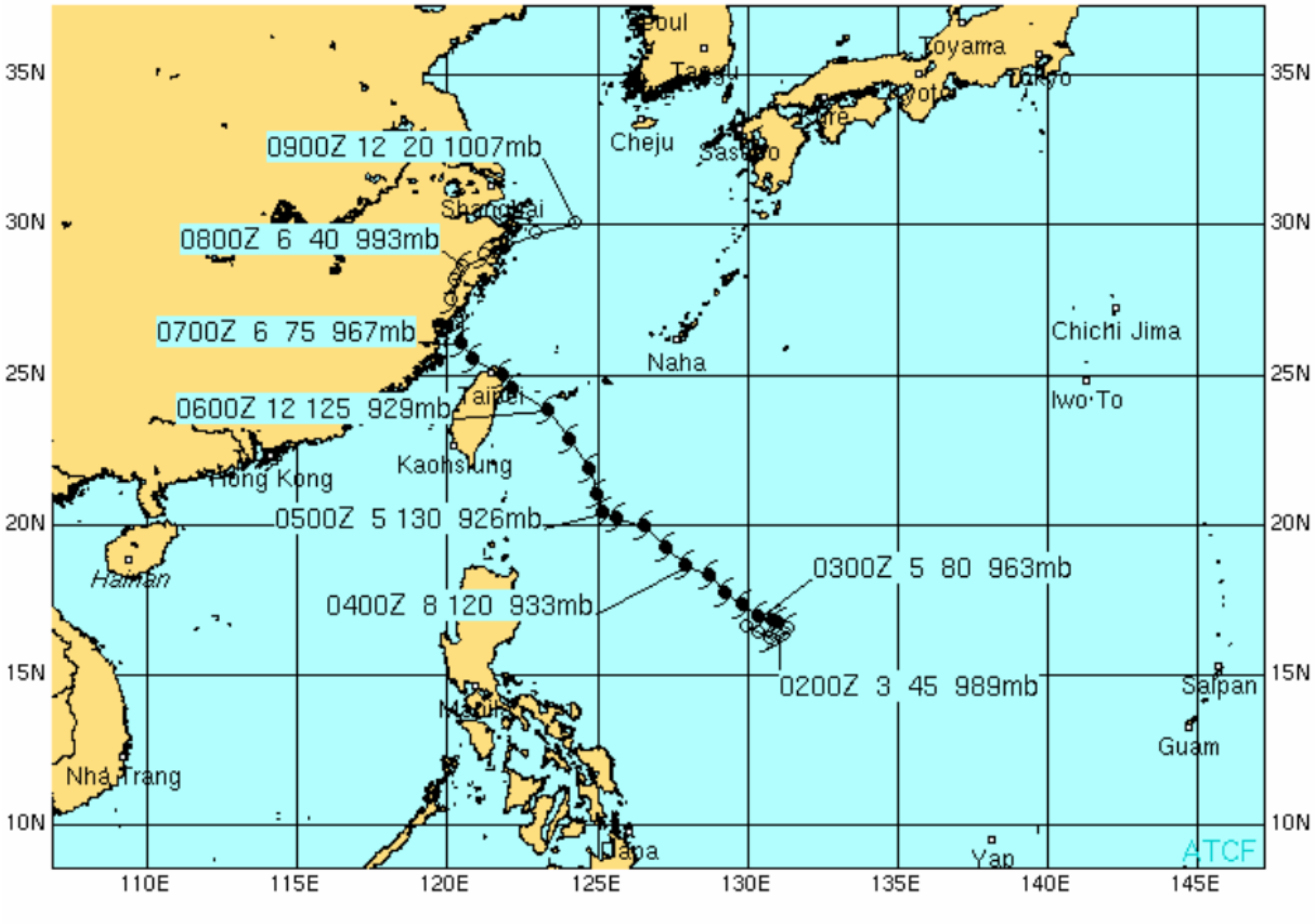
$$U_{10} = 14.1(H_s\text{^}0.5) \qquad (11)$$

Now, from Eq. (3), we have

$$H_{smax} = 0.20\,(1013 - P_o) \qquad (12)$$

Where H_{smax} is the maximum significant wave height in meters.


y = 0.9428x
R = 0.82
Hsmax = 0.2 (1013 - Po), in m
Largest Hs, in m, based on 68 hurricanes, from Abel et al (1989)
0
5
10
15
20
25


0900Z 12 20 1007mb
0800Z 6 40 993mb
0700Z 6 75 967mb
0600Z 12 125 929mb
0500Z 5 130 926mb
0400Z 8 120 933mb
0300Z 5 80 963mb
0200Z 3 45 989mb
Cheju
Naha
Chichi Jima
Iwo To
Kaohsiung
Hong Kong
Haiman
Nha Trang
Manila
Palau
Yap
Saipan
Guam
Toyama
Tokyo
35N
30N
25N
20N
15N
10N
110E
115E
120E
125E
130E
135E
140E
145E
ATCF

According to the Joint Typhoon Warning Center (see Fig. 11), on 6 October 2007, Super Typhoon Krosa was near northeastern Taiwan. The minimum sea-level pressure, P_o = 929mb. Substituting this value into Eq. (12), we get the maximum significant wave height to be approximately 17m.

Now, according to Liu et al. (2008), the maximum trough-to-crest wave height was measured to be 32.3m by a data buoy near northeast Taiwan in the western Pacific that was operating during the passage of Krosa. According to the World Meteorological Organization (1998), the maximum trough-to-crest wave height may be statistically approximated by 1.9 times the significant wave height. Therefore, the maximum significant wave height is 32.3/1.9 = 17m during Typhoon Krosa near NE Taiwan. This value is identical to the result using Eq. (12).

4. Estimating Maximum Wave Setup

According to Dean and Dalrymple (2002, page 84), the wave setup is a phenomenon that occurs primarily within the wave breaking zone and results a super elevation of the water level. According to Guza and Thornton (1981), the max wave setup, W_{setmax}, is approximately,

$$W_{setmax} = 0.17\ H_{smax} = 0.034\ (1013 - P_o) \qquad (13a)$$

$$W_{setmax}\ \text{(in feet)} = 0.11\ (1013 - P_o) \qquad (13b)$$

Where H_{smax} is the maximum significant wave height in deep-water before shoaling and P_o in mb.

