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Quantum discord protection of a two-qutrit V-type atomic system from decoherence by partially collapsing measurements

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Abstract In this paper, by exploiting the weak measurement and quantum measurement reversal procedure, we propose a scheme to show how one can protect the geometric quantum discord (GQD) of a two-qutrit V-type atomic system each of which interacts with a dissipative reservoir independently. We examine the scheme for the GQD of the initial two-qutrit Werner and Horodecki states for different classes of weak measurement strengths. It is found out that the presented protocol enables us to suppress decoherence due to the amplitude damping channel and preserve the quantum discord of the two-qutrit system successfully.

Keywords Geometric quantum discord · Decoherence · Weak measurement · Quantum measurement reversal

1 Introduction

One of the most prominent aspects of quantum mechanics is quantum correlation which plays a vital role in quantum information tasks [1]. For a long time, the only known source of quantum correlations in composite quantum systems has been entanglement which is responsible for various quantum information processing tasks, such as, quantum cryptography [2], quantum teleportation [3] and quantum dense coding [4]. Recently, it has been shown that entanglement is just one form of quantum correlations and some applications in quantum information theory do not require it. Ollivier and Zurek in [5] claimed that there is a new measure of quantum correlations other

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than entanglement. Despite entanglement, the new non-classical correlation which was named quantum discord can even exist in separable states. Some tasks that benefit from quantum discord are state merging [6,7], remote state preparation [8], entanglement distribution [9,10], transmission of correlations [11] and information encoding [12].

Quantum correlations may be corrupted by decoherence because the quantum system unavoidably coupled to its surrounding environment and lose coherence [13]. Thus, protecting quantum correlations from decoherence due to environmental noises is critical in quantum information theory. Several methods have been explored to suppress decoherence, including decoherence-free subspace [14,15], entanglement distillation [16–18], quantum Zeno effect [19] and quantum error correction [20,21]. Additionally, weak measurement and quantum measurement reversal (WMQMR) have been offered to deal with decoherence due to the amplitude damping (AD) channel.

The concept of weak measurements was suggested by Aharonov et al. [22] for the first time. Later on, by making use of weak measurement and its reversal, Katz et al. [23] presented a way to suppress the decoherence of AD for a single phase qubit. Recently, it has been shown that fragile quantum states of a single-qubit system [24–26] and quantum entanglement of two-qubit systems [27–29] can be protected by WMQMR. Xiao et al. [30] applied this method to preserve the entanglement of a two-qutrit system from decoherence. Recently, it has been demonstrated in [31] that WMQMR is suitable for protecting quantum discord from decoherence of the AD noise. In a quantum measurement, by performing the so-called von Neumann measurement on a system, the state of concern instantaneously collapses to one of the eigenstates of the system. Since von Neumann measurement destroys the initial state, it is impossible to recover it from the outcome of the measurement. In contrast to this type of quantum measurement, there is a partial collapsing or weak measurement in which the state of the system does not totally collapse, such that the final state has some information of the original state. In this case, the exact initial state can be recovered by applying the reversed weak measurement operator.

Motivated by the above assumptions, our aim of this paper is the investigation of the role of WMQMR in protecting the two-qutrit discord, measured by the geometric quantum discord, from destruction caused by the AD noise. We examine the performance of this method for the initial two-qutrit V-type Werner and Horodecki states. In [32], the exact value of geometric discord for families of three-level states with maximally mixed marginals such as Werner state, Bell state and Horodecki state is evaluated; however, both of Werner and Horodecki states after undergoing the AD channel do not have maximally mixed marginals. Thus, instead of calculating the exact values of GQD for these states, we exploit the lower bound of GQD to this aim.

The remainder of the paper is organized as follows: In Sect. 2, after a brief review of quantum discord and the lower bound of GQD, we investigate the dynamics of a V-type three-level atom as an open quantum system. In Sect. 3, the quantum discord protection protocol for a two-qutrit system, by using the method of weak measurement and its reversal, is presented and it is examined for the Werner and Horodecki states. Finally, we summarize our research in Sect. 4.

2 Basic theory

2.1 Quantum discord

At the first time, quantum discord which is one of the interesting measures of quantum correlations beyond entanglement was defined by the notion of the mutual information [33]. In classical information theory, the mutual information measures correlation between two random variables of a classical system, A and B

$$I(A:B) = H(A) + H(B) - H(A,B), \tag{2.1}$$

where $H(A) = -\sum_a p_a \log_2 p_a$ is known as the Shannon entropy of the random variable A. In addition, $H(A,B)$ shows the joint Shannon entropy of two variables A and B. Another equivalent definition of the classical mutual information is given by

$$J(A:B) = H(B) - H(B|A). \tag{2.2}$$

Notice that $H(B|A)$ indicates the information gained about the subsystem B by measuring the subsystem A. The mutual information in quantum theory is defined as

$$I(A:B) = S(A) + S(B) - S(A,B), \tag{2.3}$$

where $S(A) = -Tr[\rho_A \log_2 \rho_A]$ stands for the von Neumann entropy of the subsystem A with the reduced density operator $\rho_A = tr_B \rho$. Two definitions of the classical mutual information are the same but for quantum systems differ from each other and the difference introduces quantum discord. An extension of the classical conditional entropy is $S(B|\{\Pi_A\})$ which shows the von Neumann entropy of B conditioned on the outcome of the measurement on A. The quantum version of Eq. (2.2) is as

$$J(A:B) = S(B) - S(B|\{\Pi_A\}). \tag{2.4}$$

Ollivier and Zurek [5] called the difference between $I(A:B)$ and $J(A:B)$ quantum discord as

$$\Delta^{B|A} = I(A:B) - J(A:B). \tag{2.5}$$

According to the above definition, quantum discord is a function of the applied measurement on the subsystem A. To make quantum discord independent of such measurements, Henderson and Vederal [34] performed a minimization of Eq. (2.5) over all von Neumann measurements and obtained the following formula for quantum discord

$$\delta^{B|A} = \min_{\{\Pi_A\}}[I(A:B) - J(A:B)]. \tag{2.6}$$

However, in general, Eq. (2.6) for evaluating quantum discord is very difficult from calculational point of view. To overcome this difficulty, a measure for quantum discord named as geometric quantum discord (GQD) was introduced by Dackić et al. [35], as follows

$$D(\rho) = \min_{\chi \in \Omega_0} \|\rho - \chi\|^2. \tag{2.7}$$

Here, Ω_0 shows the set of zero-discord states and $\|A-B\|^2 = Tr(A-B)^2$ expresses the square of Hilbert–Schmidt norm of Hermitian operators. Using the Bloch representation

$$\rho = \frac{1}{4}\big(I_2 \otimes I_2 + \sum_{i=1}^{3} x_i \sigma_i \otimes I_2 + \sum_{j=1}^{3} y_j I_2 \otimes \sigma_j + \sum_{i,j=1}^{3} t_{ij} \sigma_i \otimes \sigma_j\big), \tag{2.8}$$

