Tunnelling as Fermionic Emission of Black Strings

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Abstract

Hawking radiation of relativistic particles from uncharged and charged black strings in 3+1 dimensions is studied. We have compared the above result with the temperature of the thermal radiation of quantum tunneling of Dirac particles, we implied the Dirac's equation in end, we consider the problem of uncharged relativistic particles emitting from the event horizon of the BS.The quantum tunneling approach within the framework of the Hamilton-Jacobi method is applied. It is shown that the radial function of the action leads to the tunneling rate of the outgoing relativistic particles. By the aid of the Boltzmann formula, we derive the Hawking temperatures of the black strings. We also showed that the Hawking temperature of the black strings for the massive and massless particles leads to the same answer. It is because the gesture and behavior of those particles near the horizon are alike.

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I. INTRODUCTION

Black hole, in the classical picture of general relativity theory, is an incredibly dense object where space and time are so warped that nothing can escape from its gravitational capture. However this outlook on the black holes was dramatically changed by Stephen Hawking with his astonishing papers [1–3]. Hawking showed that black holes are able to radiate when the quantum mechanics is accompanied with general relativity, which is the so-called quantum gravity theory. From that day on, physics community has entitled this phenomenon as Hawking radiation: creation of thermal energy and emission of sub atomic particles by black holes, which can last until their complete evaporation. Besides, each black hole has its own characteristic temperature: black holes radiate with temperature $\frac{\hbar\kappa}{2\pi}$ where κ denotes the surface gravity [4]. In conclusion, it is understood that black holes are not completely dark and therefore they do not last forever [5].

In the process of the Hawking radiation, virtual pairs (particles and anti-particles) are formed in the vicinity of near horizon vacuum. According to this scenario, the particles having positive energy escape - apperceived as thermal radiation - while the anti-particles (with its negative mass) can fall back into the black hole, which cause decreasing in the mass of the black hole . On the other hand, Hawking's thermal radiation is not unitary and leads to a mixed density matrix for the emitted radiation, which leads to loss in the physical information: "Information Paradox" or "Hawking Paradox" [6–8]. Namely, according to Hawking's model of black hole evaporation, the information stored in the black hole appears to be completely lost in the universe. But, this is against to the laws of physics. The debate

on the information loss paradox is still continuing [9]. In the recent years, the studies on Hawking radiation has gained momentum in the framework of quantum tunneling (see for example [10–23] and references therein).

In this paper, we consider particular black strings (BSs), which embody cylindrical black holes with negative cosmological constant $\frac{-\Lambda}{3}$ [24]. These BSs have an immense range of cylindrically symmetric and spherically symmetric black holes possessing axis of symmetry. These asymptotically anti-de Sitter solutions admit the background of supergravity, topological defects, and low energy string theories [24–27]. We shall apply the semi-classical Hamilton-Jacobi method (HJM) together with WKB approximation for studying the quantum tunneling of charged particles from the BSs. [28, 29]

The outline of the paper is as follows. In Sec. II, we briefly review some geometrical and thermodynamical features of the considered BSs. Next, we employ the HJM with a suitable ansatz and calculate the tunneling rate of the chargeless relativistic particles radiating form the BS. Then, we compute the Hawking temperature of the BS. Section III is devoted to the Hawking radiation of charged particles from the charged BS. To this end, we consider the charged Hamilton-Jacobi equation [30, 31] for the charged relativistic waves propagating in the charged BS background. We draw our conclusions in Sect. IV. (We use the natural units with $c = G = \hbar = k_B = 1$).

II. FERMION EMISSION OF BLACK STRINGS

In this section, we shall present some physical features of the BS geometry having the negative cosmological constant (Λ). BSs are solution to the Einstein's field equations in the form of cylindrical symmetric spacetime. BS line-element is given by [32]

