The 1st International Conference on Smart Grid Technologies

Keynote Presentation

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Solving Stochastic Optimization by Shaping the Probability Density Function of Cost Functions and Constraint Equations

HONG WANG, SHAOBU WANG AND ZHENYU HUANG

EMAIL: HONG.WANG@PNNL.GOV

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U.S. DEPARTMENT OF ENERGY

Outline

- Power grid as complex systems – challenges and characteristics
- Equivalence between optimization and control
- Source of uncertainties and their processing
- Probability density function shaping based optimization – a generic solution

Background

- *Complex industrial systems (steel making, power systems, petro-chemical, papermaking, etc) are important sectors in the world economy*
- *There are two issues in their operation :*
  - Operational quality
  - Operational safety

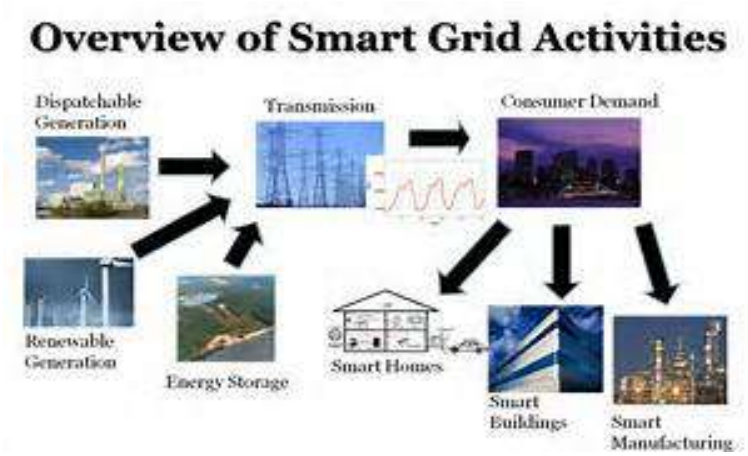


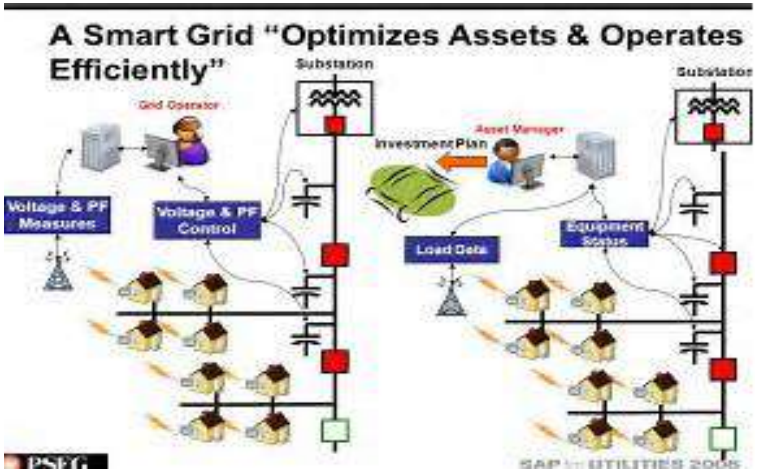


Both these issues are realized via Automation Science

2. Problems – smart grid case

- An ecosystem of asset owners, manufacturers, service providers, etc
- Reduce the frequency and duration of power outages, reduce storm impacts, and restore service faster when outages occur.
- Human operators are involved in each information layer for decision making and grid operation





2. Problems - manufacturing

- Production structure + control = optimized quality + reduced energy
- High-dimensional loop control tracking error affects the production quality and energy efficiency
- Human operators are involved in each information layer for decision making and plant operation

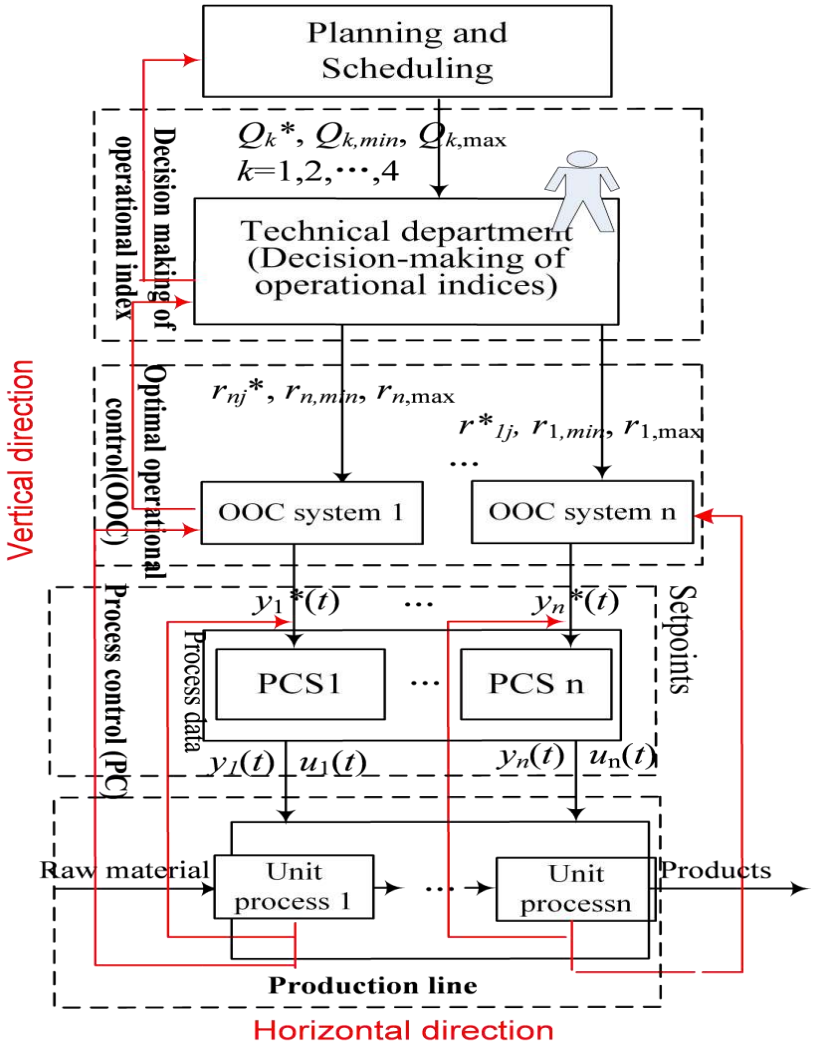


Fig. 1 2D operational mode

Issues Related to Decision Making

For complex systems such as industrial processes and power grid, optimization is always required