GQD can be evaluated for every general two-qubit state ρ as

$$D(\rho) = \frac{1}{4}(\|x\|^2 + \|T\|^2 - \mu_{\max}), \tag{2.9}$$

where $x = (x_1, x_2, x_3)^t$ is a column vector, $\|x\|^2 = x^t x$, $T = (t_{ij})$ is the correlation matrix and $\mu_{\max}$ shows the largest eigenvalue of the matrix $xx^t + TT^t$.
We can extend this result for the general bipartite system belonging to $H^a \otimes H^b$ with $\dim(H^a) = m$ and $\dim(H^b) = n$. The vector space of all linear operators on H^a with the Hilbert–Schmidt inner product

$$\langle X|Y\rangle = Tr(X^\dagger Y), \tag{2.10}$$

constitutes a Hilbert space $L(H^a)$ (as well as $L(H^b)$ and $L(H^a \otimes H^b)$). Two sets of Hermitian operators $\{X_i : i = 1, 2, \ldots, m^2\}$ and $\{Y_j : j = 1, 2, \ldots, n^2\}$ which satisfy the orthonormality conditions

$$TrX_i X_{i'} = \delta_{ii'}, \qquad TrY_j Y_{j'} = \delta_{jj'}, \tag{2.11}$$

can be chosen to construct orthonormal bases for $L(H^a)$ and $L(H^b)$, respectively. Then, $\{X_i \otimes Y_j\}$ is considered as an orthonormal basis for $L(H^a \otimes H^b)$. The expansion of an arbitrary bipartite density matrix ρ on $H^a \otimes H^b$ can be represented by

$$\rho = \sum_{ij} c_{ij} X_i \otimes Y_j, \tag{2.12}$$

where $c_{ij} = Tr\rho X_i \otimes Y_j$.
Let us consider λ_i $(i = 1, 2, \ldots, m^2 - 1)$ and λ_j $(j = 1, 2, \ldots, n^2 - 1)$ each of which satisfy $Tr(\lambda_i \lambda_j) = 2\delta_{ij}$ as the generators of SU(m) and SU(n), respectively. By using this set of basis, we can rewrite the bipartite density matrix ρ on $H^a \otimes H^b$ as

$$\rho = \frac{1}{mn}\left(I_m \otimes I_n + \sum_i x_i \lambda_i \otimes I_n + \sum_j y_j I_m \otimes \lambda_j + \sum_{i,j} t_{ij} \lambda_i \otimes \lambda_j\right), \tag{2.13}$$

where the $x_i s$ and $y_j s$ are the elements of the vectors $\mathbf{x} \in R^{m^2-1}$ and $\mathbf{y} \in R^{n^2-1}$ corresponding to the subsystems A and B, respectively, obtained as

$$x_i = \frac{m}{2} Tr(\rho \lambda_i \otimes I_n) = \frac{m}{2} Tr(\rho_A \lambda_i), \tag{2.14}$$

$$y_j = \frac{n}{2} Tr(\rho I_m \otimes \lambda_j) = \frac{n}{2} Tr(\rho_B \lambda_j). \tag{2.15}$$

Note that

$$T = \frac{mn}{4} [Tr(\rho \lambda_i \otimes \lambda_j)], \tag{2.16}$$

is the correlation matrix then we have

$$C = \begin{pmatrix} \frac{I}{\sqrt{mn}} & \frac{\sqrt{2}}{n\sqrt{m}} \mathbf{y}^t \\ \frac{\sqrt{2}}{m\sqrt{n}} \mathbf{x} & \frac{2}{mn} T \end{pmatrix}. \tag{2.17}$$

This matrix can also be written in terms of the basis $\{X_i\}$ and $\{Y_j\}$ as $C = [c_{ij}] = Tr(\rho X_i \otimes Y_j)$. Since evaluation of $D(\rho)$ in Eq. (2.9) is a difficult procedure for high-dimensional bipartite systems ($m, n \geq 3$), we can use the lower bound of GQD introduced by Luo et al. [36] as

$$D(\rho) \geq Tr(CC^t) - \sum_{i=1}^{m} \mu_i = \sum_{i=m+1}^{m^2} \mu_i, \tag{2.18}$$

instead of $D(\rho)$ in which μ_is show the eigenvalues of CC^t sorted in non-increasing order. So in this way, the lower bound of GQD in Eq. (2.18) is used as GQD in Sect. 3.

2.2 Dynamics of V-type three-level open systems

This section is devoted to investigate the interaction between a V-type three-level atom and a zero-temperature bosonic Lorentzian reservoir [37,38]. In this configuration of the three-level atom, each of the excited levels $|2\rangle$ and $|1\rangle$ can decay to the ground state $|0\rangle$ and emit a single photon but the transition between $|2\rangle$ and $|1\rangle$ is dipole-forbidden. The total Hamiltonian for the three-level system is composed of three terms

$$H = H_S + H_E + H_I, \tag{2.19}$$

where the first and second terms represent the free Hamiltonian

$$H_0 = H_S + H_E = \sum_{l=1}^{2} \omega_l \sigma_+^l \sigma_-^l + \sum_k \omega_k b_k^\dagger b_k, \tag{2.20}$$

and the third term is responsible for the atom–environment interaction Hamiltonian

$$H_I = \sum_{l=1}^{2} \sum_{k} \left(g_{lk} \sigma_+^l b_k + g_{lk}^* \sigma_-^l b_k^\dagger \right). \tag{2.21}$$

In Eq. (2.21), $\sigma_+^l = |l\rangle\langle 0|$ and $\sigma_-^l = |0\rangle\langle l|$ $(l = 1, 2)$ indicate atomic transition operators and the kth field mode of the environment is described by the frequency ω_k, the atom–field coupling constant g_{lk} and creation and annihilation operators b_k, $b_k^\dagger$, respectively. It is convenient to investigate the atom–field interaction problem in the interaction picture. In the interaction picture, the interaction term in Eq. (2.21) is given by

$$H_{\text{int}} = \sum_{l=1}^{2} \sum_{k} \left(g_{lk} \sigma_+^l b_k e^{i(\omega_l - \omega_k)t} + g_{lk}^* \sigma_-^l b_k^\dagger e^{-i(\omega_l - \omega_k)t} \right), \tag{2.22}$$

and the time evolution of the total system is determined by the Schrödinger equation