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \alpha^2 dz^2), \tag{1}$$

where $-\infty < t < \infty$, $z < \infty$, $0 \leq r < \infty$, $0 \leq \theta \leq 2\pi$, and $\alpha = \frac{-\Lambda}{3}$. Metric function $F(r)$ is given by [10]

$$F(r) = (\alpha^2 r^2 - \frac{M}{r}). \tag{2}$$

The event horizon of the BS (1) is located at

$$r_h = (\frac{M}{\alpha^2})^{\frac{1}{3}}. \tag{3}$$

Using the well-known expression for the surface gravity [4]:

$$\kappa = [\lim_{r\to r_h}(\frac{-1}{4}g^{tt}g^{ij}g_{tt,i}g_{tt,j})]^{\frac{1}{2}}, \tag{4}$$

one can compute the statistical Hawking temperature of the BS as follow

$$T_H = \frac{\kappa}{2\pi} = \frac{3}{4\pi}M^{\frac{1}{3}}\alpha^{\frac{4}{3}}. \tag{5}$$

To compare the above result with the temperature of the thermal radiation of quantum tunneling of Dirac particles, we implement the Dirac's equation is end, we consider the problem of uncharged relativistic particles emitting from the event horizon of the BS. While doing this, **we ignore the back-reaction and self-gravitational effects** [33–35] **and thus the particles tunneling the event horizon are not contingent upon a potential barrier** [35].

We assume that the massless spinor field Ψ surrendered the general covariant Dirac's equation

[45–49]

$$i\gamma^{\mu}D_{\mu}\Psi + \frac{\mu_d}{\hbar}\Psi = 0, \tag{6}$$

where μ_d is the mass of the Dirac particle and

$$D_{\mu} = \partial_{\mu} + \Omega_{\mu}, \tag{7}$$

$$\Omega_{\mu} = \frac{1}{2}i\Gamma_{\mu}^{\alpha\beta}\Sigma_{\alpha\beta}, \tag{8}$$

$$\Sigma_{\alpha\beta} = \frac{1}{4}i\left[\gamma^{\alpha},\gamma^{\beta}\right]. \tag{9}$$

The γ^{μ} matrices satisfy $\left[\gamma^{\alpha},\gamma^{\beta}\right] = -\left[\gamma^{\beta},\gamma^{\alpha}\right]$, when $\alpha \neq \beta$, and $\left[\gamma^{\alpha},\gamma^{\beta}\right] = 0$ for $\alpha = \beta$. γ^{μ} matrices can be represented in the following chiral form:

$$\gamma^{t} = \frac{1}{\sqrt{F(r)}}\gamma^{0},\ \gamma^{r} = \sqrt{F(r)}\gamma 3,\ \gamma^{\theta} = \frac{1}{r}\gamma^{1},\ \gamma^{\phi} = \frac{1}{r\sin\theta}\gamma^{2}, \tag{10}$$

where

$$\gamma^0 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \qquad \gamma^1 = \begin{bmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{bmatrix}, \tag{11}$$

$$\gamma^2 = \begin{bmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{bmatrix}, \qquad \gamma^3 = \begin{bmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{bmatrix}, \tag{12}$$

while $\sigma^i s$ $(i = 1, 2, 3)$ are the well-known Pauli matrices [45]. As is known to all, Dirac particles have spin-$\frac{1}{2}$ and the wave function must be well described with both spin-up $(\uparrow)$ and spin-down $(\downarrow)$. The wave functions of the Dirac particles can be expressed as follows

$$\Psi_\uparrow(t, r, \theta, \phi) = \begin{pmatrix} A(t, r, \theta, \phi) \\ 0 \\ B(t, r, \theta, \phi) \\ 0 \end{pmatrix} \exp\left(\frac{i}{\hbar} I_\uparrow(t, r, \theta, \phi)\right), \tag{13}$$

$$\Psi_\downarrow(t, r, \theta, \phi) = \begin{pmatrix} 0 \\ C(t, r, \theta, \phi) \\ 0 \\ D(t, r, \theta, \phi) \end{pmatrix} \exp\left(\frac{i}{\hbar} I_\downarrow(t, r, \theta, \phi)\right), \tag{14}$$

in which $I_\uparrow(t, r, \theta, \phi)$ and $I_\downarrow(t, r, \theta, \phi)$ are the action of Dirac particles with spin up $(\uparrow)$ and spin down $(\downarrow)$ tunneling across BH horizon, respectively. Substituting the wave function with spin-up $(\uparrow)$ into Dirac equation (6) [meanwhile, the spin-down case $(\downarrow)$ is completely analogous to the spin-up case $(\uparrow)$], then dividing by the exponential term and multiplying by $\hbar$, the resulting Dirac equations in leading order $\hbar$ become:

$$-\left(\frac{iA}{\sqrt{F(r)}}\partial_t I_\uparrow + B\sqrt{F(r)}\partial_r I_\uparrow\right) + \mu_d A = 0, \tag{15}$$

$$\left(\frac{iB}{\sqrt{F(r)}}\partial_t I_\uparrow - A\sqrt{G(r)}\partial_r I_\uparrow\right) + \mu_d B = 0, \tag{16}$$

$$-B\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\phi I_\uparrow\right) = 0, \tag{17}$$

$$-A\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\phi I_\uparrow\right) = 0. \tag{18}$$