- *Model based approach – optimal power flow design*
- *Data-driven approach – machine learning based optimization*

Human in the loop is a common feature, challenges are:

- *How to quantify the human operators knowledge and embedded uncertainties?*
- *How to link the quantified human knowledge with process dynamics?*
- *How to deal with decision making with human-in-the-loop cases?*

Possible solutions:

Development of knowledge automation

The measure of uncertainties in knowledge automation

Probability density function shaping based optimization.

Objective – knowledge automation

Knowledge Automation for Control and Optimization of Process Industries

- *New concept proposed by Hong Wang and his colleagues in 2015*
- *Realize quantitative approach to solve human-in-loop*

Basic Knowledge Automation Components

- *Automation of knowledge acquisition*
- *Automation of knowledge usage in decision making*
- *Automation of performance assessment*

Objectives:

Develop a method on how the uncertainties in knowledge automation can be measured and used in operational optimization for process control.

Scientific Challenges

The decision making phase of human operators contains uncertainties in the time-horizon in terms of

observation,

use of knowledge

decision realization*.*

The uncertainties of human operator's decision making can be measured through brain signals (EEGs).

It is therefore imperative that such a measure be used by well-defined process variables in the minimization of the impact of uncertainties embedded in human operator's decision making.

Challenges:

How can the negative effect of these uncertainties on the optimization be quantified and subsequently minimized.

Optimization is a special case of feedback control design

Deterministic optimization tries to solve the following problem

$$\min_x J(x) \quad (1)$$
$$s.t.\ f(x)=0 \quad (2)$$

where

- $J(x)$ *is the performance function (e.g., energy consumption),*
- x *is the decision variable,*
- $f(x)=0$ *is the constraints .*

- If we denote x_k as the value of x at sample number k, then the following recursive optimization algorithm is generally used

$$x_k = \pi(x_{k-1}, J(x_{k-1}), \theta)$$

- $\pi(.,.,.)$ = a functional operator representing the optimization algorithm
- θ = a group of learning rates.

Optimization is a special case of feedback control design

Taking the optimization algorithm as a "control" algorithm, this means that to optimize J subjected to the required constraints one needs to design a "controller" that can ensure *ideally* that the following tracking error

$$\lim_{k\to+\infty} \boldsymbol{e_k} = \lim_{k\to+\infty} [\,0 - J(x_k)] = 0 \qquad (3)$$

This observation has therefore broadened the scope of the optimization in the following,

- Multi-objective optimization = a multi-input and multi-output (MIMO) controller design problem;
- The rather rich stability and robustness analysis (Lyapunov stability theory) in control theory can be employed to analyze the performance of the concerned optimization,
- For example the convergence and dynamic performance of the tracking error given in (6) can be readily assessed using Lyapunov stability theory.

Source of uncertainties

- For complex processes, human operators at different layer in Figure 1 generally participate in the optimization.
- An experienced operator would exhibit less uncertainties in his/her mind when making the decision.
- Such uncertainties can be represented as a disturbance to the "controller" in figure 2.
- The "plant" are generally subjected to some degree of uncertainties in terms of the uncertainties embedded in both the performance function and the constraints.

  - Uncertainties in the controller caused by human decision makers
  - Uncertainties in the plant

Technical Approach – Uncertainties in the "controller"

Use methods from different disciplines including techniques from Brain Computer Interface (BCI) to look into the following:

1. Investigate the ways to quantify knowledge quality and randomness using signals collected from the brain via EEG;

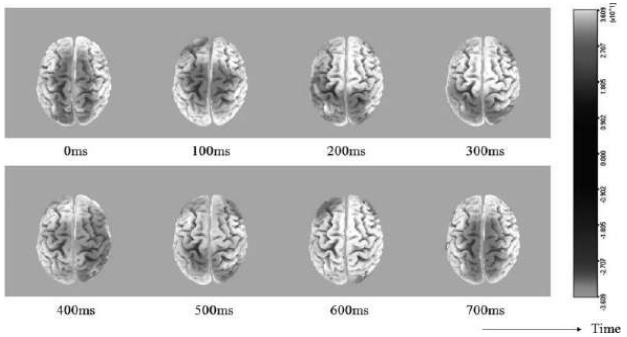
2. Develop a method on performance index modeling that can describe the relationships between uncertainties of human decision variables and the well-defined process variables.

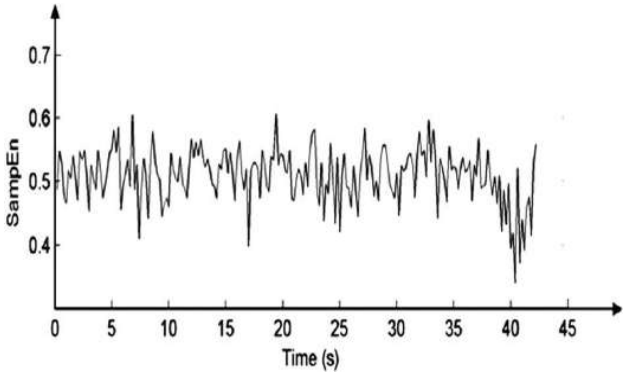
3. Explore the framework that can be used to realize the risk-minimization of human decision making by adjusting a group of well-defined process variables;

Technical Approach

A preliminary study by Hong Wang [1] has shown that the uncertainties of human operator's decision making can be measured through EEG (brain) signals,







[1] Zhang, C., Wang, H., & Wu, M. H. (2013). EEG-based expert system using complexity measures and probability density function control in alpha sub-band. *Integrated computer-aided engineering*, *20*(4), 391-405.