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle = H_{\text{int}} |\psi(t)\rangle. \tag{2.23}$$

At any time t, $|\psi(t)\rangle$ is expressed as a linear combination of the states $|l\rangle_S \otimes |0\rangle_E$ and $|0\rangle_S \otimes |1_k\rangle_E$. Here, $|l\rangle_S \otimes |0\rangle_E$ indicates that the atom is in the excited level and the environment has no photon. A similar description exists for the state $|0\rangle_S \otimes |1_k\rangle_E$; therefore, the state vector is

$$|\psi(t)\rangle = \sum_{l=0}^{2} c_l(t) |l\rangle_S \otimes |0\rangle_E + \sum_{k} c_k(t) |0\rangle_S \otimes |1_k\rangle_E. \tag{2.24}$$

Using the Schrödinger equation, the time evaluation of probability amplitudes $c_1(t)$ and $c_2(t)$ is governed by the following coupled differential equations

$$\dot{c}_l(t) = -i \sum_{k} g_{lk} c_k(t) e^{i(\omega_l - \omega_k)t}, \quad (l = 1, 2) \tag{2.25}$$

$$\dot{c}_k(t) = -i \sum_{l=1}^{2} g_{lk}^* c_l(t) e^{-i(\omega_l - \omega_k)t}. \tag{2.26}$$

Then, substituting $c_k(t)$ from the solution of Eq. (2.26) into Eq. (2.25) with the initial condition $c_k(0) = 0$, i.e., we have no photon in the initial state of the environment, the time-varying probability amplitudes can be obtained as

$$\dot{c}_l(t) = -\sum_{m=1}^{2} \int_0^t f_{lm}(t - t') c_m(t') dt', \quad (l = 1, 2) \tag{2.27}$$

where

$$f_{lm}(t-t') = \int d\omega J_{lm}(\omega) e^{i(\omega_l-\omega)t - i(\omega_m-\omega)t'}. \tag{2.28}$$

It should be noted that $c_0(t)$ remains unchanged because $H_{\rm int}|0\rangle_S \otimes |0\rangle_E = 0$. In Eq. (2.28) $J_{lm}(\omega)$ denote the spectral density of the reservoir given by the following Lorentzian function

$$J_{lm}(\omega) = \frac{1}{2\pi} \frac{\gamma_{lm}\lambda^2}{(\omega_0 - \Delta - \omega)^2 + \lambda^2}, \tag{2.29}$$

where Δ is defined as the detuning between the atomic transition frequency and the central frequency of the reservoir and λ shows the spectral width of the coupling. Here, $\gamma_{lm} = \sqrt{\gamma_l \gamma_m}\theta$ ($l \neq m$ and $|\theta| \leq 1$) causes the spontaneously generated interference (SGI) between the two decay channels $|2\rangle \longrightarrow |0\rangle$ and $|1\rangle \longrightarrow |0\rangle$ with θ denoting the relative angle between two dipole moment elements of these decay channels and $\gamma_{ll} = \gamma_l$ corresponds to the relaxation rate of the lth excited state. For $\theta = 0$, dipole moment elements of the two decay channels are perpendicular to each other and we do not have any SGI between them while $\theta = \pm 1$ corresponds to the case of two parallel (or antiparallel) decay channels and shows the strongest SGI. Let $\tilde{c}_l(p) = L[c_l(t)] = \int_0^\infty c_l(t) e^{-pt} {\rm d}t$ ($l = 1, 2$), be the Laplace transform of $c_l(t)$. Taking the Laplace transform of Eq. (2.27), we have

$$\begin{pmatrix} p\tilde{c}_2(p) - c_2(0) \\ p\tilde{c}_1(p) - c_1(0) \end{pmatrix} = -\frac{\lambda}{2(p+\lambda)} \begin{pmatrix} \gamma_2 & -\sqrt{\gamma_1\gamma_2}\theta \\ \sqrt{\gamma_1\gamma_2}\theta & \gamma_1 \end{pmatrix} \begin{pmatrix} \tilde{c}_2(p) \\ \tilde{c}_1(p) \end{pmatrix}. \tag{2.30}$$

We need to apply the following unitary transformation on Eq. (2.30)

$$\upsilon = \begin{pmatrix} \sqrt{\frac{h+\gamma_1-\gamma_2}{2h}} & -\sqrt{\frac{h-\gamma_1+\gamma_2}{2h}} \\ \sqrt{\frac{h-\gamma_1+\gamma_2}{2h}} & \sqrt{\frac{h+\gamma_1-\gamma_2}{2h}} \end{pmatrix}, \tag{2.31}$$

and perform the inverse Laplace transform. Consequently, it is obtained

$$c_\pm(t) = G_\pm(t) c_\pm(0), \tag{2.32}$$

in which $c_\pm(0) = \frac{1}{\sqrt{2h}}\left(c_2(0)\sqrt{h \pm \gamma_1 \mp \gamma_2} \mp c_1(0)\sqrt{h \mp \gamma_1 \pm \gamma_2}\right)$ are the initial probability amplitudes with $h = \sqrt{(\gamma_1 - \gamma_2)^2 + 4\gamma_1\gamma_2\theta^2}$ and

$$G_\pm(t) = e^{\frac{-\lambda t}{2}} \left(\cosh\left(\frac{d_\pm t}{2}\right) + \frac{\lambda}{d_\pm} \sinh\left(\frac{d_\pm t}{2}\right) \right). \tag{2.33}$$

In Eq. (2.33), $d_\pm = \sqrt{\lambda^2 - 2\lambda\gamma_\pm}$ and $\gamma_\pm = \frac{\gamma_1+\gamma_2 \pm h}{2}$. Notice that the unitary transformation mentioned above corresponds to the space spanned by $\{|2\rangle, |1\rangle\}$. It can be extended on the whole space of the system spanned by $\{|2\rangle, |1\rangle, |0\rangle\}$ as

$$U = \begin{pmatrix} \sqrt{\frac{h+\gamma_1-\gamma_2}{2h}} & -\sqrt{\frac{h-\gamma_1+\gamma_2}{2h}} & 0 \\ \sqrt{\frac{h-\gamma_1+\gamma_2}{2h}} & \sqrt{\frac{h+\gamma_1-\gamma_2}{2h}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.34}$$