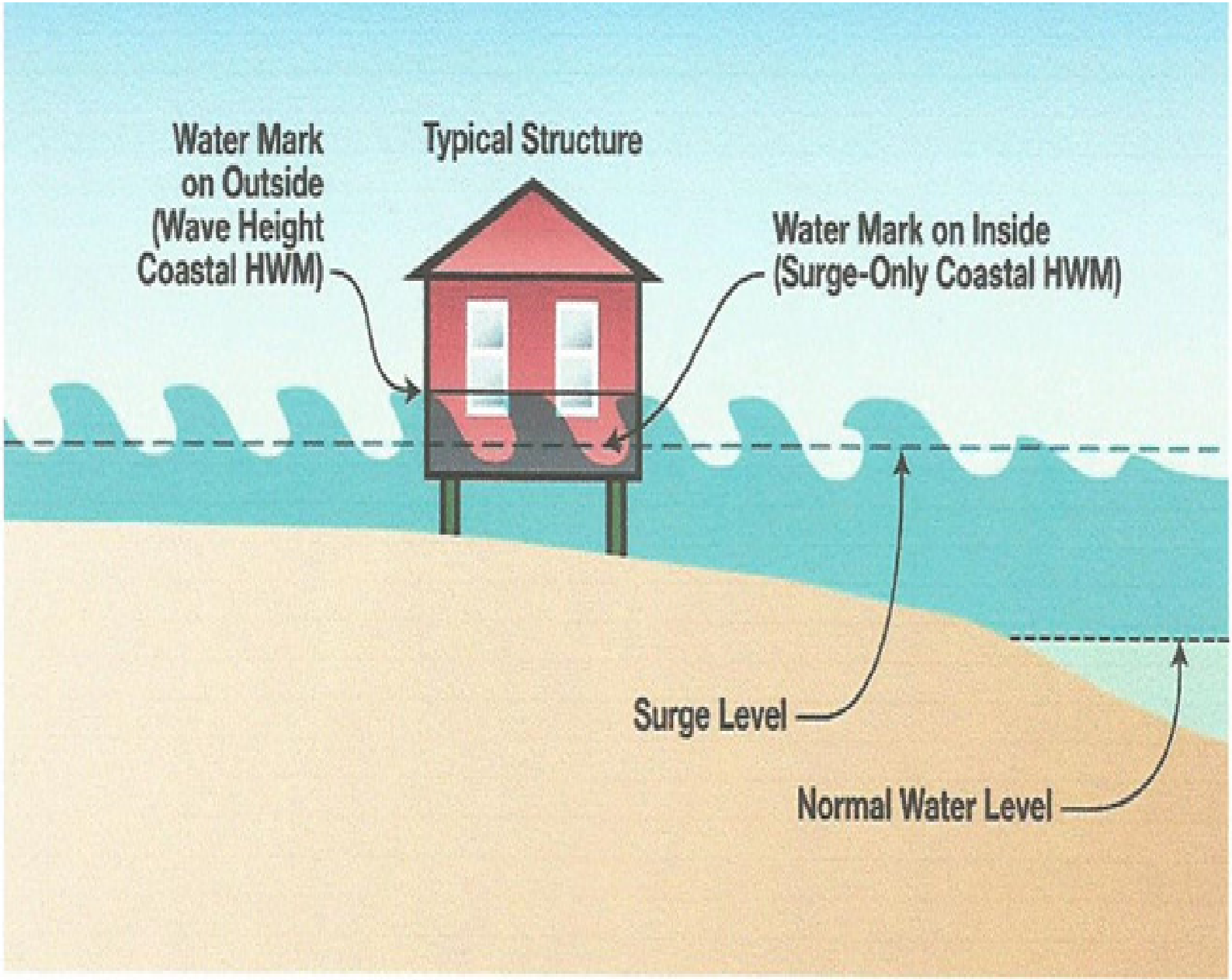


Figure 13 – Coastal HWM Resulting from Wave Height

During Ivan in 2004 and Katrina in 2005, values of H_{smax} are available from NDBC as shown in Figs. (12) and (13), respectively. Substituting the average value of 16m into Eq. (13a), the maximum wave setup was about 2.72m or 8.9ft. This value is in good agreement with ADCIRC modeling (see Douglass, 2006) (see Fig.14). Note that the value of 8ft for the wave setup has been used in wave force estimation for the failure of the Biloxi Bridge during Katrina (Fig.5) (see, e.g., McPherson (2008). For simplicity, it is illustrated as follows:

Since Force = pressure*area, we have

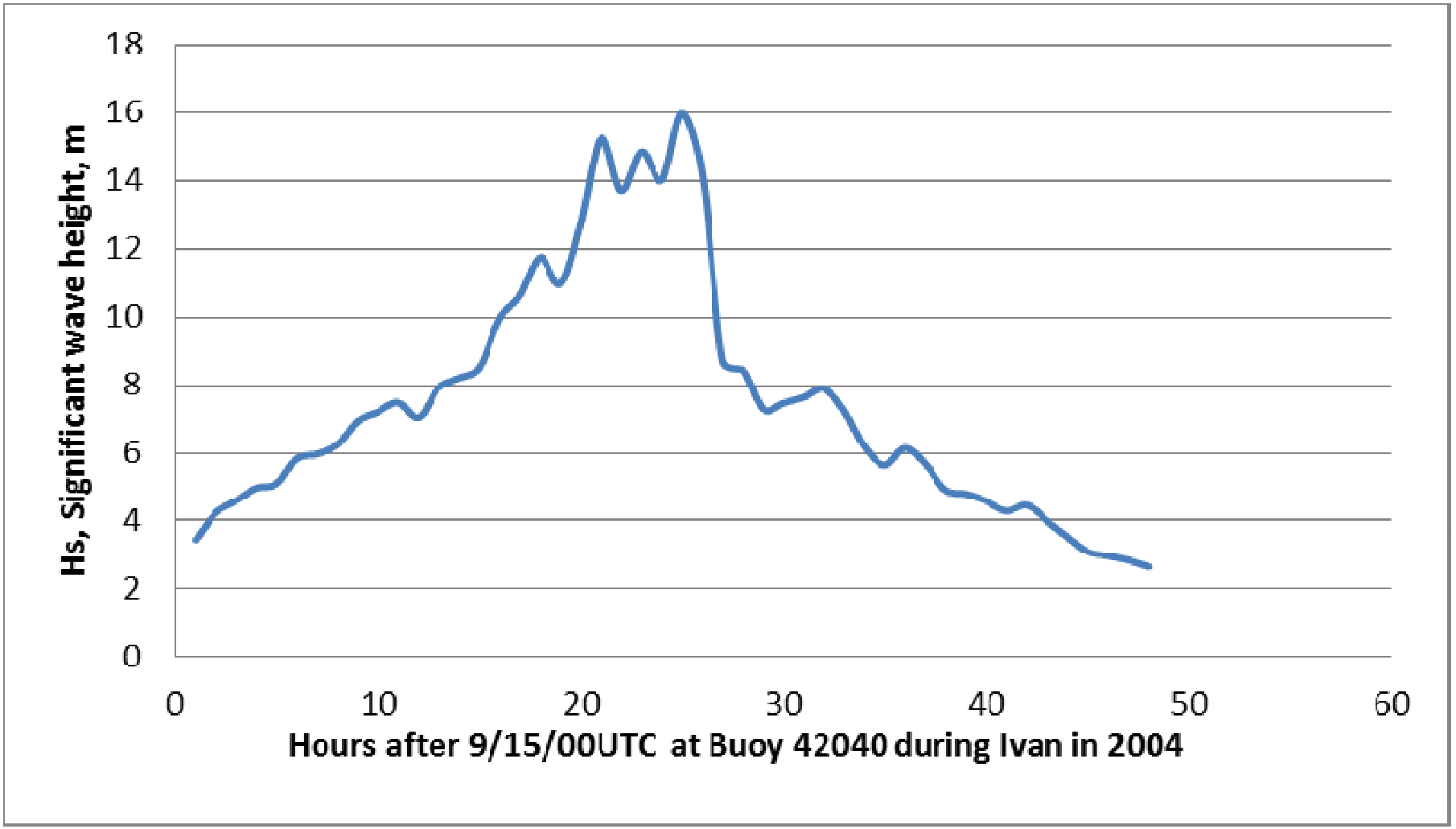
Force per unit area = pressure

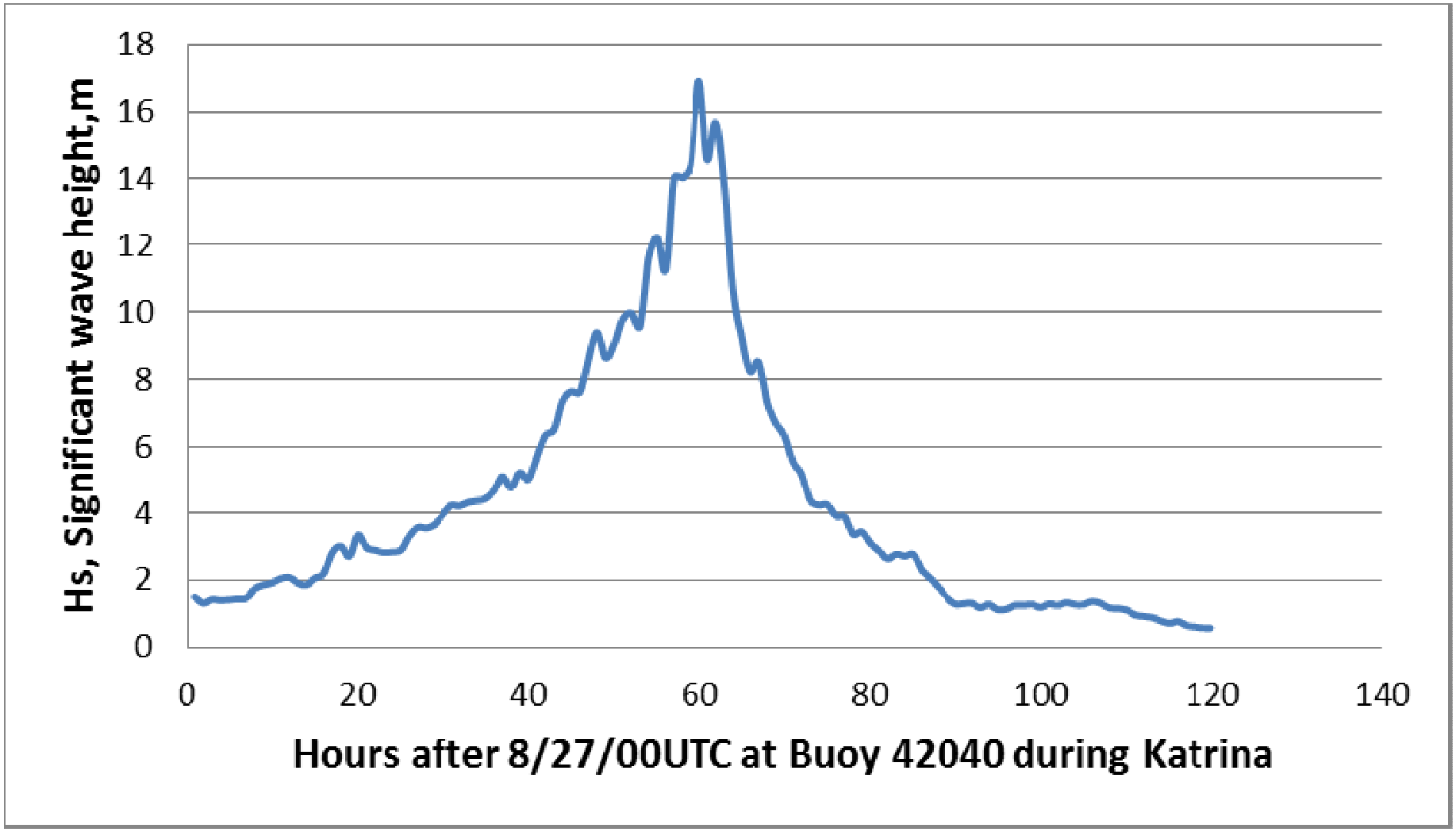
= density*gravitational acceleration*height