Technical Approach

the brain signals of inexperienced operators would generally exhibit large portion of uncertainties and randomness which lead to a widely distributed probability density functions (pdf) as shown in the following Figure.

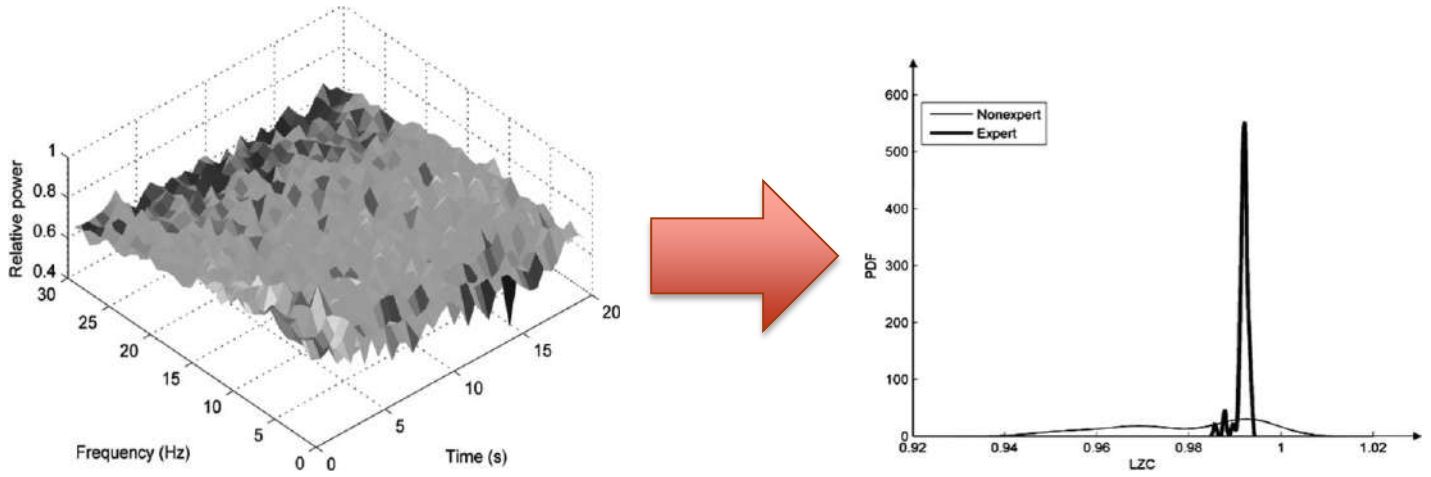
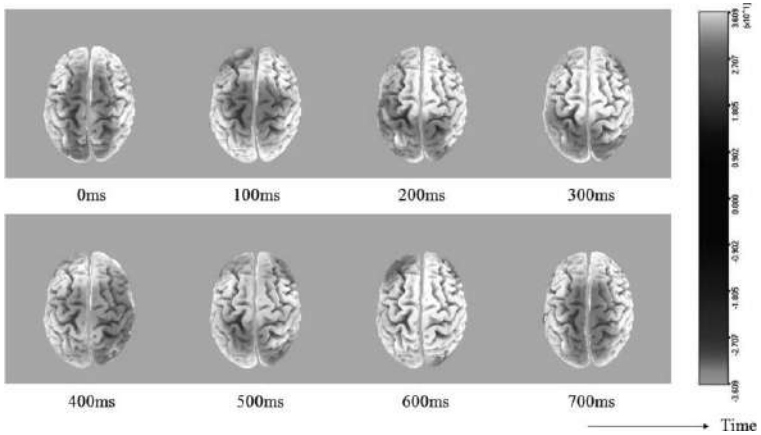


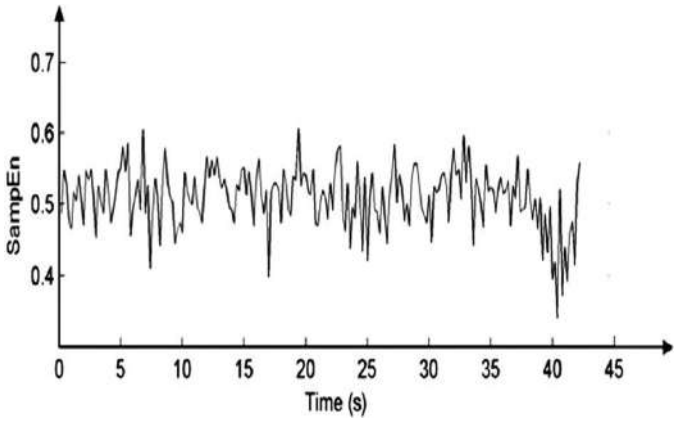
Figure 2. PDFs of EEG signals for experienced and non-experienced operators

Results – uncertainties quantification

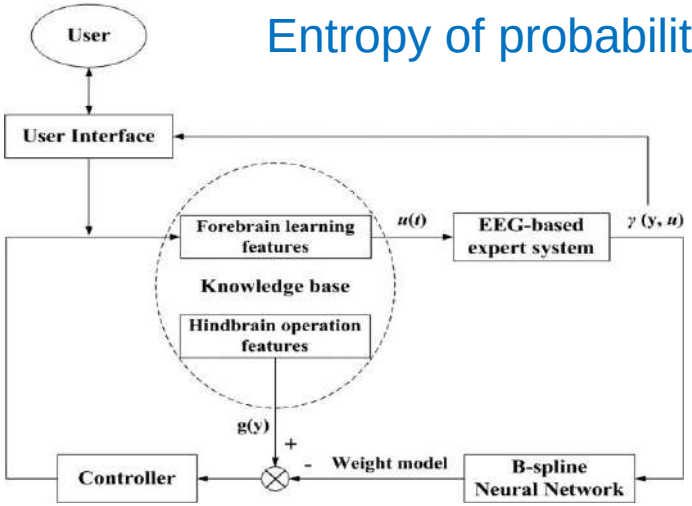
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- *Previous study by Hong Wang et, al in 2013 shows the following [1]*