With these assumptions, the density matrix of a three-level V-type atom at time t takes the form

$$\varrho_s(t) = \begin{pmatrix} |G_+(t)|^2|c_+(0)|^2 & G_+(t)c_+(0)G_-^*(t)c_-^*(0) & G_+(t)c_+(0)c_0^* \\ G_+^*(t)c_+^*(0)G_-(t)c_-(0) & |G_-(t)|^2|c_-(0)|^2 & G_-(t)c_-(0)c_0^* \\ G_+^*(t)c_+^*(0)c_0 & G_-^*(t)c_-^*(0)c_0 & 1-|G_+(t)|^2|c_+(0)|^2-|G_-(t)|^2|c_-(0)|^2 \end{pmatrix}, \tag{2.35}$$

which corresponds to the time development of the state $\varrho_s(0) = U|\psi(0)\rangle\langle\psi(0)|U^\dagger$. Here, $|\psi(0)\rangle$ shows the state vector of the total system at time $t = 0$.
Let us introduce Kraus operators as

$$\kappa_1 = \begin{pmatrix} G_+(t) & 0 & 0 \\ 0 & G_-(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.36}$$

$$\kappa_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{1-|G_+(t)|^2} & 0 & 0 \end{pmatrix}, \tag{2.37}$$

$$\kappa_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sqrt{1-|G_-(t)|^2} & 0 \end{pmatrix}. \tag{2.38}$$

It is convenient and useful to rewrite $\varrho_s(t)$ in terms of Kraus operators

$$\varrho_s(t) = \sum_{i=1}^{3} \kappa_i \varrho_s(0) \kappa_i^\dagger, \tag{2.39}$$

which satisfy the condition $\sum_{i=1}^{3} \kappa_i^\dagger \kappa_i = I_3$. Finally, the original density matrix is given by

$$\rho_s(t) = U^\dagger \varrho_s(t) U. \tag{2.40}$$

The above results can be applied to the case of a system involves two three-level V-type atoms each of which couples independently to a Lorentzian reservoir. For such a two-qutrit system, the original density matrix at time t is defined as

$$\rho_s(t) = (U \otimes U)^\dagger \varrho_s(t)(U \otimes U), \tag{2.41}$$

where

$$\varrho_s(t) = \sum_{k,l=1}^{3} \kappa_{k,l} \varrho_s(0) \kappa_{k,l}^\dagger, \tag{2.42}$$

with $\kappa_{k,l} = \kappa_k \otimes \kappa_l$ and $\sum_{k,l=1}^{3} \kappa_{k,l}^\dagger \kappa_{k,l} = I_3 \otimes I_3$.

3 Quantum discord protection

In this section, based on WMQMR [39], we present a decoherence suppression scheme to protect GQD from the decoherence due to the AD channel [40]. We consider an arbitrary density matrix $\rho_s(0)$ of the qutrit as the initial state

$$\rho_s(0) = \begin{pmatrix} a & b & c \\ b^* & d & e \\ c^* & e^* & f \end{pmatrix}, \tag{3.1}$$

where $a + d + f = 1$. Notice that $\rho_s(0)$ can represent the density matrix of a pure state with the state vector $|\psi(0)\rangle$ or a mixed state.
In the first step, we perform the unitary transformation of Eq. (2.34) with $\gamma_1 = \gamma_2 = \gamma$ on the initial density matrix of the qutrit

$$\varrho_s(0) = U\rho_s(0)U^\dagger. \tag{3.2}$$

Next, a non-unitary operation as a prior weak measurement, M_w, is applied on the qutrit before it suffers decoherence arisen from the AD noise

$$\varrho_s^w(0) = M_w\varrho_s(0)M_w^\dagger, \tag{3.3}$$

with

$$M_w = \begin{pmatrix} \sqrt{1-p} & 0 & 0 \\ 0 & \sqrt{1-q} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.4}$$

where $0 \leq p,\ q < 1$ are called the weak measurement strengths which correspond to transitions $|2\rangle \longrightarrow |0\rangle$ and $|1\rangle \longrightarrow |0\rangle$, respectively. In the next step, the state is altered by the AD channel and the consequent state is given as

$$\varrho_s^w(t) = \sum_{i=1}^{3} \kappa_i \varrho_s^w(0)\kappa_i^\dagger. \tag{3.5}$$

According to the previous section, κ_i $(i = 1, 2, 3)$ are Kraus operators. To recover the initial state, the system is subject to a post-weak measurement reversal operation, M_r, which is also denoted by the following non-unitary operator

$$M_r = \begin{pmatrix} \sqrt{1-q_r} & 0 & 0 \\ 0 & \sqrt{1-p_r} & 0 \\ 0 & 0 & \sqrt{(1-q_r)(1-p_r)} \end{pmatrix}, \tag{3.6}$$

where $0 \leq p_r,\ q_r < 1$ indicate strengths of the reversing measurement. The density matrix of the qutrit with respect to this measurement is defined by

$$\varrho_s^r(t) = M_r\varrho_s^w(t)M_r^\dagger. \tag{3.7}$$

It is demonstrated that $p_r = 1-(1-p)|G_+|^2$ and $q_r = 1-(1-q)|G_-|^2$ provide the best restoration of the original state [41]. Moreover, it should be noted that $G_+(t)$ and $G_-(t)$ are in fact two complex functions, as a result we have

$$G_+(t) = |G_+|e^{i\varphi_+(t)}, \tag{3.8}$$
$$G_-(t) = |G_-|e^{i\varphi_-(t)}. \tag{3.9}$$