It is not an easy task to obtain the analytical solution to the action $I_\uparrow$ by using the above differential system. Because of this, we employ the HJm and consider the symmetries of the

Grumiller BH. Namely, we perform the separation of variables technique:

$$I_{\uparrow} = -Et + W(r) + J\phi + N(\theta), \tag{19}$$

where E is the energy of the emitted fermions measured by the static observer. As is well-known, the surface gravity is thought of as the acceleration due to gravity experienced by a hypothetical test particle which is very close to the surface of the BH horizon as seen by a static observer whose 4-vector velocity U^{μ} is proportional to the Killing vector ξ^{μ}. The location of the observer is found from the normalization condition: $U^{\mu}U_{\mu} = -1$. For example, the static observer's location for the Schwarzschild black hole is located at spatial infinity. If we apply the same normalization condition for the Grumiller BH, one obtains

$$r_{observer} = \sqrt{\frac{M}{a}}. \tag{20}$$

Besides, J seen in Eq. (19) denotes the angular quantum number about ϕ and $W(r)$ and $N(\theta)$ are the undetermined functions. Substituting Eq.(19) into Eqs.(15)-(18), we get

$$\left(\frac{iA}{\sqrt{F(r)}}E - B\sqrt{F(r)}\partial_r W(r)\right) + \mu_d A = 0, \tag{21}$$

$$-\left(\frac{iB}{\sqrt{F(r)}}E + A\sqrt{F(r)}\partial_r W(r)\right) + \mu_d B = 0. \tag{22}$$

$$-B\left(\frac{1}{r}\partial_{\theta}N(\theta) + \frac{i}{r\sin\theta}J\right) = 0, \tag{23}$$

$$-A\left(\frac{1}{r}\partial_{\theta}N(\theta) + \frac{i}{r\sin\theta}J\right) = 0. \tag{24}$$

From Eqs.(23) and (24), one can get the following remark: the contributions coming from J and $N(\theta)$ are the same for both the outgoing and ingoing probabilities. Thus, when finding the factor of the outgoing probability to the incoming probability, we get unity. In other words, the contributions of and $N(\theta)$ to the tunneling rate are canceled out. So, we understand that although there are four Dirac equations (21-24), only Eqs. (21) and (22) are the master equations, which give remarkable contribution to the imaginary part of the action. Eqs. (21) and (22) have non-trivial solutions for A and B if and only if the determinant of the coefficient matrix vanishes. Thus, we get

$$W_{\pm}(r) = \pm\int\frac{\sqrt{E^2 + \mu_d^2 F(r)}}{F(r)}dr, \tag{25}$$

where positive/negative sign corresponds to the outgoing/ingoing wave of the fermions. The integrand of Eq. (25) has a simple pole at $r = r_h$. Following the contour as a half loop passing around this pole from left to right, we evaluate the integral (25) as follows

$$W_{\pm}(r_h) = \pm i\pi \frac{E}{F'(r_h)} = \pm i\pi \frac{Er_h}{2a(r_h - rn)}. \tag{26}$$

Therefore, the tunneling rate for the fermions crossing the horizon of the Grumiller BH becomes

$$\begin{aligned} \Gamma &= \frac{P_{\text{emission}}}{P_{\text{absorption}}} = \frac{\exp\left[-2\text{Im}W_{+}(r_h)\right]}{\exp\left[-2\text{Im}W_{-}(r_h)\right]}, \\ &= \exp\left(-2\pi \frac{Er_h}{a(r_h - r_n)}\right). \end{aligned} \tag{27}$$

Recalling the definition of the Boltzmann factor $[\Gamma = \exp\left(\frac{-\hbar E}{T}\right)]$ [28], the fermion emission yields the surface temperature of the Grumiller BH as follows

$$T_h = \frac{\hbar a(r_h - r_n)}{2\pi r_h}, \tag{28}$$

which is fully in accordance with the Hawking temperature obtained in Eq. (**??**).