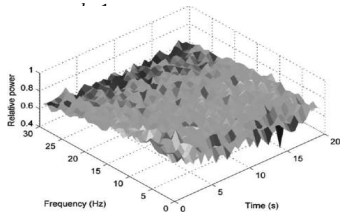


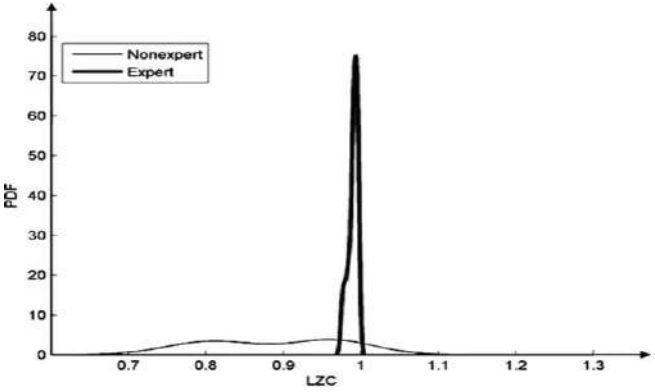
Entropy of probability density function measures the uncertainties



$$P\{x \in \Omega\} = \int_{\Omega(x)} \gamma(\tau,\theta)d\tau$$

$$\gamma(\tau,\theta) = \sum^{N} \eta_k(\theta)B_k(\tau) + \Delta(\tau,\theta)$$



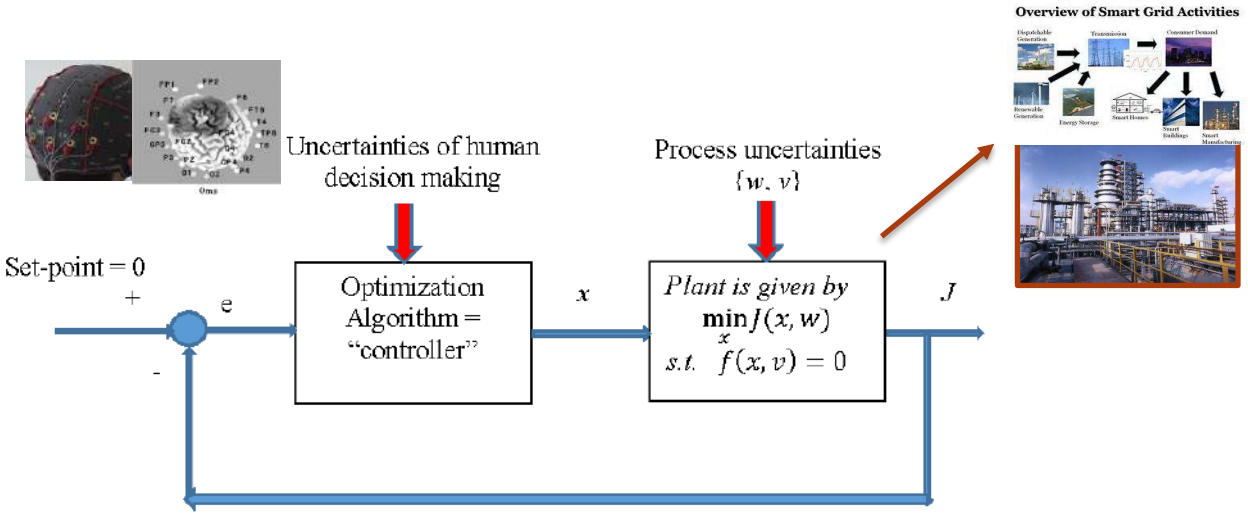


[1] Chi Zhang, Hong Wang, *, Hong Wang and Mian-Hong Wu, EEG-based expert system using complexity measures and probability density function control in alpha sub-band, *Integrated Computer-Aided Engineering* 00 (2013) 1–15

Results – a generalized framework

We have shown that decision making (optimization) is a special case of general feedback control design

The only difference is that the controller now is subjected to uncertainties and randomness of the human decision making phase



Stochastic distribution control theory can now be used to solve the problem

Dealing with uncertainties in decision making – probability density shaping

Taking into account the uncertainties in human operator's decision making, the following stochastic optimization needs to be solved

$$\min_x J(x, \mathrm{w}) \quad (4)$$

$$s.t.\ f(x, v) = 0 \quad (5)$$

where

- $J(x)$ *is the performance function (e.g., energy consumption),*
- x *is the decision variable,*
- $f(x, v) = 0$ *is the constraints*
- $\{w, v\}$ *groups the uncertainties of (BCI) human decision making phase.*

Existing theory has solved the above problem in the mean-valued sense with minimized variance for Gaussian uncertainties.

Dealing with uncertainties in decision making – probability density shaping

- *Since the probability density function is the comprehensive measure of random variable, if we can shape it then we can control the random variable totally*

- *Probability density function shaping of both the objective function and the constraints provide A TOTAL SOLUTION*

Dealing with uncertainties in decision making – probability density shaping

Novelty: we developed a novel approach that solves the above problem by shaping the probability density functions (pdf) of $J(x)$ and $f(x, v)$

Key Idea:

Select the decision variable x so that the probability density function of $J(x)$ is made as left and as narrow as possible;

Select the decision variable x so that the probability density function of $f(x, v)$ can converge to a δ- distribution;