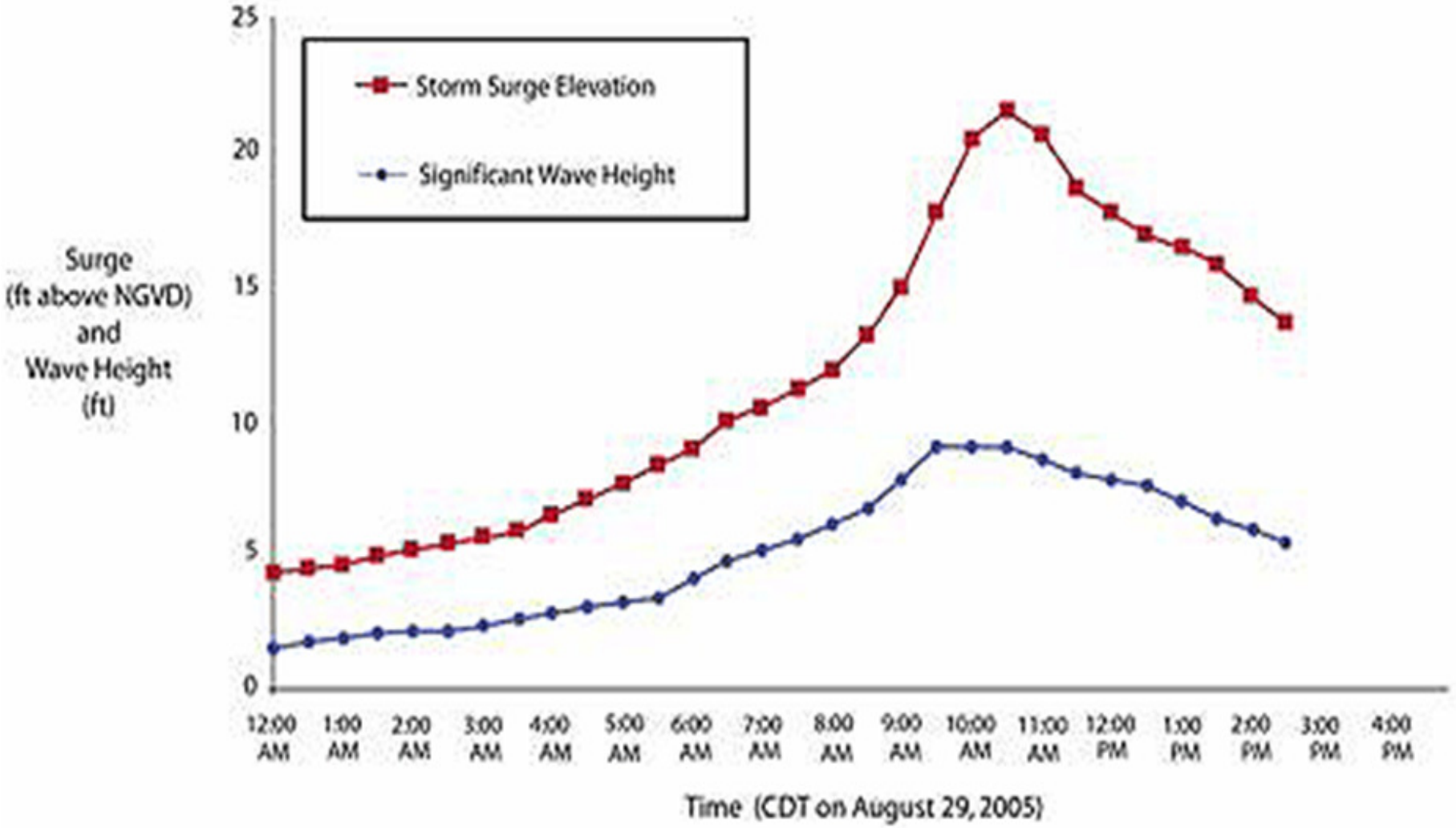
= unit (or specific) weight of water*height

= 62.4 (lb /ft^3)*W_{setmax} = 62.4*8 =499lb/ft^2.

Therefore, this 8ft wave setup can exert approximately 500 pound wave force per square foot impacted on the Biloxi Bridge during Katrina.


Hs, Significant wave height, m
18
16
14
12
10
8
6
4
2
0
0
10
20
30
40
50
60
Hours after 9/15/00UTC at Buoy 42040 during Ivan in 2004


Hs, Significant wave height,m
18
16
14
12
10
8
6
4
2
0
0
20
40
60
80
100
120
140
Hours after 8/27/00UTC at Buoy 42040 during Katrina


Storm Surge Elevation
Significant Wave Height
Surge
(ft above NGVD)
and
Wave Height
(ft)
25
20
15
10
5
0
12:00 AM
1:00 AM
2:00 AM
3:00 AM
4:00 AM
5:00 AM
6:00 AM
7:00 AM
8:00 AM
9:00 AM
10:00 AM
11:00 AM
12:00 PM
1:00 PM
2:00 PM
3:00 PM
4:00 PM
Time (CDT on August 29, 2005)