III. CONCLUSION

There are many various methods for concluding Hawking radiation and for calculating its temperature[0][**?**][**?**]. The exquisite method considered the creation of a black hole in the background of a collapse geometry, computing the Bogoliubov transformation among the prime and ultimate states of incoming and outgoing radiation. The most reputable method of analytic resumption to a Eucliden section emerged soon after.Based on the manners of infinite temperature quantum field theory, an analytic continuation $t \to i\tau$ of the black hole metric is accomplished and the alternation of τ (signified by β) is selected in order to eliminate a conical singularity that would differently be willing at fixed points of U(1) isometry generated by $\frac{\partial}{\partial\tau}$ (the event horizon in the original Lorentzian part). The black hole is then investigated to be in equilibrium with a scalar field that has reverse temperature β at infinity. We apply semi-classical approach as a tunnelling effect, which involves with calculation of the imaginary part of the action for the procedure of scalar- wave emission

across the horizon, this process is classically forbidden. Afterwards we applied WKB approximation the tunnelling probability for the classically impermissible path of the scalar wave coming from inside to outside the horizon is appointed by $\Gamma \propto \exp(-2ImI)$ where I known as a classical action of the path to leading order in $\hbar$. We expanded the action in terms of energy of the particle, at the linear order the Hawking temperature is recovered, meant $2I = \beta E + \bigcirc(E^2)$ which drives $\Gamma \sim \exp(-2I) \simeq \exp(-\beta E)$, in which β denotes the Boltzmann factor and $T = \frac{1}{\beta}$ which is temperature of the horizon.

We have shown that computation of the Unruh and Hawking temperature with the tunnelling method approach is valid for fermions. We have drived that fermionic propagation has the same temperature as scalar propagation due to the presence of these horizons appears in the literature[11]. In the tunnelling precisianism the probability of particles crossing the black hole horizon on either way with application of complex path integrals are computed. In this approach the particles are allowed to traverse geodesics which are forbidden in classical trajectory. Though the absorption probability for incoming particles is equal to1 as this is a permitted trajectory in classic.This enables an impressive way of estimating the hawking temperature as well. We have extended its application to some Singular and Non-Singular Coordinate System to compare the result whether the Hawking temperature is truly recovered in either system or not. Solving Hamilton-Jacobi equation in the background of uncharged Grumiller black hole and applying WKB approximation, we have estimated the tunnelling probability of fermions. Hawking temperature in both Singular and Non-Singular Systems is recovered truly.

Solving equations (30) and (32) for A and B in the case that $\mu_d \neq 0$ conducting to the result: $(\frac{A}{B})^2 = \frac{-iE+\sqrt{f(r)}+\mu_d}{iE+\sqrt{f(r)}+\mu_d}$ and near the horizon we can drive that $\lim_{r\to r_n}(\frac{A}{B})^2 = -1$. Subsequently, we reached to the same result for the Hawking Temperature in the massless case. The spin-down calculation is very analogous to the spin-up case negotiated above. For the spin down case are equations are same as spin up case, only there is some changes of sign. It is implied that both spin up and spin down particles are propagated at the same rate. This is established with the initial assumption that there are as much spin up as spin down fermions propagated. We examined that Dirac equation on the Grummiler geometry within the WKB approximation yields the T_H for fermionic emission.

In this paper, we studied the HR of virtual particles emitting from the event horizon of a four dimensional BS metric. For this purpose, we separated the Hamilton-Jacobi equation on

the BS geometry using particular ansatz for the classical action I of a relativistic particle. We calculated the quantum tunneling rates of the uncharged and charged relativistic particles employing the HJM in the concept of first-order WKB approximation. Remarkably, we showed that both tunneling rates have similar structures. However, the energy seen in the Boltzmann formula is altered when the charge is taken into account [see Eqs. (27) and (33)]. Ultimately, we recovered the Hawking temperatures of the uncharged and charged BS metrics.Furthermore, the result takes the same practical form as that of massless particles.

Finally, whether the results are modified in other BSs, such as higher dimensional [43] BSs [44], is an intriguing question and will be explored in our next study.

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