Functional distance concept has been used

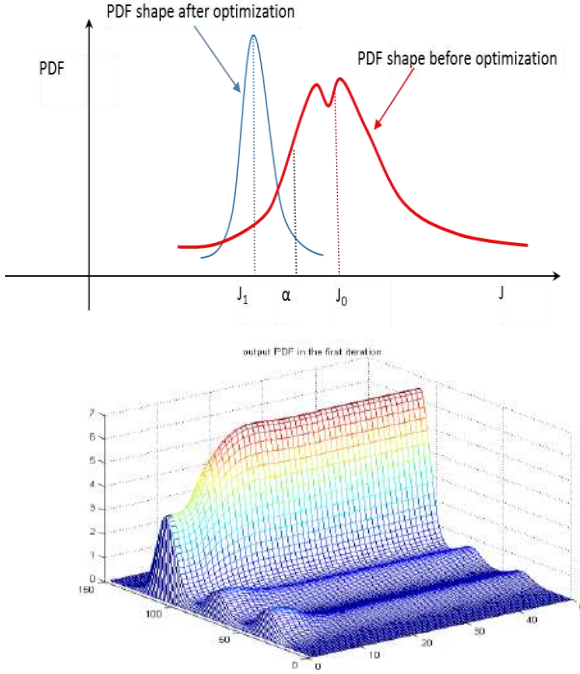


Figure 5. PDFs of the performance function before and after the optimization

Our proposed solution - PDF shaping for the performance function

Denote "as left and as narrow as possible function", $\delta(\tau - a)$ which is an impulse probability density function defined as follows:

$$\delta(\tau - a) = \begin{cases} +\infty, & \tau = a \\ 0, & otherwise \end{cases} \quad (6)$$

Denote the probability density function of J as $\gamma_J(x, \tau)$, $\tau \in [a, b]$, then to realize the above purpose of optimization we need to minimize the following induced functional distance

$$\pi_0(x) = \left\| \delta(\tau - a) - \gamma_J(x, \tau) \right\| = \sqrt{\int_a^b [\delta(\tau - a) - \gamma_J(x, \tau)]^2 d\tau} = min \quad (7)$$

This is equivalent to minimize the following,

$$\pi(x) = \int_a^b [\delta(\tau - a) - \gamma_J(x, \tau)]^2 d\tau = min \quad (8)$$

Our proposed solution - Dealing with the constraint

Denote the pdf of *f(x, v)* as $\gamma_f(x, \varphi)$, $\varphi \in [c, d]$, then to accomplish the constraint, we need to select the decision variable *x so that*

$$\varepsilon(x) = \int_c^d [\delta(\varphi) - \gamma_f(x, \varphi)]^2 d\varphi \longrightarrow \underline{\text{ZERO}}.$$

We just need to select $\{x_k\}$ so that the following is minimized.

$$J_\sigma = \pi(x_k) + \sum_{j=1}^{k} \varepsilon(x_j), \quad k = 1, 2, 3, \ldots$$

Minimizing $J_\sigma(x)$ for *k* that goes to infinite, the constraint *f(x, v) = 0* can be strictly guaranteed if the following condition is satisfied,

$$\sum_{j=1}^{+\infty} \varepsilon(x_j) < +\infty$$

This constitutes a sufficient condition for $\{x_k\}$ to converge to its optimal value.

Our proposed solution - Dealing with the constraint

Minimizing $J_\sigma(x)$ for k that goes to infinite, the constraint $f(x, v) = 0$ can be strictly guaranteed,

$$\sum_{j=1}^{+\infty} \varepsilon(x_j) < +\infty \quad \Longrightarrow \quad \lim_{k\to+\infty} \int_c^d [\delta(\varphi) - \gamma_f(x_k, \varphi)]^2 d\varphi = 0$$

$$\lim_{k\to+\infty} \gamma_f(x_k, \varphi) = \delta(\varphi) \quad \rightarrow \quad \lim_{k\to+\infty} f(x_k, v) = 0$$

THEOREM 1 (The Discrete-time Case). A *sufficient condition for solving the optimization problem (4) – (5) is that there is a sequence of decision variable* $\{x_k\}$ *for* $k = 1, 2, 3, \ldots$ *so that the following inequality holds*

$$\sum_{k=1}^{+\infty} \varepsilon(x_k) < +\infty$$

where it has been denoted that

$$\varepsilon(x_k) = \int_c^d [\delta(\varphi) - \gamma_f(x_k, \varphi)]^2 d\varphi$$

Our proposed solution - Dealing with the constraint

When $\Delta x = x_k - x_{k-1}$is very small, a continuous-time format can be obtained to give

$$J_\sigma(x) = \pi(x_t) + \int_0^t \int_c^d [\delta(\varphi) - \gamma_f(x(\tau), \varphi)]^2 d\varphi d\tau$$

In this case we need to find a profile $x(\tau)$ for $\tau \in [0, t]$ that can minimize (14) for any $t > 0$, this means that we need to solve the following optimization problem

$$\min_{x(\tau), \tau \in [0,t]} \left\{ \pi(x_t) + \int_0^t \int_c^d \left[\delta(\varphi) - \gamma_f(x(\tau), \varphi)\right]^2 d\varphi d\tau \right\} \quad \forall t \in [0, +\infty)$$

This is an optimization problem, if it is solvable for any $t > 0$, then it can be concluded that

$$\int_0^t \int_c^d [\delta(\varphi) - \gamma_f(x(\tau), \varphi)]^2 d\varphi d\tau < +\infty, \qquad for \; \forall \; t > 0$$

Our proposed solution - Dealing with the constraint

THEOREM 2 (The Continuous-time Case). *A sufficient condition for solving the optimization problem (4) – (5) is that there is a function* $x(\tau)$ *for* $\tau \in [0, t]$ *so that the following inequality holds*

$$\int_0^t \int_c^d [\delta(\varphi) - \gamma_f(x(\tau), \varphi)]^2 d\varphi d\tau < +\infty, \qquad for\ \forall\ t > 0$$

As a result, it can be obtained that

$$\lim_{t \to +\infty} \gamma_f(x(t), \varphi) = \delta(\varphi), \qquad for\ \ \forall \varphi \in [c, d]$$

$$\lim_{t \to +\infty} x(t) = x_{opt}$$

Relations to chance constrained optimization

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Problem (4)-(5) can also be regarded as a chance constrained optimization (CCO) problem studied in optimal power flow where we need to find an optimal *x* for.

$$\min_x J(x, w) \tag{9}$$

$$s.t. \quad Prob\{f(x, v) = 0\} \geq 1 - \beta \tag{10}$$

where $\beta \approx 0$ is a very small positive number. The above can be further relaxed into

$$\min_x E_w\{J(x, w)\} \tag{11}$$

$$s.t. \quad Prob\{f(x, v) \leq 0\} \geq 1 - \beta = \alpha \tag{12}$$

where $E_w\{.\}$ is the mathematical expectation operator over random variable *w*.