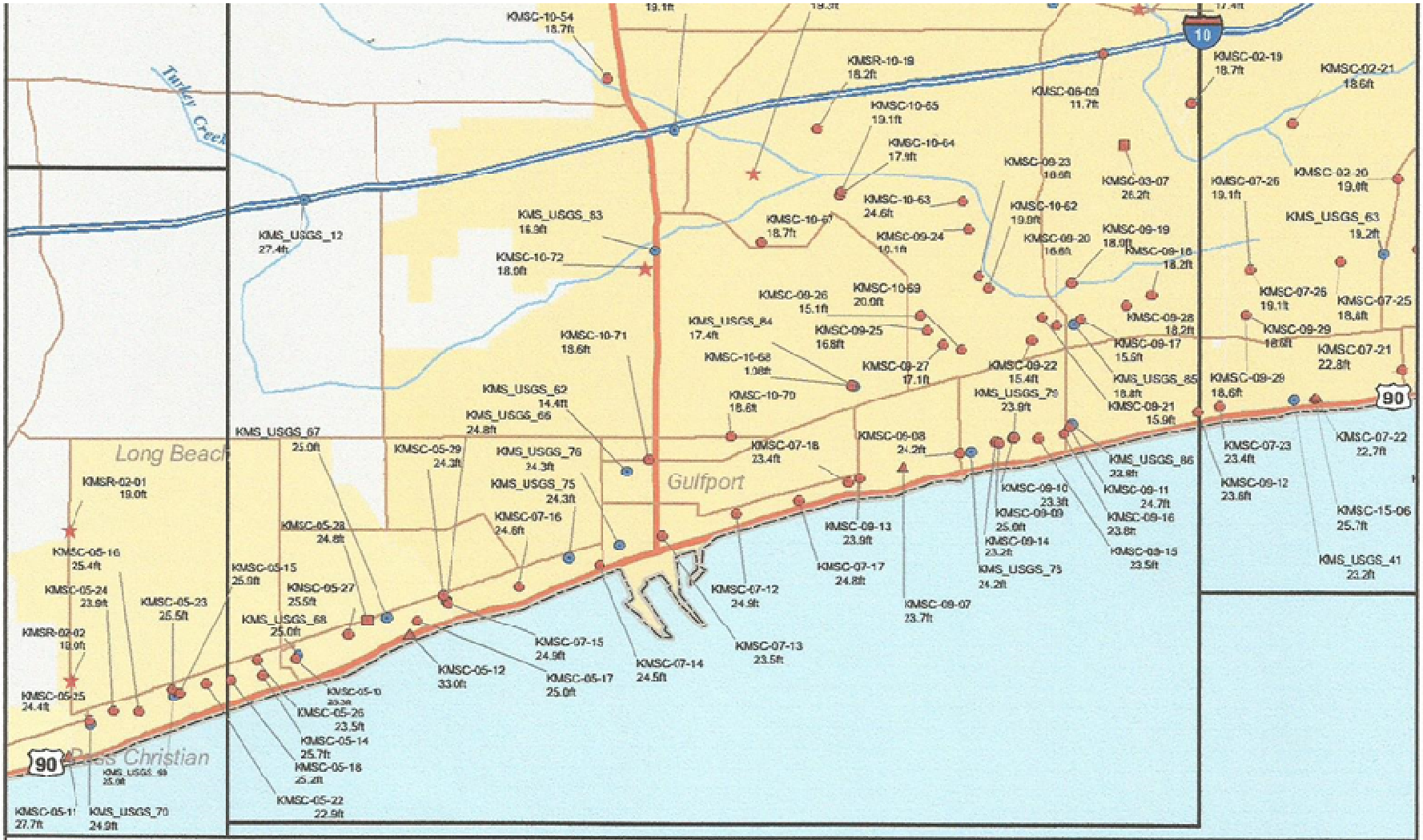
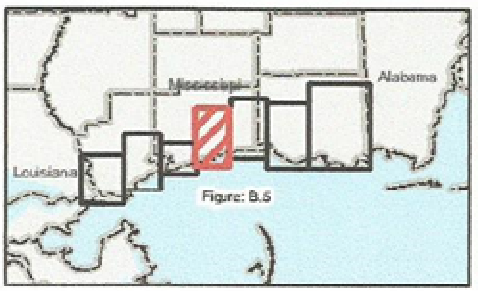


Figure: B.5
Harrison County, Mississippi
Hurricane Katrina
High Water Mark (HWM)
Surveyed Locations of High Water Mark Elevations
Final

Flooding Type

- Coastal - Surge Only
- Coastal - Wave Height
- Coastal - Wave Runup
- Riverine
- USGS High Water Marks
- City Limit
- County

VERTICAL DATUM: NAVD88
HORIZONTAL DATUM: NAD83





FEMA

5. Estimating Hurricane-generated Currents

According to Hsu (2003), the magnitude of surface drift velocity, U_{sea} is

$$U_{sea} = 0.22\ P\ U_{10} \qquad (14)$$

Where P is the turbulence intensity which is related to the gust factor, G, as follows:

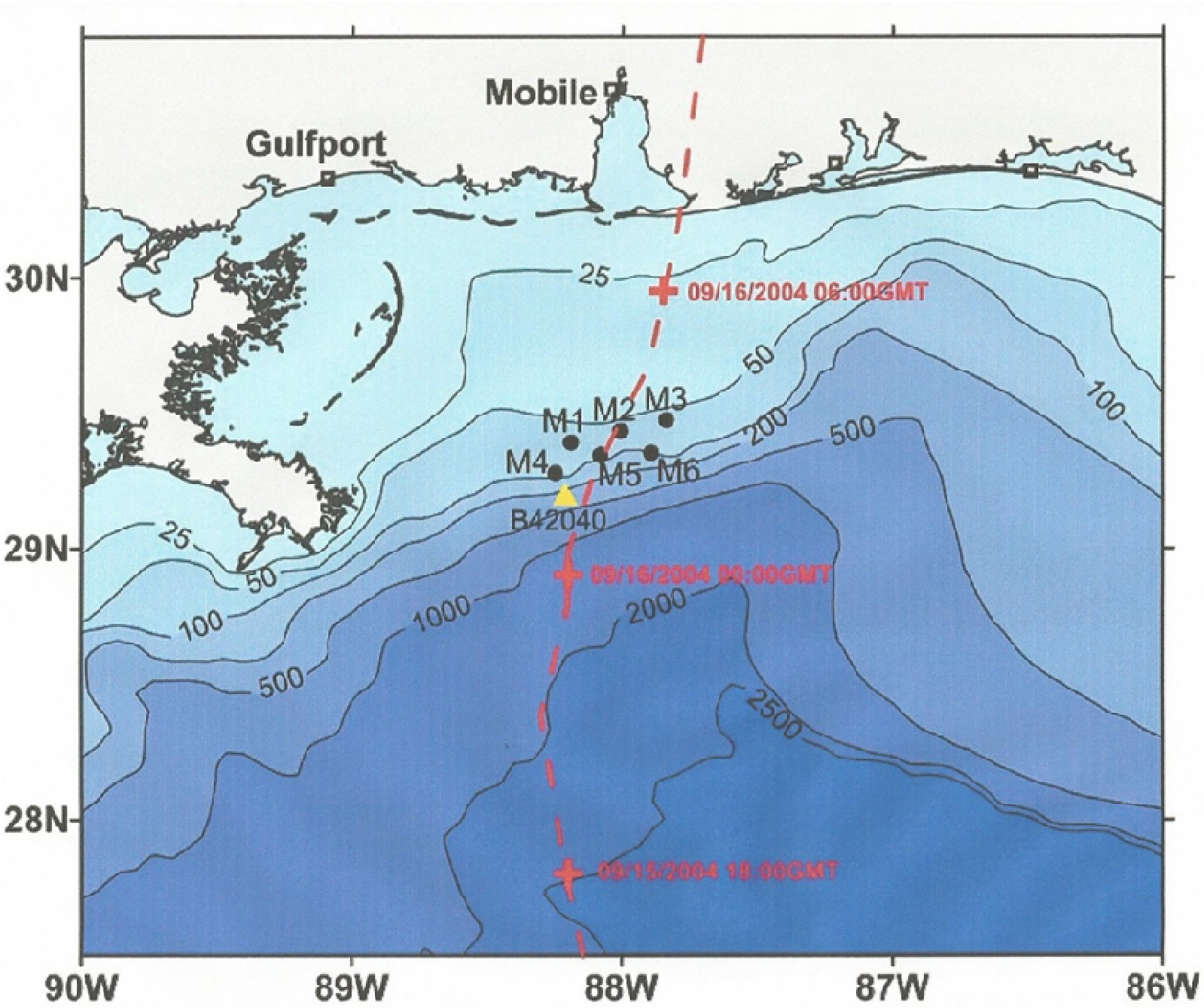
$$G = 1 + 2P \qquad (15)$$

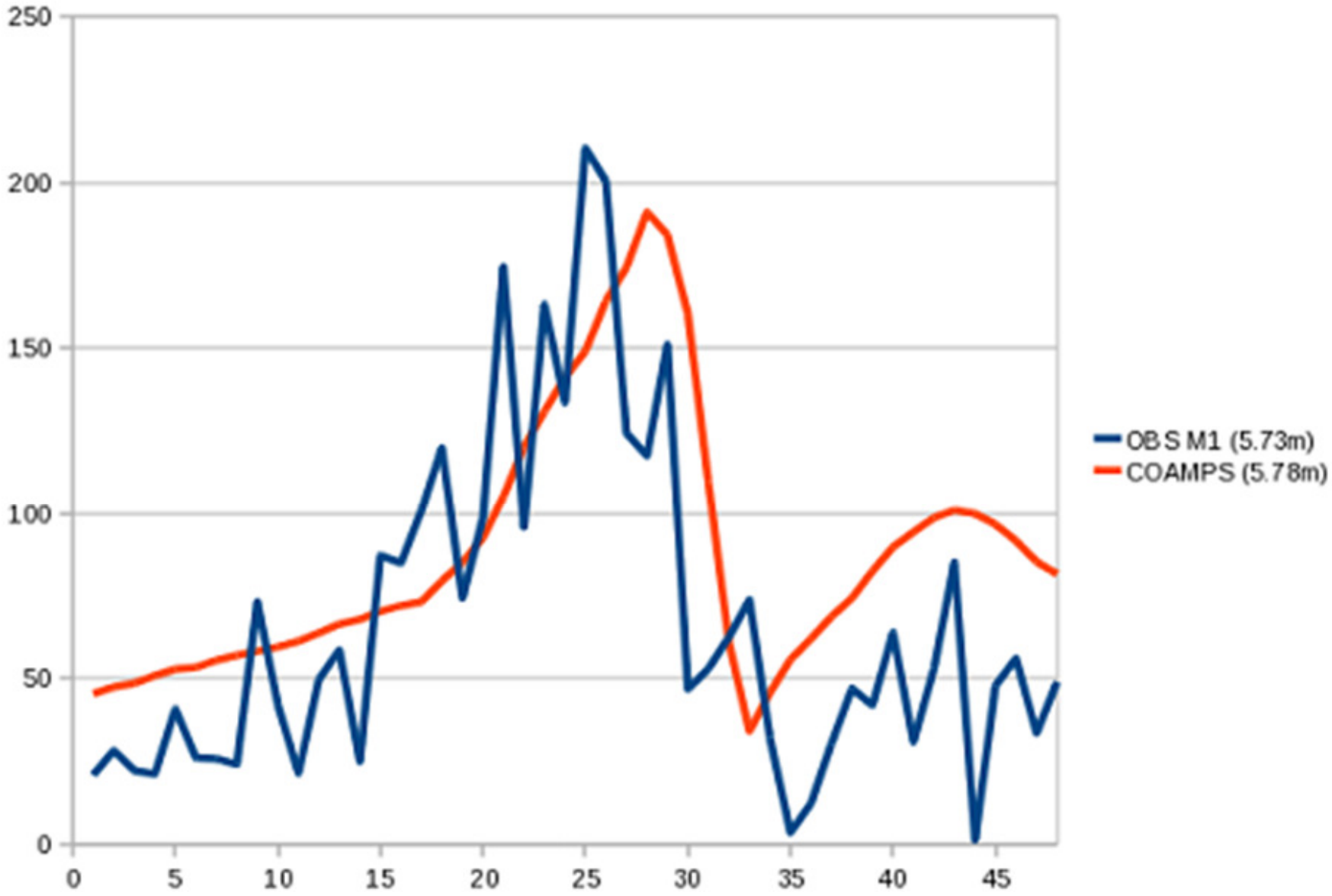
According to Stewart (2004) and Fig.15, during Ivan, G= 73kts/55kts = 1.327 at 10m at Buoy 42040 and G = 135kts/102kts = 1.324 at 122m at a nearby oil rig (NDBC station #42364, see www.ndbc.noaa.gov). Since the G values at 10 and 122m are nearly identical, we substitute either value into Eq.(15) and get P = 0.16. Substituting this P value into Eq. (14) and applying Eq. (3), we get

$$U_{sea} = 0.22\,(1013\text{-}P_o)^{0.5} \qquad (16)$$

Now, according to the Tropical Cyclone Report for Hurricane Ivan (see p.9 in Stewart, 2004 at www.nhc.noaa.gov), P_o = 931 mb. Substituting this value into Eq.(16), we have U_{sea} = 2.0 m/s.

Comparisons this value against both measurements and modeling results (Fig.16) show that Eq.(16) is consistent with both measurements and numerical modeling.


Mobile
Gulfport
30N
29N
28N
90W
89W
88W
87W
86W
25
50
100
200
500
1000
2000
2500
M1
M2
M3
M4
M5
M6
B42040
09/16/2004 06:00GMT
09/16/2004 00:00GMT
09/15/2004 18:00GMT


250
200
150
100
50
0
0
5
10
15
20
25
30
35
40
45
OBS M1 (5.73m)
COAMPS (5.78m)

6. Estimating Shoaling Depth

From Taylor and Yelland (2001) and Equations (7) and (12), the shoaling depth is

$$D_{shoaling} = 0.2\ L_p = 0.2\ gT_p^2/2\pi$$

$$= 4.7\ Hs = 4.7\ *0.2\ (1013 - P_o) \qquad (17)$$

Where L_p is the wave length.

Therefore,

Shoaling depth $\approx (1013 - P_o)$, in meters (18)

According to Wijesekera et al (2010, see www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA523020), during the passage of Ivan (see Fig. 15), the bottom stress was dominated by the wind-induced stresses, and exceeded critical levels at depths as large as 90 meters. Now, substituting P_o = 931mb into Eq. (18), we get that the shoaling depth was 82 m during Ivan. Since this estimate is consistent with the measurements, Eq. (18) may be useful as a first approximation.

7. Estimating Storm Surges

According to Hsu (2013), for estimating the storm surges caused by the wind-stress tide,

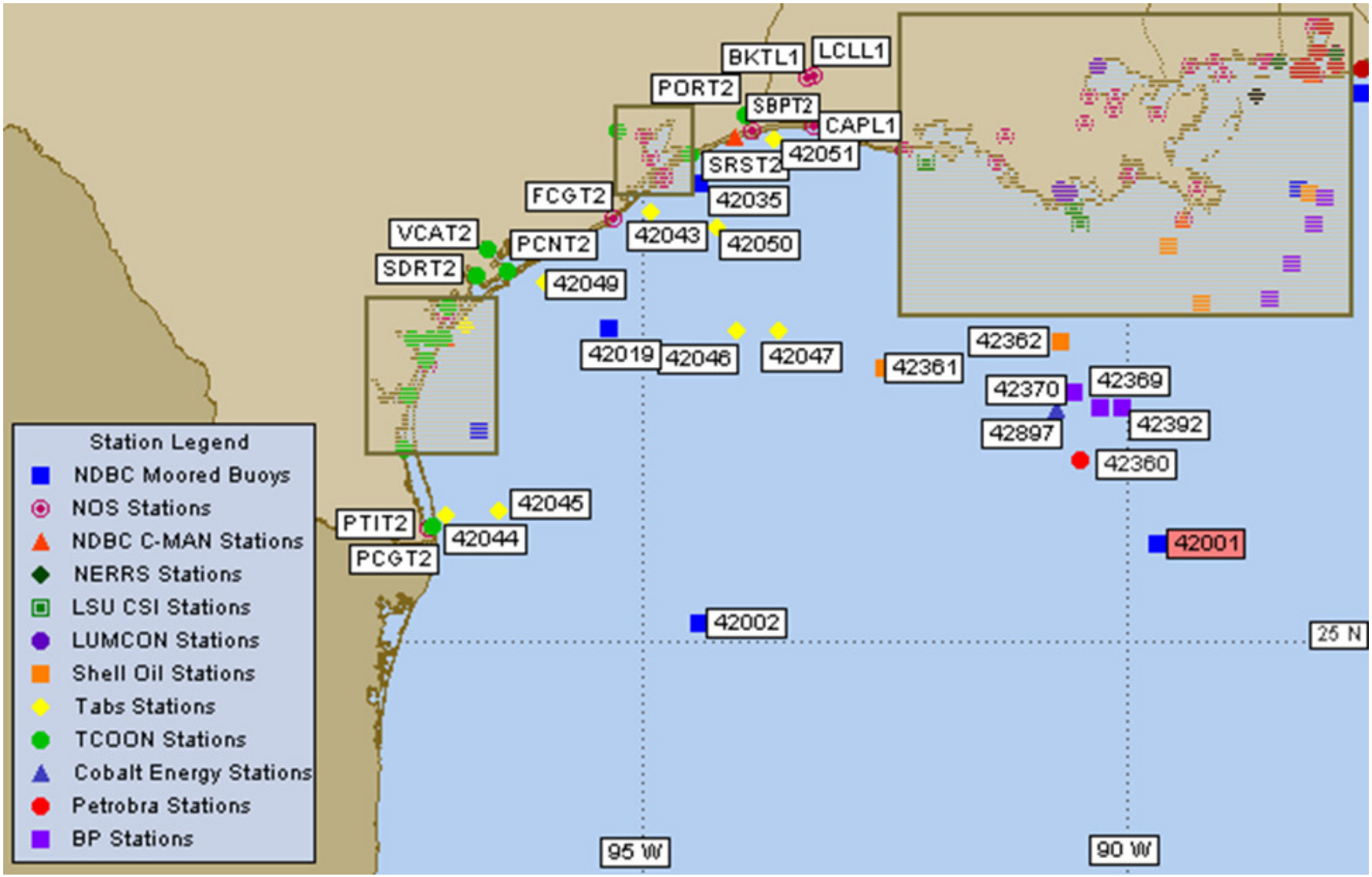
$$g\,D\,(dS/dx) = \tau_{sx}\,/\rho_w \qquad (19)$$

