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Hawking Radiation from Non-stationary Rotating de Sitter Black Hole

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Introduction

🕗Classical theory:

Nothing including light can ever escape from a black hole.

➢ R. Penrose (Riv. Nuovo Cimento **1**, 1969, 252):

③ A particle entering into the ergosphere of a rotating black hole disintegrates into two particles –

(i) A positive energy particle – escapes to infinity with an energy greater than that of the original particle,

(ii) A negative energy particle w.r.t. an observer at infinity (but positive locally) – tunnels inside the black hole.

- Rotational energy and angular momentum of the black hole get extracted in the process
- The positive and negative energy states are the classical correspondents of the positive and negative energy states of a quantized field; level crossing might occur inside the ergosphere leading to possible pair creation and discharge from the black hole. (Deruelle and Ruffini, Phys. Lett. B., 52B, 1974, 437)

S. W. Hawking (Nature, **248**, 1974, 30; Comm. Math. Phys. 43, 1975, 199):

- Black holes emit particles and the energy spectrum of the emitted particles is thermal
- Hawking radiation may be considered to be produced by vacuum fluctuations near the event horizon.

- A pair of particles is created just outside the horizon –

(i) the negative energy virtual particle tunnels into the black hole

(ii) the positive energy virtual particle escapes to infinity where it constituting a part of the thermal emission.

- Hawking radiation may also be considered as tunneling of particles across the event horizon. And the imaginary part of the action of the tunneling particle is related to the Boltzman factor for emission at the Hawking temperature. (Hartle & Hawking, Phys. Rev. D 13, 1976, 2188; Kraus & Wilczek, Nucl. Phys. B 433, 1995, 403; Parikh & Wilczek, Phys. Rev. Lett. 85, 2000, 5042; (Srinivasan & Padmanabhan, Phys. Rev. D 60, 1999, 24007; Angheben et al. JHEP 05, 2005, 014; Kerner & Mann, Class. & Quant. Grav. 25, 2008, 095014; 73, 2008, 104010)

- Non-zero and positive cosmological constant:

  Studies of type Ia supernova (Perlmutter et al. 517, 565, 1999) and also of the anisotropy of the cosmic microwave background radiation suggest a non-zero, positive cosmological constant.

Non-stationary rotating de Sitter solution

- Line element:

$$ds^2 = \left[1 - \frac{\Lambda(u) r^4}{3R^2}\right] du^2 + 2 du dr$$

$$+ \frac{2a\Lambda(u) r^4}{3r^2} \sin^2\theta \, du d\phi$$

$$-2a \sin^2\theta \, dr d\phi - R^2 d\theta^2$$

$$-\frac{}{2}[(r^2 + a^2)^2 - \otimes \quad a^2 \sin^2\theta] R^{-2} \sin^2\theta \, d\phi$$

--------- (1)

where

$$\otimes \qquad = r^{2} \frac{}{\Lambda(r)r} + a^{2}$$

$$R = r + ia\cos\theta$$

u is the retarded time coordinate.

- Introduce the generalized tortoise coordinate transformation (GTCT):

$$r_* = r + \frac{1}{2\kappa}\ln\frac{[r - r_h(u,\theta)]}{r_h(u,\theta)} \qquad \text{---------- (2)}$$

$$u_* = u - u_o$$

$$\theta_* = \theta - \theta_o$$

- Null surface condition:

$$g^{\mu\nu}\partial_{\mu}F\partial_{\nu}F=0 \qquad \text{--------- (3)}$$

In the limit: $r \to r_h(u_o,\theta_o), \quad u \to u_o, \quad \theta \to \theta_o$

$$\otimes_h \quad \dot{r}_h \quad \dot{r}_h - \dot{r}_o \dot{r}_h + r'^2_h = 0 \qquad \text{--(4)}$$

$$+ \quad 2(r_h^2 + a^2)\dot{r}_h + a^2 \sin^2 \theta_h \frac{\partial r_h}{\partial u}, \quad r'_h - \frac{\partial r_h}{\partial \theta}$$

where

- Four roots:

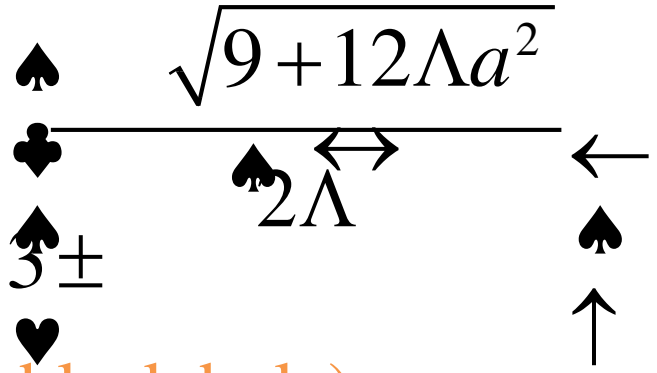
$$r_h = +\sqrt{\frac{\Upsilon}{2\Lambda(u)}\left(\dot{r}_h + \sqrt{W}\right)} \qquad \text{------- (5)}$$

where

$$W = 9 + 12a^2\Lambda(u) + 3H$$

$$H = 12\dot{r}_h(1+\dot{r}_h) + 4\Lambda(u)\left(\dot{r}^2 + 2a^2\dot{r}_h + a^2\dot{r}_h^2\sin^2\theta'_{\infty}\right)$$

- When $\dot{r}_h = 0 = r_h' :$ $r_h = \pm \dfrac{\sqrt{9+12\Lambda a^2}}{2\Lambda}$



(horizons of stationary de Sitter black hole)

- When $a = 0 :$ $r_h = \pm\sqrt{\dfrac{3}{\Lambda}}$

(horizons of stationary non-rotating de Sitter black hole)

Hamilton Jacobi Equation

- A minimally coupled scalar field ϕ with mass μ satisfies

$$\left(\Box - \frac{\mu^2}{\hbar^2}\right)\Phi = 0 \qquad \text{------- (6)}$$

- Putting $\Phi = \exp\left(\frac{iS}{\hbar}\right)$ where S is the action

functional, we get the Hamilton-Jacobi equation satisfied by a article of mass μ:

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^{\mu}}\right)\left(\frac{\partial S}{\partial x^{\nu}}\right)-\mu^{2}=0 \qquad \text{------- (7)}$$

- Using the Generalised Tortoise Coordinate Transformation

$$A\left(\frac{\partial S}{\partial r_{*}}\right)^{2}-4\kappa(r-r_{h})r_{h}B\left(\frac{\partial S}{\partial r_{*}}\right)+\{2\kappa(r-r_{h})r_{h}\}^{2}C=0 \qquad \text{------- (8)}$$

where

$$A = a^2 \sin^2\theta \dot{r}_h^{\,2} r^2 + 2(a^2 + r^2) r_h r \dot{r}_h \{2\kappa(r - r_h) + 1\}$$

$$+ \otimes \quad r^2 \{2\kappa(r - r \;) + 1\}^2 + r$$

$$^2 r'^2;$$

$$h \qquad h \qquad h$$

$$B = -a^2 \sin^2\theta \omega r_h r - (a^2 + r^2) r_h \omega \{2\kappa(r - r_h) + 1\}$$

$$+ amr_h' r + amr_h \{2\kappa(r - r_h) + 1\} + lrr_h';$$

$$C = a^2 \sin^2\theta\omega^2 - 2am\omega + l^2 + m^2 \sin^2\theta$$

$$+\mu^2(r^2 + a^2\cos^2\theta).$$

$$= -\frac{\partial S}{\partial u_*}, \quad \text{(energy of the particle)}$$

$$l = \frac{\partial S}{\partial \theta_*}, \quad \text{(angular momentum of the particle)}$$

$$\left(\frac{\partial}{\partial \phi}\right)^\alpha = m, \quad \text{(a Killing vector)}$$

From Eq. (8):

$$\frac{\partial S}{\partial r_*} = 2\kappa(r - r_h) r_h \leq \frac{\Upsilon \; B \pm \sqrt{B^2 - AC}}{A} f$$