- In pdf shaping sense, this means that we need to find a good *x* so that the pdf of *J* is moved as left and as narrow as possible together with the pdf of *f(x, v)*.
- As a result, the formulation and sufficient conditions stated in Theorems 1 and 2 can all be applied to seek the solutions to the above chance constrained optimization problem.

Relations to chance constrained optimization

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Using the B-spline expression for $\gamma_f(x,\varphi)$, we have

$$\gamma_f(x,\varphi) = \sum_{i=1}^{N} \theta_i(x) B_i(\varphi), \quad \forall \varphi \in [c,d] \tag{13}$$

$B_i(\varphi), \{i = 1, 2, \dots, N\}$ = the set of pre-specified basis functions on $[c,d]$,

$\theta_i(x), \{i = 1, 2, \dots, N\}$ = B-spline expansion weights related to x.

bility constraint (12) is equivalent to

$$Prob\{f(x,v) \le 0\} \ge 1 - \beta = \alpha \quad \Rightarrow \quad \int_c^0 \gamma_f(x,\varphi)d\varphi = \sum_{i=1}^{N} \theta_i(x) \int_c^0 B_i(\varphi)d\varphi \ge \alpha$$

Denote $\gamma_w(\tau)$ as the pdf of w with $\tau \in [a,b]$, then it can be obtained that

$$E_w\{J(x,w)\} = \int_a^b J(x,\tau)\gamma_w(\tau)d\tau = J_1(x) \tag{15}$$

In this case the chance constrained optimization (11) – (12) can be transferred into the following deterministic nonlinear optimization problem

$$\min_x J_1(x) \tag{16}$$

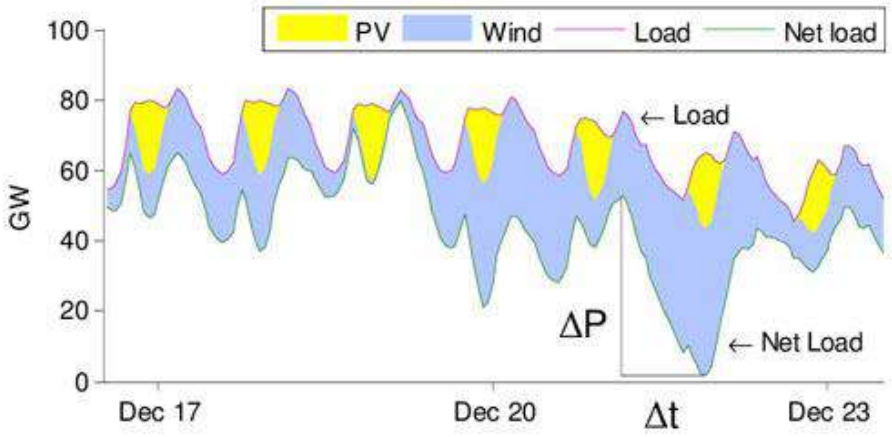
$$s.t. \quad \sum_{i=1}^{N} \theta_i(x) \int_c^0 B_i(\varphi)d\varphi \ge \alpha \tag{17}$$

Application PDF shaping to power grid operation

- Uncertainties in power grid (Wind, solar, etc.)