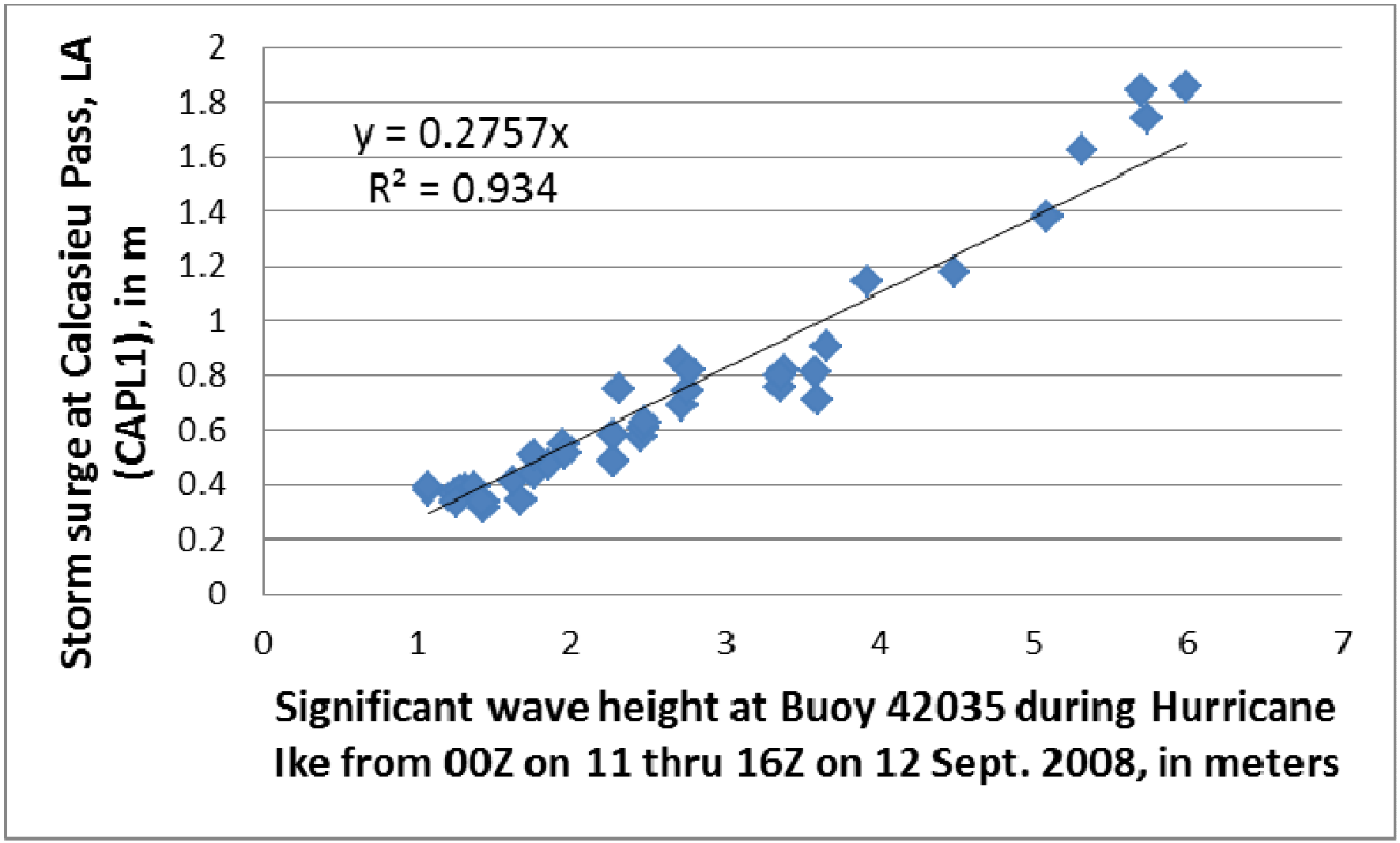
$$\tau_{sx} = \rho_a\,C_d\,V^2 \qquad (20)$$

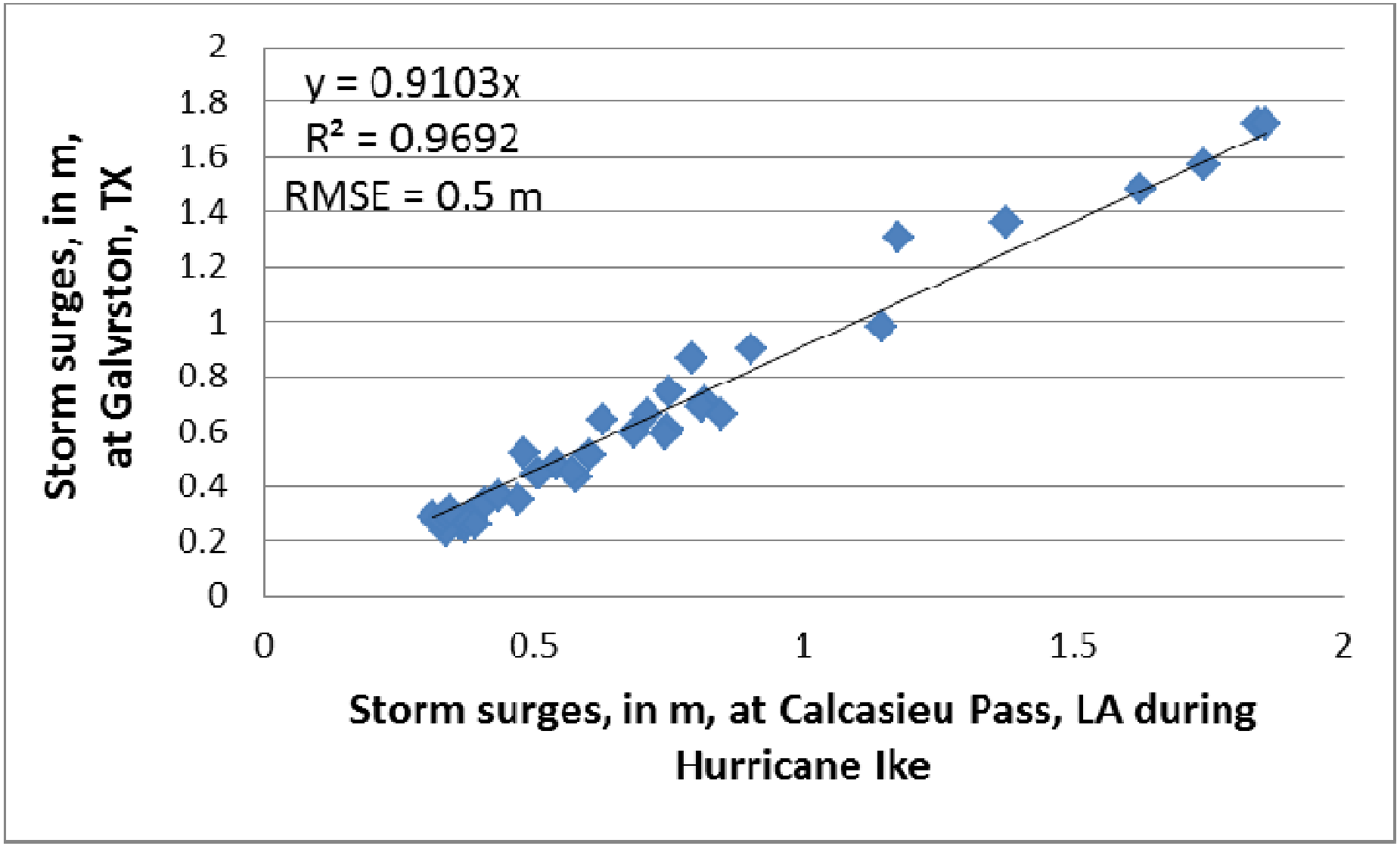
$$S - S_0 = [\rho_a\,C_d/(\rho_w\,g)](F/D)\,V^2 \qquad (21)$$

$$S = K_1\,V^2 = K_2\,(1013 - P_o) \qquad (22)$$

$$S = K_3\,H_s \qquad (23)$$


BKTL1
LCLL1
PORT2
SBPT2
CAPL1
42051
SRST2
42035
FCGT2
42043
42050
VCAT2
PCNT2
SDRT2
42049
42019
42046
42047
42362
42361
42370
42369
42392
42897
42360
42045
PTIT2
42044
PCGT2
42001
42002
25 N
95 W
90 W
Station Legend
NDBC Moored Buoys
NOS Stations
NDBC C-MAN Stations
NERRS Stations
LSU CSI Stations
LUMCON Stations
Shell Oil Stations
Tabs Stations
TCOON Stations
Cobalt Energy Stations
Petrobra Stations
BP Stations