Substituting p_r, q_r, $G_+(t)$ and $G_-(t)$ into the $\varrho_s^r(t)$, we obtain

$$\varrho_s^r(t) = R\begin{pmatrix} \frac{1}{2}(a-b^*-b+d) & \frac{1}{2}(a-b^*+b-d)e^{i(\varphi_+(t)-\varphi_-(t))} & \frac{1}{\sqrt{2}}(c-e)e^{i\varphi_+(t)} \\ \frac{1}{2}(a+b^*-b-d)e^{-i(\varphi_+(t)-\varphi_-(t))} & \frac{1}{2}(a+b^*+b+d) & \frac{1}{\sqrt{2}}(c+e)e^{i\varphi_-(t)} \\ \frac{1}{\sqrt{2}}(c^*-e^*)e^{-i\varphi_+(t)} & \frac{1}{\sqrt{2}}(c^*+e^*)e^{-i\varphi_-(t)} & f+f' \end{pmatrix}, \tag{3.10}$$

with $R = (1-p)(1-q)|G_+|^2|G_-|^2$ and $f' = \frac{1}{2}(a-b^*-b+d)(1-p)(1-|G_+|^2) + \frac{1}{2}(a+b^*+b+d)(1-q)(1-|G_-|^2)$. In order to achieve the initial state of the qutrit, first we have to use the matrix

$$V = \begin{pmatrix} e^{i\varphi_+(t)} & 0 & 0 \\ 0 & e^{i\varphi_-(t)} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.11}$$

to omit phase factors from Eq. (3.10) in the following way

$$\varrho_s(t) = V^\dagger \varrho_s^r(t) V. \tag{3.12}$$

Then, we need to apply the unitary transformation on $\varrho_s(t)$ as

$$\rho_s(t) = U^\dagger \varrho_s(t) U. \tag{3.13}$$

Finally, the normalized qutrit state is described by

$$\rho(t) = \frac{R}{N}(\rho_s(0) + f'|0\rangle\langle 0|), \tag{3.14}$$

where

$$N = R(1+f'), \tag{3.15}$$

represents the normalization factor.
It is easy to extend this scheme to a two-qutrit system. The efficiency of our geometric discord protection protocol will be examined for two different families of two-qutrit systems such as Werner and Horodecki states. Let us begin with the Werner state.

3.1 Werner state

In this section, the initial state of the system is assumed to be prepared in a two-qutrit Werner state

$$\rho_\eta(0) = (1-\eta)\frac{I_9}{9} + \eta|\psi_0\rangle\langle\psi_0|, \tag{3.16}$$

where $|\Psi_0\rangle = \frac{1}{\sqrt{3}}\big(|22\rangle + |11\rangle + |00\rangle\big)$, I_9 shows the identity matrix and $\eta \in [0,1]$ is called the purity parameter. Notice that for $\eta \leq \frac{1}{4}$, the Werner state is PPT, whereas it will be NPT for $\eta > \frac{1}{4}$ [32,42]. The density matrix of a two-qutrit Werner state under the protection protocol discussed above is given by

$$\begin{aligned}\rho_\eta(t) =& \frac{R^2}{N_w}\Big\{\rho_\eta(0) + \Big(\frac{\eta}{3}s_1 + \frac{(1-\eta)}{9}(s_2+2s_3)\Big)|00\rangle\langle 00| \\ &+ \frac{(\eta+2)}{18}s_3\Big(|20\rangle\langle 20| + |02\rangle\langle 02| + |10\rangle\langle 10| + |01\rangle\langle 01|\Big) \\ &+ \frac{\eta}{6}s_4\Big(|10\rangle\langle 20| + |01\rangle\langle 02| + |20\rangle\langle 10| + |02\rangle\langle 01|\Big)\Big\},\end{aligned} \tag{3.17}$$

where

$$N_w = R^2\left(1 + \frac{\eta}{3}s_1 + \frac{(1-\eta)}{9}s_2 + \frac{2}{3}s_3\right), \tag{3.18}$$

is the normalization factor of $\rho_\eta(t)$ and

$$s_1 = (1-p)^2(1-|G_+|^2)^2 + (1-q)^2(1-|G_-|^2)^2, \tag{3.19}$$

$$s_2 = s_1 + 2(1-p)(1-q)(1-|G_+|^2)(1-|G_-|^2), \tag{3.20}$$

$$s_3 = (1-p)(1-|G_+|^2) + (1-q)(1-|G_-|^2), \tag{3.21}$$

$$s_4 = -(1-p)(1-|G_+|^2) + (1-q)(1-|G_-|^2). \tag{3.22}$$

Then using of Eq. (2.18), we can numerically calculate the lower bound of GQD for $\rho_\eta(t)$. In Figs. 1, 2 and 3, the variations of GQD versus γt and η are plotted for different values of λ and p when the two-qutrit system is initially in the Werner state. It should be noted that, without loss of generality, $p = q$. One can observe that for the case of $p = 0$, the GQD is decreased with time (for both cases $\lambda = 0.1$ and $\lambda = 1$) although the decrement rate for $\lambda = 0.1$ is smaller than the corresponding case of $\lambda = 1$, as is evident from Fig. 1. Under this condition, the GQD, ultimately, approaches to zero. As the weak measurement strength is nonzero ($p = 0.5$), the GQD can be protected against the dissipation but not completely (see Fig. 2). When the weak measurement strength $p \to 1$ (for example consider $p = 0.99$), the two-qutrit GQD for the Werner state becomes to be well protected against decoherence arisen from AD channel, as observed from Fig. 3. It is interesting to note that in the case of $p \to 1$, the efficiency of protection protocol is independent of the values of λ. This, in fact, leads to provide the protection of the GQD for the two-qutrit Werner state from decoherence in both Markovian and non-Markovian dynamics.