- Need to optimize probabilistic Power Flow

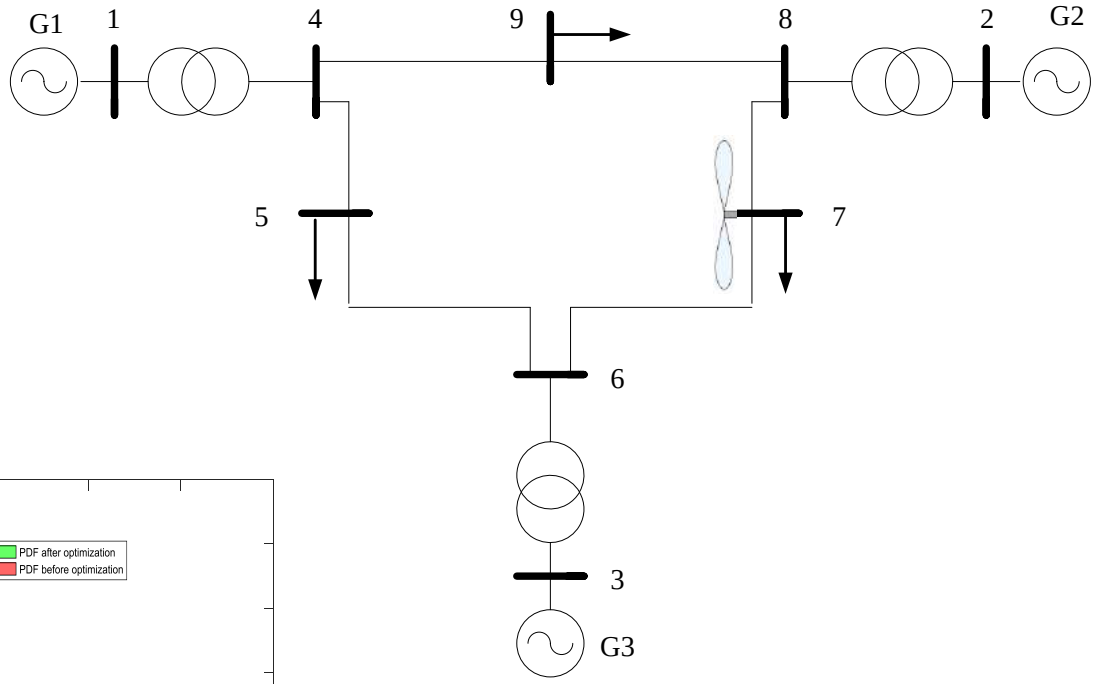


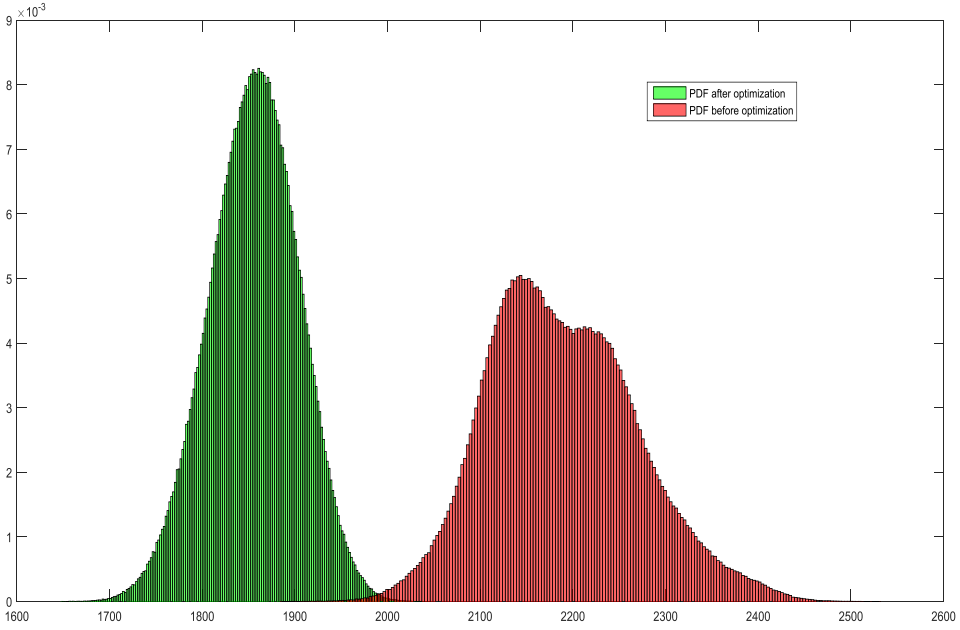
Case 1: Economic dispatch for power systems with intermittent generation

Cost function and constrains

$$F_c = \sum_{t=1}^{24} \sum_{i=1}^{3} f_{it}(P_{it})$$

$$\sum_{i=1}^{3} P_{it} + P_t^W - P_{Dt} = 0$$



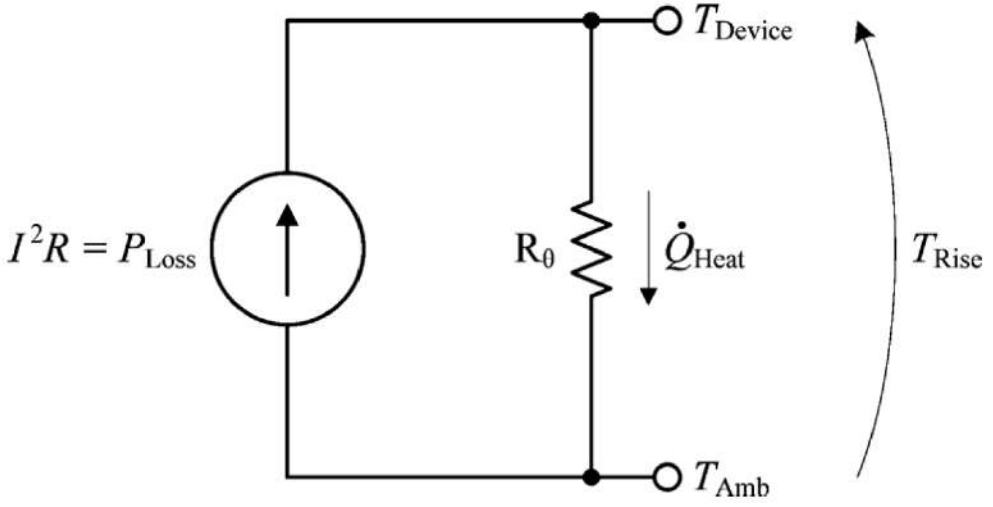


P_t^W and P_{Dt} are stochastic variables, representing the output of wind farm and load demand in the system. F is a nonlinear function of P_{it}

Case 2:P-PF (Probabilistic Power Flow) optimization for power loss under uncertainty of temperature

- Non-neglectable impact of temperature on power loss [1]
- The resistance of metallic conductors is proportional to temperatures.



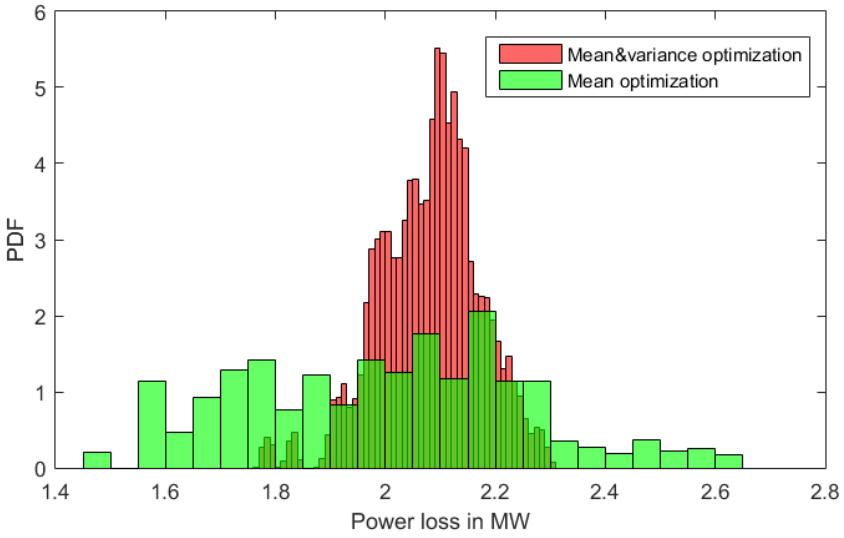


$$R = R_{\text{Ref}} \cdot \frac{T + T_{\text{F}}}{T_{\text{Ref}} + T_{\text{F}}}$$