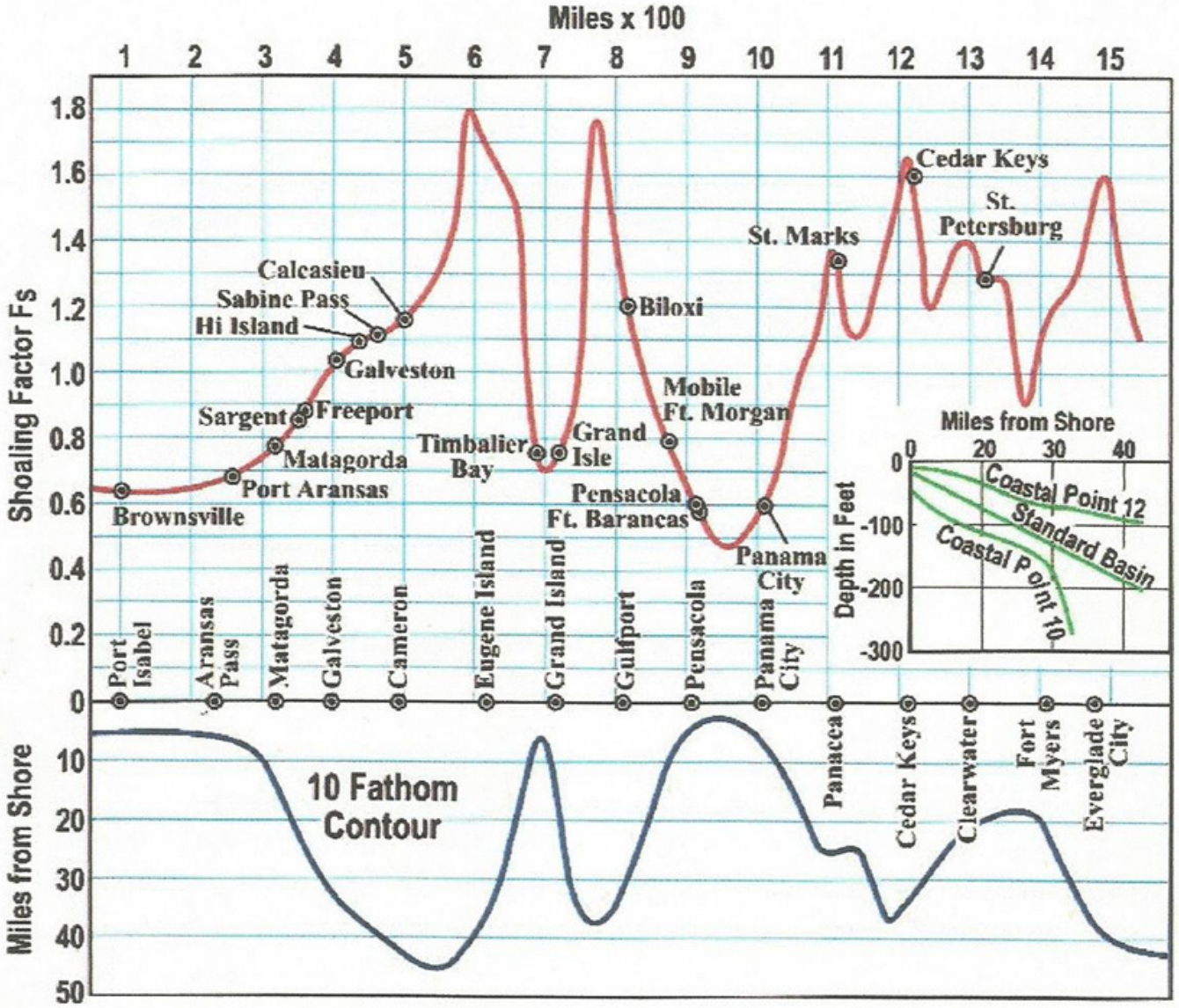

Storm surge at Calcasieu Pass, LA (CAPL1), in m
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
y = 0.2757x
R² = 0.934
0
1
2
3
4
5
6
7
Significant wave height at Buoy 42035 during Hurricane Ike from 00Z on 11 thru 16Z on 12 Sept. 2008, in meters


y = 0.9103x
R² = 0.9692
RMSE = 0.5 m
Storm surges, in m, at Galvrston, TX
Storm surges, in m, at Calcasieu Pass, LA during Hurricane Ike
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.5
1
1.5
2

Maximum storm surge elevation without wave setups, S, can also be estimated analytically (see Hsu, 1988 and 2004, and Hsu et al., 2006) as

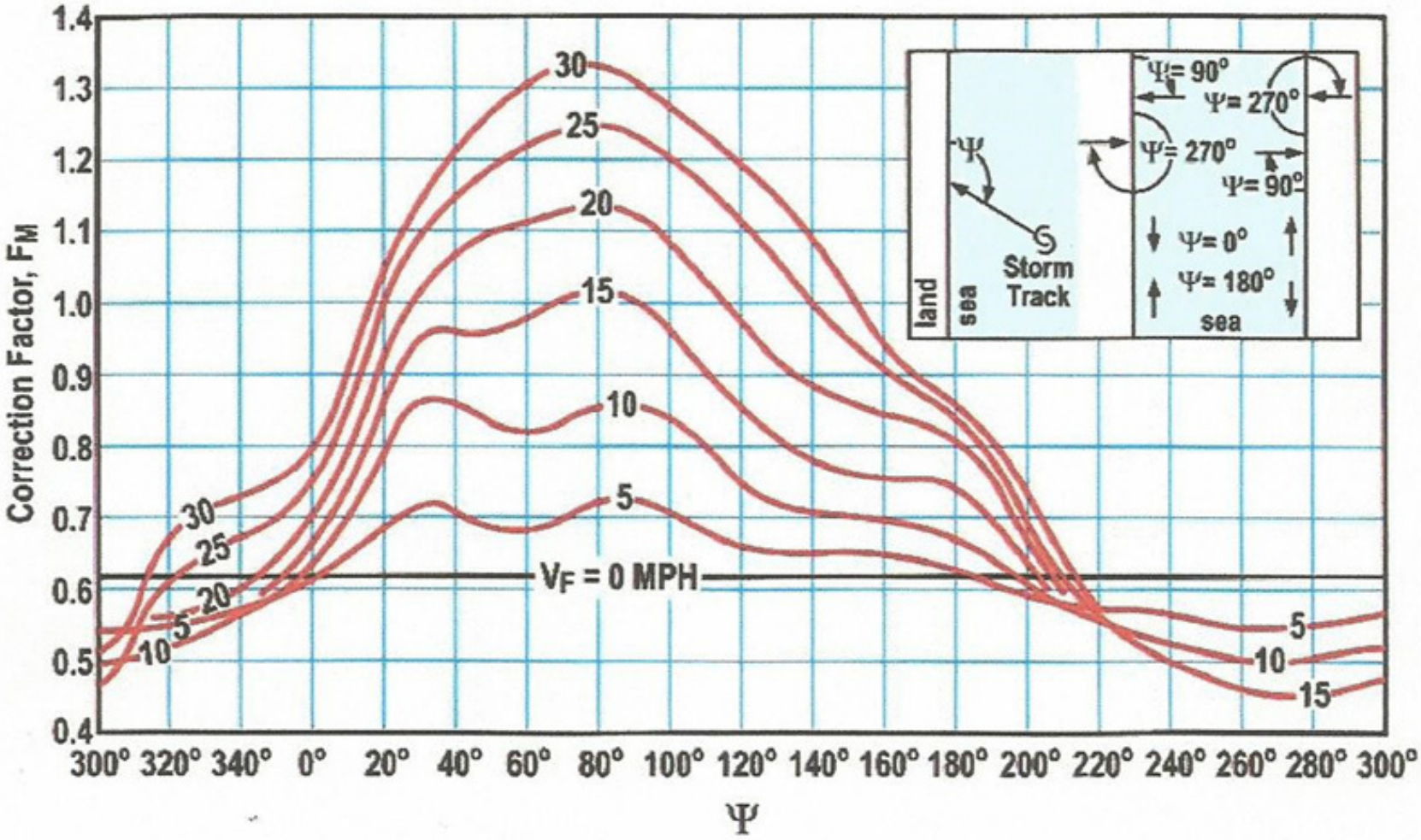
$$S \text{ (in feet)} = 0.23*(1010 - P_o)* F_s* F_m \qquad (24)$$

Where P_o is the minimum sea-level pressure in mb, F_s is a shoaling factor (see Fig. 20), and F_m is a correction factor for storm motion (see Fig. 21).