Using the GTCT, the above equation reduces to

$$\frac{\partial S}{\partial r} = r_h \{2\kappa(r - r_h) + 1\} \leq \frac{\Upsilon \; B \pm \sqrt{B^2 - AC}}{A} \qquad \text{----- (9)}$$

- For regions outside the event horizon, the contribution to action is real:

$$B^2 - AC \geq 0$$

$$\Rightarrow F_1\omega^2 + 2F_2\omega + F_3 \geq 0 \qquad \text{------- (10)}$$

where

$$F_1 = \frac{\Upsilon}{\rho^2}\left[a^2\sin^2\theta\,\dot{r}_h\, r + (a^2+r^2)\,r_h\left\{2\kappa(r-r_h)+1\right\}^{1/2}\right]$$

$$\leq \oint$$

$$-A^2a^2\sin^2\theta$$

$$F_2 = \frac{\Upsilon}{\leq} -a^2 \sin^2\theta \dot{r}_h r - (a^2 + r^2) r_h \{2\kappa(r - r_h) + 1\} \frac{'}{\infty} \times$$

$$[am\dot{r}_h r + amr_h \{2\kappa(r - r_h) + 1\} + lrr_h' + 2Aam]$$

$$F_3 = \frac{\Upsilon}{'} \quad am\dot{r}_h r + amr_h \{2\kappa(r - r_h) + 1\} + \frac{'^2}{\infty}$$

$$lrr_h$$

$$+ A\{l^2 + m^2 \sin^2\theta + \mu^2(r^2 + a^2 \cos^2\theta)\} \quad f$$

➢ Using equality, the solutions are:

$$\omega = \omega^{\pm} = \frac{-F_2 \pm \sqrt{F_2^2 - F_1 F_3}}{F_1} \qquad \text{------ (11)}$$

➢Energy states of particles in the vicinity of the event horizon satisfy:

$$\omega \geq \omega^{+} \qquad \text{or} \qquad \omega \leq \omega^{-}$$

- Thus, there exist seas of positive and negative energy states of particles close to the event horizon.
- There exists a forbidden energy gap between the seas.
- Width of the forbidden energy gap is:

$$\delta\omega = \omega^{+} - \omega^{-} = \frac{2\sqrt{F_2{}^2 - F_1 F_3}}{F_1} \qquad \text{------ (11)}$$

➢ At $r = r_h$, $\otimes$ ω

- The width of the forbidden energy gap vanishes on the surface of the event horizon.

- The positive and negative energy particles might coexist on the surface of the event horizon.

➢ Energy of a particle on the surface of the event horizon:

$$o = \frac{am + am\,\dot{r}_h + r_h' l}{a^2 \sin^2 \theta_o + (r_h^2 + a^2)} \qquad (12)$$

- The width of the forbidden energy gap and the energy of the particle on the surface of the event horizon depends on:

- the cosmological constant, Λ;
- position of the particle;
- angular momenta of the particle and of the black hole;
- evaporation rate;
- shape of the event horizon.

$$\frac{\partial S}{\partial r} = r_h \left\{2\kappa(r - r_h) + 1\right\} \frac{B \pm \sqrt{B^2 - AC}}{A} f \qquad \text{----- (9)}$$

$$S(r_2, t_2; r_1, t_1) = \int_{r_1}^{r_2} r_h \{2\kappa(r - r_h) + 1\} \frac{[B \pm \sqrt{B^2 - AC}]}{A}$$

- For a non-rotating de Sitter black hole and for μ=0 at θ=0,

$$S = \int_{r_1}^{r_2} \omega r \left[\frac{1}{r - r_h} \right]$$

- The imaginary part of the action is given by

$$\mathrm{Im}\, S[emission] = i\pi r_h \omega$$

- The tunneling probability is

$$\Gamma \propto \exp\left[-\frac{2\pi r_h \omega}{\square}\right]$$

$$\Rightarrow T = \frac{1}{2\pi r_h}$$

(standard Hawking temperature of de Sitter black hole)

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