[1] S. Frank, *etal*, "Temperature-Dependent Power Flow," IEEE transactions on power systems, 2013

Case 2: Formulation and results

- Objective: Power loss P_L
- Decision variables: P, V at PV bus
- Uncertainties: g_k subjected to temperature change
- Constrains: power flow at each bus



$$\min P_L = \sum_{k=1}^{nl} g_k \left[V_i^2 + V_j^2 - 2V_iV_j \cos(\delta_i - \delta_j) \right]$$

$$s.t.$$

$$P_i = V_i \sum_{i=1}^{n} V_j \left(G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij} \right)$$

$$Q_i = V_i \sum_{i=1}^{n} V_j \left(G_{ij} \cos\delta_{ij} - B_{ij} \sin\delta_{ij} \right)$$

$$V_i^{\min} \le V_i \le V_i^{\max}, \qquad i = 1, 2, \cdots n$$

$$Q_j^{\min} \le Q_j \le Q_j^{\max}, \qquad j = 1, 2, \cdots n$$

Case 2 (cont.): Formulation

- Objective: Power loss P_L

$$\min P_L = \sum_{i,j \in L} G_{ij} \left[V_i^2 + V_j^2 - 2 V_i V_j \cos(\delta_i - \delta_j) \right]$$

- Constrains: power flow at each bus

$$P_i = V_i \sum_{i=1}^{n} V_j \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right)$$

$$Q_i = V_i \sum_{i=1}^{n} V_j \left(G_{ij} \cos \delta_{ij} - B_{ij} \sin \delta_{ij} \right)$$

$$V_i^{\min} \le V_i \le V_i^{\max}, \qquad i = 1, 2, \cdots n$$

$$Q_j^{\min} \le Q_j \le Q_j^{\max}, \qquad j = 1, 2, \cdots n$$

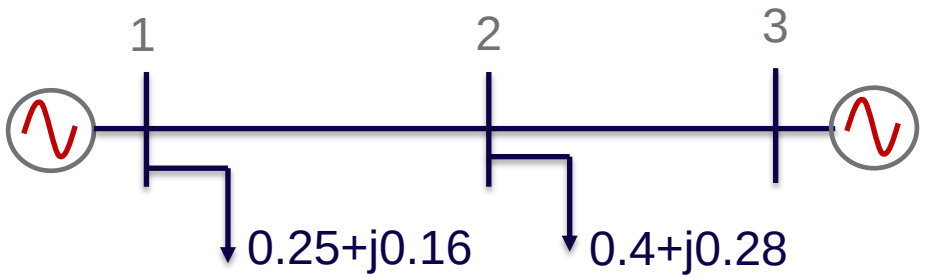
- Decision variables: P, V at PV bus

- Uncertainties: G_{ij} subjected to temperature uncertainty

Case 2 (cont.): results

- Power loss in a simple power system.

Adjust decision variables (P V at Bus 3) to optimize the PDF of power loss.



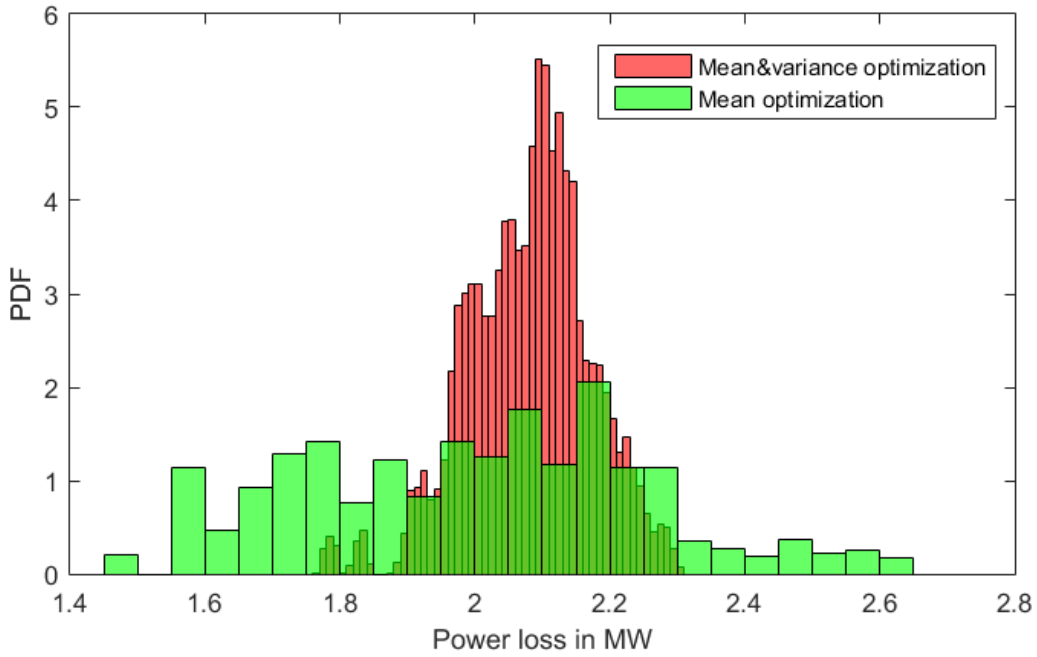
Bus 1: Slack bus

Bus 2: PQ bus

Bus 3: PV bus

G12~N(0.11, 0.0165^2)

G23~N(0.09, 0.0045^2)



Conclusion

- A general framework has been obtained that shows
  - The optimization is a special case of closed loop control system design where rather rich control theory can be directly applied to deal with optimization issues;
  - Uncertainties of human operator's decision making phase have been characterized and embedded into the stochastic optimization scenario
- A necessary condition on pdf shaping based stochastic optimization has been derived
- Encouraging results have been obtained for the power grid cases

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Thank you

Questions?