Shoaling Factors on Gulf Coast

(From Jelesnianski, 1972)



Correction Factor for Storm Motion

(From Jelesnianski, 1972)

According to the Tropical Cyclone Report for Hurricane Katrina issued by the National Hurricane Center (NHC) (see http://www.nhc.noaa.gov/pdf/TCR-AL122005_Katrina.pdf). The lowest pressure was 927.4mb (see NHC, Page 32) recorded at Buoy 42007, which was located about 25 miles due south of Biloxi.

Now, substituting P_o = 927.4mb, F_s =1.2 for Biloxi, MS, and F_m =1.0, according to the NHC Advisories at the time of Katrina landfall near LA/MS border, which was approximately 15 mph, into Eq. (24), we have

$$S = 0.23^* (1010 - 927.4)^*1.2^*1.0 = 23 \text{ feet.}$$

Since this value is in excellent agreement with the results of ADCIRC modeling (Fig. 14) and high-water mark survey by FEMA (2006), we can say that Eq. (24) is useful for practical use.

8. Estimating the Stress on Seabed

According to Wijesekera et al (2010, see www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA523020), strong surface waves and currents generated by major hurricanes can produce extreme forces at the seabed that scour the sea floor and cause massive underwater mudslides. The combined current-wave stress, $\tau_{cw,}$ on the sea floor is approximately related to the wind stress, U^2, so that from Eq. (3), we have

$\tau_{cw} = 0.0004\ U_{10}\text{^}2 = 0.016\ (1013 - P_o)$ (25)

Note that the units of bottom stress are N/m^2 or Pa and P_o is in mb.

Table1. Critical stress thresholds for sand mixtures of select median grain sizes following Soulsby (1997):

Median Grain Size (d50, mm)	Median Grain Size (d50, Phi)	Critical Stress (Pa)
2.00	-1.0	1.17
1.00	0.0	0.48
0.50	1.0	0.26
0.25	2.0	0.19
0.13	3.0	0.15
0.06	4.0	0.12

The critical bottom stress to initiate the sediment movement is provided in Table 1. It can be seen that for the median grain sand of 0.06 mm and finer ones, a tropical storm force (P_o = 1005mb, approximately) wind can start these sands in motion at water depth shallower than 8 m according to Eq.(18). Now, on the basis of Eq. (25) and Fig.15, the bottom stresses caused by Ivan (when P_o = 931mb) and Katrina (P_o =927mb) could have exceeded 1.31 and 1.38 Pa, respectively, more than 10 times of the critical bottom stress needed to set the sediment in motion.

These estimates may be used to explain massive sediment transport near the seabed shallower than 80-90m that in turn caused numerous structural failure and pipeline displacements due to strong near-bottom orbital wave velocity (>2m/s) and near-bottom currents ranged from 0.40 to 1.20 m/s at all moorings (see Fig.15) during Ivan's passage (Teague et al., 2006).

9. Conclusions

On the basis of aforementioned analyses, during a tropical cyclone, several air-sea-land interaction parameters can be estimated rapidly using the minimum sea-level pressure (P_o, in mb) as the most important input.

They are:

(1). Maximum wind speed (in m/s) = $6.3\ (1013 - P_o)^{0.5}$.

(2). Max significant wave height (in m)

$= 0.20\ (1013 - P_o)$.

(3). Max wave setup (in feet) = 0.11 (1013 – P_o).

(4). Max surface drift velocity (in m/s)

= 0.22 (1013 – P_o) $^{0.5}$.

(5). Most probable shoaling depth (in m)

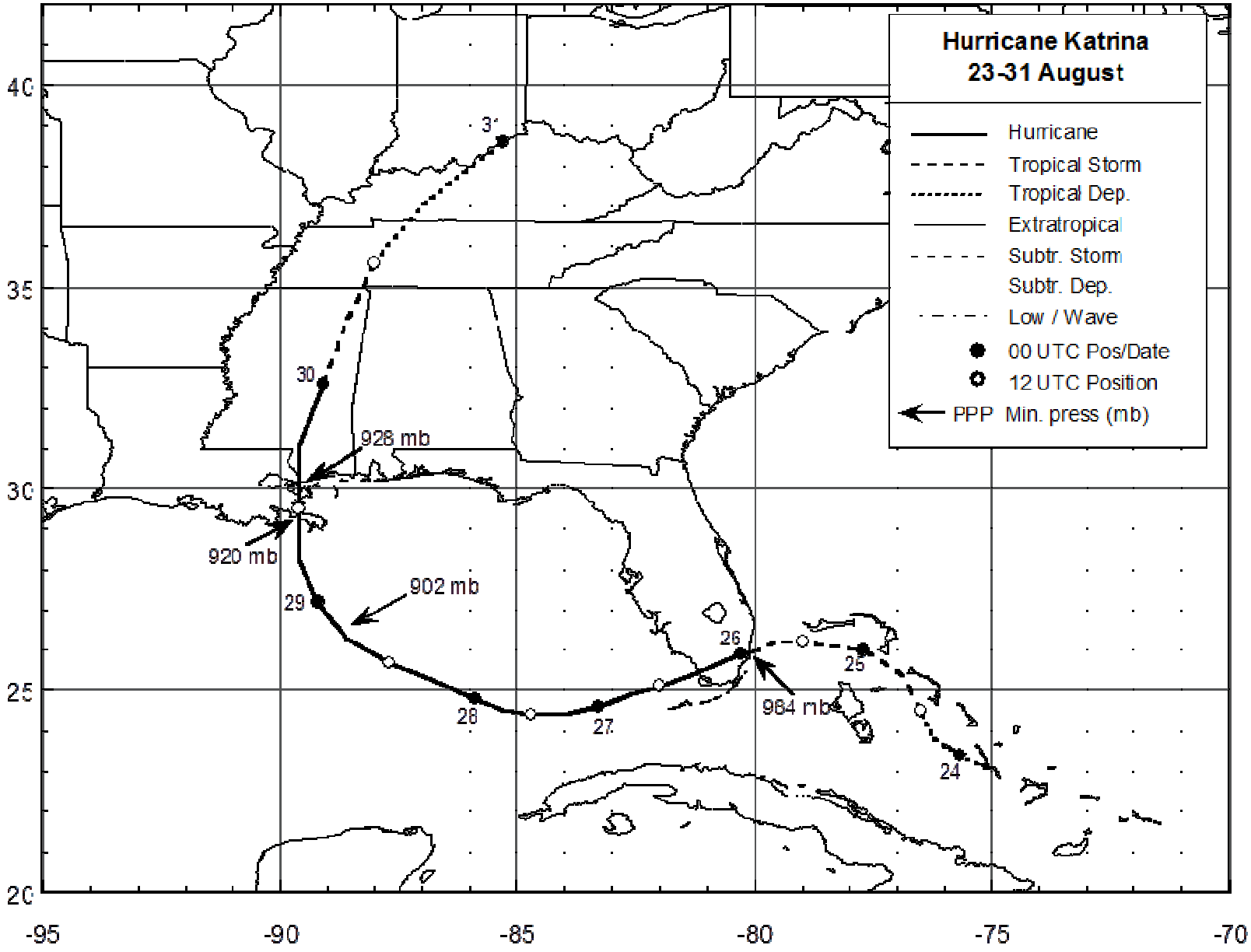
= (1013 – P_o).

(6). Max storm surge (in feet)

= 0.23*(1010 – P_o)*F_s*F_m,

(7). Max bottom (seabed) stress (in N/m^2)

= 0.016 (1013 – P_o).


Hurricane Katrina
23-31 August
Hurricane
Tropical Storm
Tropical Dep.
Extratropical
Subtr. Storm
Subtr. Dep.
Low / Wave
00 UTC Pos/Date
12 UTC Position
PPP Min. press (mb)
40
35
30
25
20
-95
-90
-85
-80
-75
-70
31
30
928 mb
920 mb
29
902 mb
28
27
26
984 mb
25
24

Now, using Katrina as an example and application (see Fig.22 and Fig.1), by setting P_o = 902mb, we have, from (1) above, max wind speed = 66 m/s = 148 mph, and (2), max significant wave height = 22.2m = 73ft. Referring back to Fig.2 and 3, since both wind speed and wave height as estimated exceeded the designed limits (140 mph winds and 70ft wave height), the designed criteria for the Gulf of Mexico need to be re-examined as suggested by many engineers (see, e.g. Cruz and Krausmann, 2008). In addition, both storm surge and wave setup estimations are verified by the high water mark survey by FEMA (see FEMA, 2006).