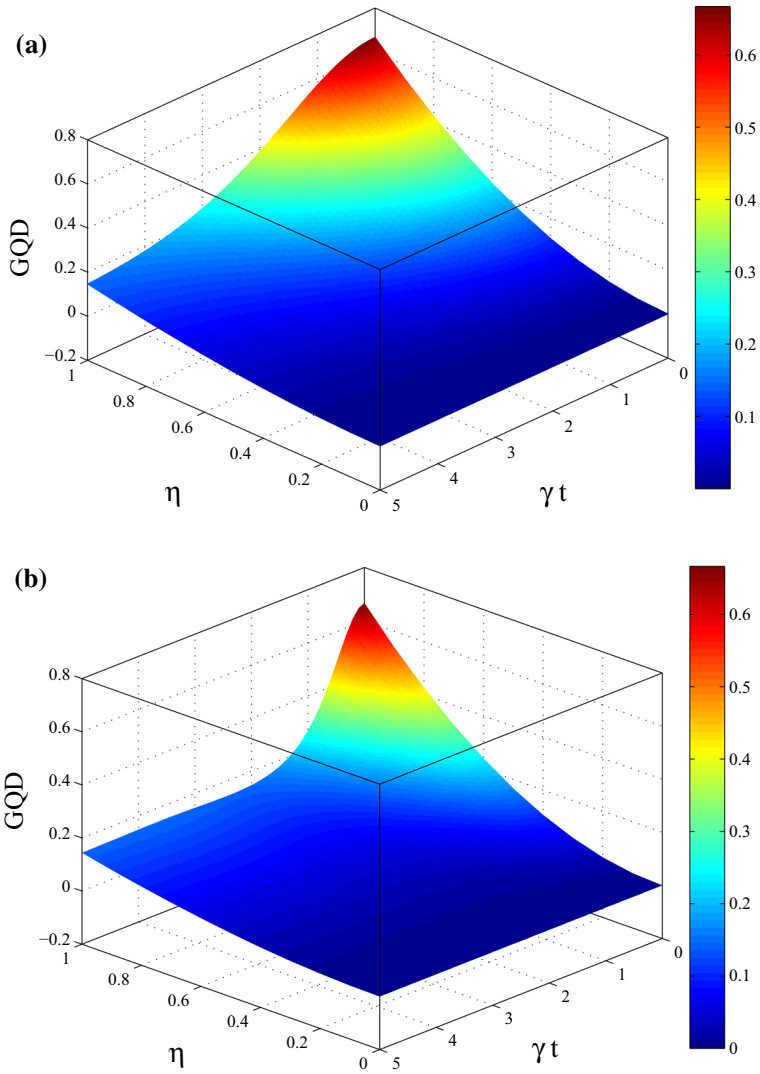


Fig. 1 Geometric discord for the Werner state as functions of η and γt without the protection protocol for **a** $\lambda = 0.1$ and **b** $\lambda = 1$, with $p = 0$

3.2 Horodecki state

In the next step, to show further the efficiency of the protocol, we take the well-known two-qutrit Horodecki state [43] as an initial state as follows

$$\rho_\alpha(0) = \frac{2}{7}|\psi_0\rangle\langle\psi_0| + \frac{\alpha}{7}\sigma_+ + \frac{5-\alpha}{7}\sigma_-, \tag{3.23}$$

where $0 \leq \alpha \leq 5$ and

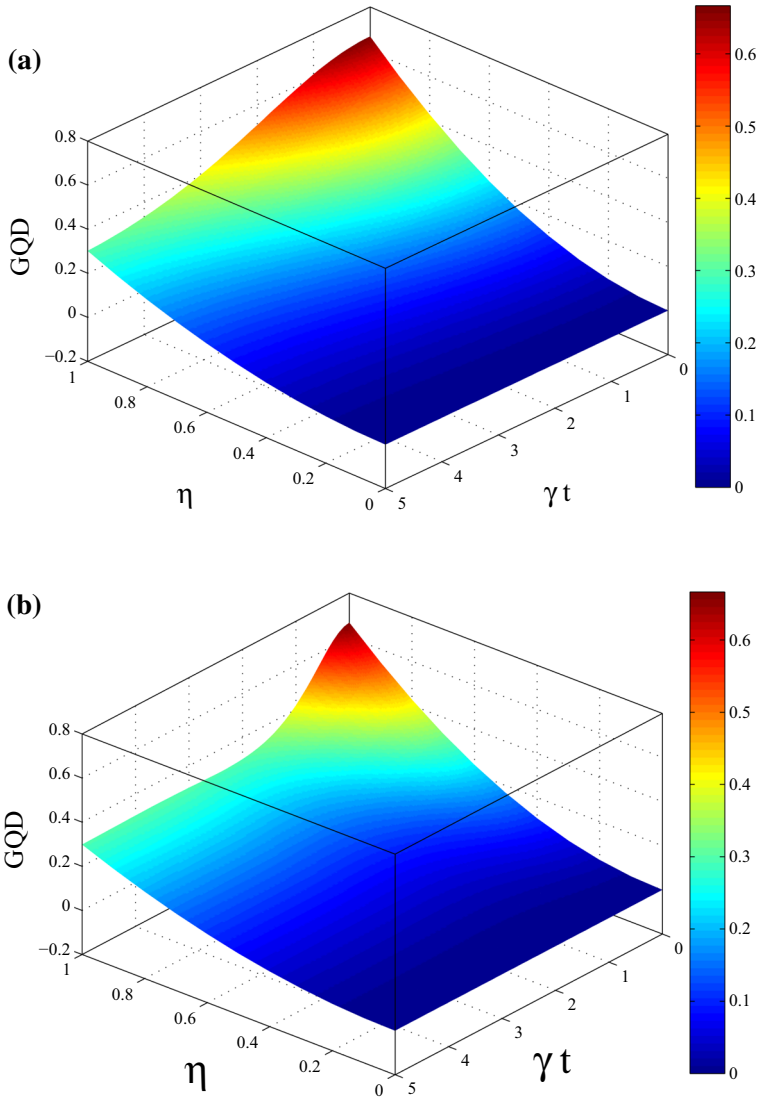


Fig. 2 Geometric discord for the Werner state as functions of η and γt under the protection protocol for **a** $\lambda = 0.1$ and **b** $\lambda = 1$, with $p = 0.5$

$$\sigma_+ = \frac{1}{3}\big(|01\rangle\langle 01| + |12\rangle\langle 12| + |20\rangle\langle 20|\big), \tag{3.24}$$

$$\sigma_- = \frac{1}{3}\big(|10\rangle\langle 10| + |21\rangle\langle 21| + |02\rangle\langle 02|\big). \tag{3.25}$$

It is known that the Horodecki state remains unchanged with the replacement of α by $(5-\alpha)$. $\rho_\alpha(0)$ is (i) separable for $2 \leq \alpha \leq 3$, (ii) bound entangled for $1 \leq \alpha < 2$ and $3 < \alpha \leq 4$, (iii) free entangled for $0 \leq \alpha < 1$ and $4 < \alpha \leq 5$. By considering the scheme for the Horodecki state, the final state after the related process becomes as

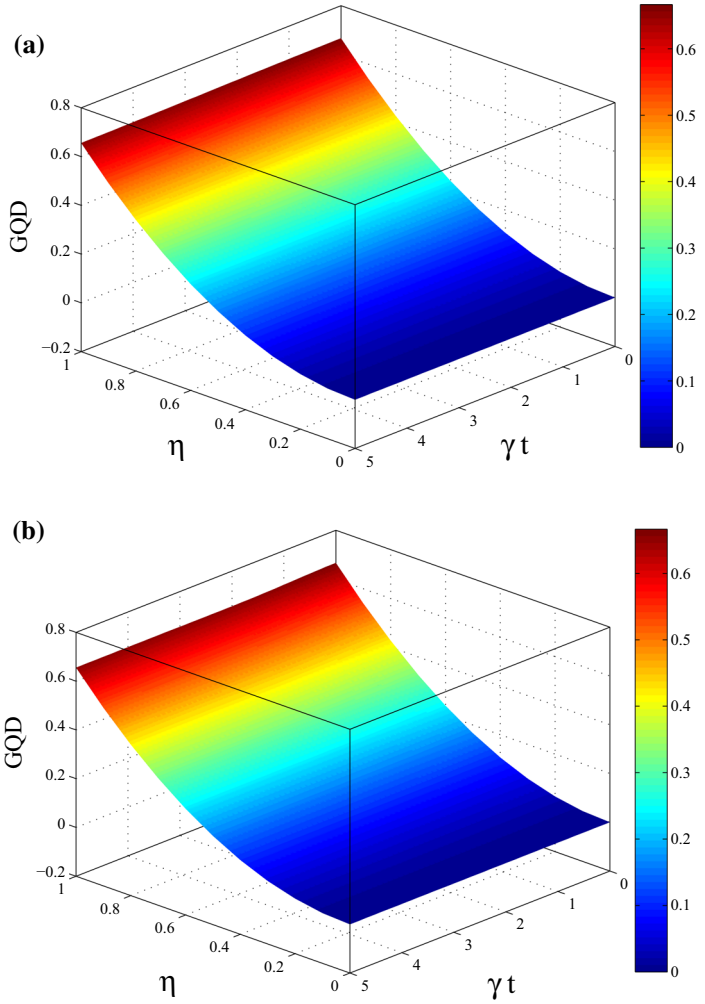


Fig. 3 Geometric discord for the Werner state as functions of η and γt under the protection protocol for **a** $\lambda = 0.1$ and **b** $\lambda = 1$, with $p = 0.99$

$$\begin{aligned}\rho_\alpha(t) =& \frac{R^2}{N_\alpha}\Big\{\rho_\alpha(0) + \frac{1}{21}\left(2s_1 + \frac{5}{4}s_2 + 5s_3\right)|00\rangle\langle 00| \\ &+ \frac{1}{21}s_3\left(|20\rangle\langle 20| + |02\rangle\langle 02| + |10\rangle\langle 10| + |01\rangle\langle 01|\right) \\ &+ \frac{1}{21}s_4\left(|10\rangle\langle 20| + |01\rangle\langle 02| + |20\rangle\langle 10| + |02\rangle\langle 01|\right) \\ &+ \frac{\alpha}{42}s_3\left(|20\rangle\langle 20| + |01\rangle\langle 01| - |10\rangle\langle 10| - |02\rangle\langle 02|\right) \\ &+ \frac{5}{42}s_3\left(|10\rangle\langle 10| + |02\rangle\langle 02|\right)\Big\}, \end{aligned} \tag{3.26}$$

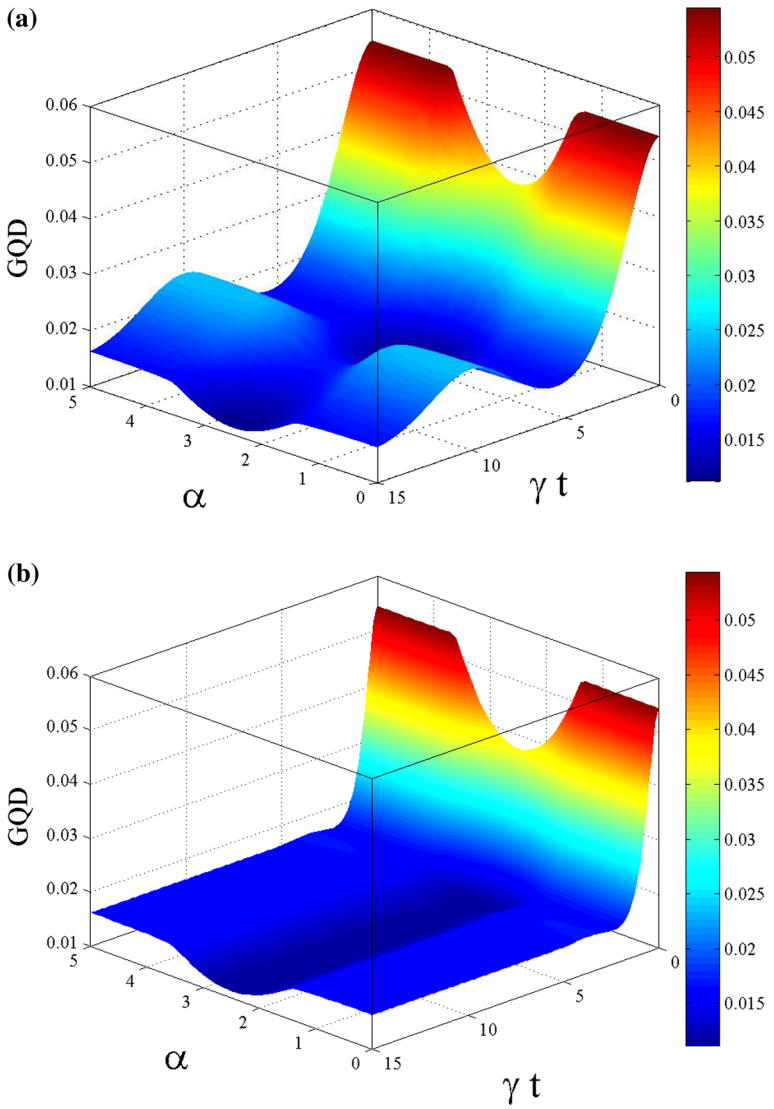


Fig. 4 Geometric discord for the Horodecki state as functions of α and γt without the protection protocol for **a** $\lambda = 0.1$ and **b** $\lambda = 1$, with $p = 0$

with the normalization factor

$$N_\alpha = R^2\left(1 + \frac{1}{21}\left(2s_1 + \frac{5}{4}s_2 + 14s_3\right)\right). \tag{3.27}$$

As it is expected, numerical results show that the increment of weak measurement strength p leads to the improvement of the protection process of the quantum discord against the decoherence of AD channel, as shown in Figs. 4, 5 and 6. As for the case of

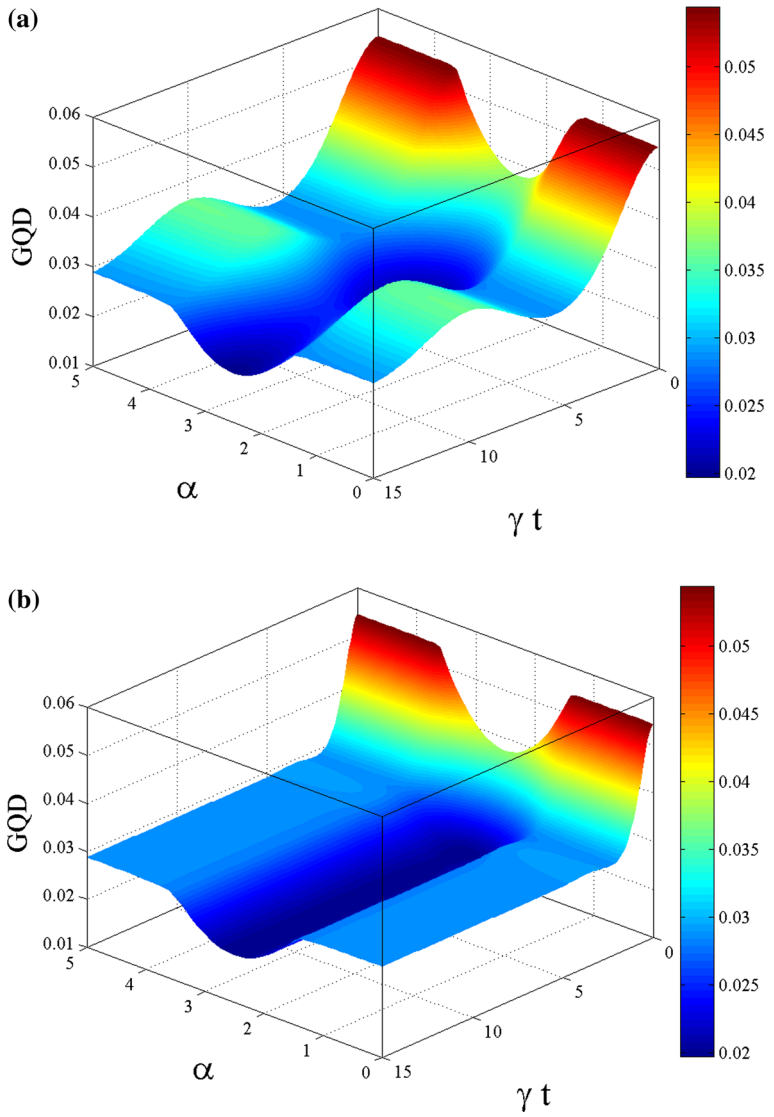


Fig. 5 Geometric discord for the Horodecki state as functions of α and γt under the protection protocol for **a** $\lambda = 0.1$ and **b** $\lambda = 1$, with $p = 0.5$

Werner state, when $p = 0$ the GQD suffers sudden death (especially for the Markovian dynamics corresponding to the Fig. 4b). Figure 5 shows that a nonzero p provides the protection of the GQD from sudden death. Consequently, for $p \to 1$, the GQD for the Horodecki state is also well-protected against the AD dissipation (see Fig. 6). It is worthwhile to mention that, apart from that the dynamical evolution of the two-qurit system is Markovian or non-Markovian, the initial time GQD of the Horodecki state can also be protected from the influences of AD channel.

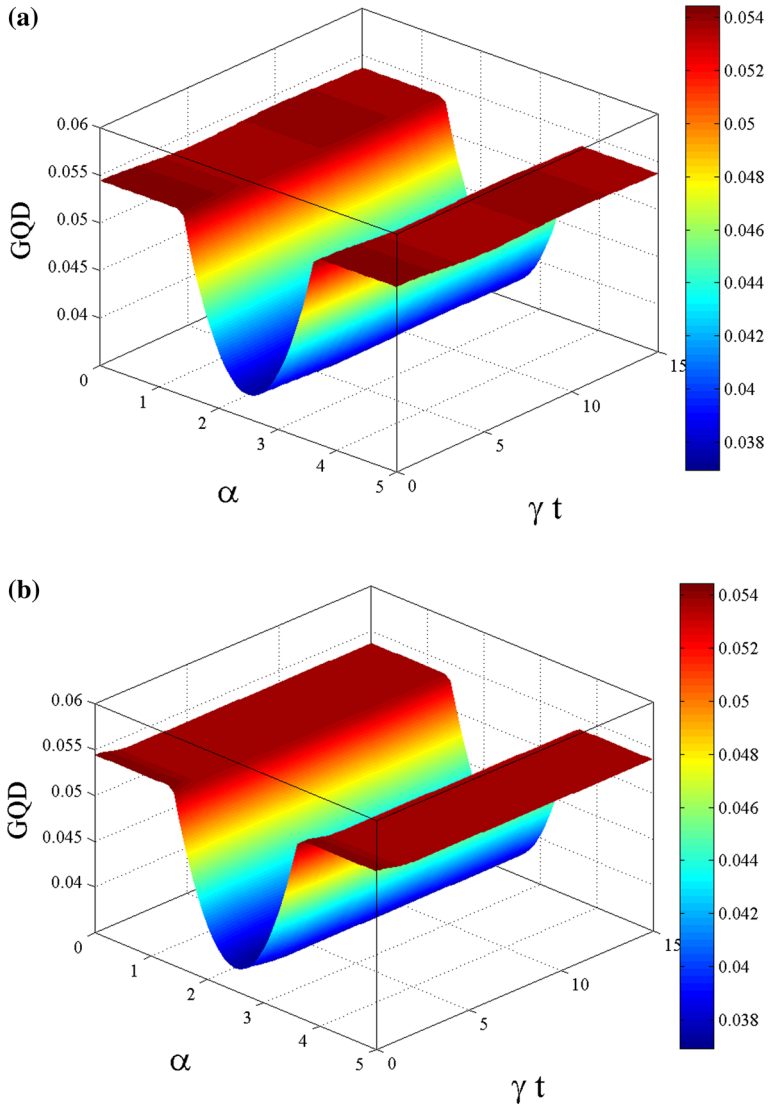


Fig. 6 Geometric discord for the Horodecki state as functions of α and γt under the protection protocol for **a** $\lambda = 0.1$ and **b** $\lambda = 1$, with $p = 0.99$

4 Conclusions

In this work, we first studied the lower bound of GQD to quantify the two-qutrit quantum discord. In the next step, we investigated the dynamics of a two-qutrit system each of which interacts with a dissipative environment independently. Then we showed that WMQMR can be used for protecting GQD of two-qutrit systems against the dissipative AD channel. To confirm the efficiency of our approach, we examined it for the two-qutrit Werner and Horodecki states. It was found that for the strength of

weak measurement with $p \to 1$, the GQD could be well preserved. In general, the efficiency of the proposed scheme is independent from weak or strong coupling of the system to the surrendering environment, i.e., Markovian or non-Markovian dynamics of the system.

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