# JOURNEY THROUGH THE DARK MONSTER

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## "THE FUTURE BELONGS TO THOSE WHO BELIEVE IN THE BEAUTY OF THEIR DREAMS"

"THE FUTURE BELONGS TO THOSE WHO BELIEVE IN THE BEAUTY OF THEIR DREAMS" is what I believe in. Setting everyday goals, living in the present and managing time polishes me each and every day. I love challenges and equally enjoy stress as it examines my capabilities to achieve my goal. I started my journey into physics at the age of 15 and was curious to understand the laws of nature. I eventually fall in love with particle physics for obvious reasons and started exploring the microscopic world. Along with this, I started my blog "Physics Mindboggler" to share my knowledge with others. The main agenda of Physics Mindboggler is to inspire, enhance and throw lights on various scientific aspects without misleading the audience. I firmly believe that research and development of technologies lead in building up a majority of the economic growth. Therefore, this field needs to be taken at the next level by today's young generation. With this and other upcoming books, I would like to inspire young physicists and make them think about the

revolution in technologies which they can bring via research. This book has been made with the help of my research diary which I have maintained from past few years. I hope you will definitely love the content and will learn something new from this book.

Thank you.

## PREFACE

Black holes have been all-time fascinating topic to study, learn and explore as a kid as well as an adult. Black hole makes us think about the possibility of interstellar travel and a path to visit ‘aliens'! It thus sometimes leads us to many unsolved paradoxes. In this book, I have put forward some useful and fascinating concepts of Blackhole. The chapters inside the book are arranged in a way to build strong pictorial and conceptual foundation to understand few aspects of Genersal Relativity and Black holes. In our journey through these cosmological monsters, we shall be halting at eighteen stops, starting from understanding the difference between supernova and hypernova we shall be seeing how a star like our ‘sun’ is born and what all things happen during their lifespan. We then shall see how a black hole is formed and will look at some mathematics (a solution to Einstein's field equations) to see how the first black hole was predicted! This solution is called a Schwarzschild metric, whose derivation would be available at our sixth stop. At seventh and eighth stops we will be talking about Minkowski spacetime diagrams to catch some grip on relativity which would be helpful for visualizing the effects and consequences of an object going inside a black hole. In our later stops, we shall be representing infinite time and space in a finite space and will visualize a white hole pictorially! Now there's only one step to achieve before we dig into the mathematics of General Relativity and Black Hole i.e. to understand and visualize Ring Singularity. We then are digging inside the mathematical description of black holes and will learn about time dilation, length contractions, tidal forces, spacetime interval, space behaving as time and time behaving as space, causal effects, etc. After departing from the $12^{th}$ station we shall be heading towards the quantum mechanics and will be seeing what quantum mechanics has to say about black holes. We shall then be going through some evidence, which explicitly explains the existence of black holes. In the latter part of our journey we shall be talking about wormholes and their mathematics and will eventually be talking about some open questions which were assigned by Wikipedia. I have not added any of my statements in "Open Questions", since I thought, that wiki could explain much better. We then get to our third last stop where we shall be talking about string theory and the Holographic principle followed by references. So without wasting further time, let's initiate our journey!

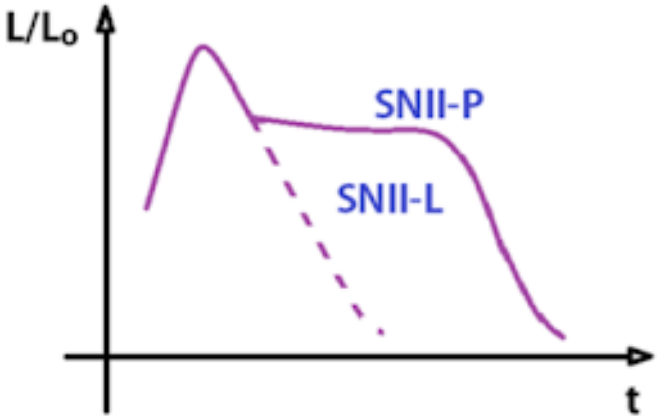
## SUPERNOVA AND HYPERNOVA

In 1054, Chinese astronomers recorded a supernova, which occurred 6,500 light years away. This supernova whose remnants we see today as crab nebula was so bright that

it was visible to the naked eye for several months. However the visible light represented only 1% of the energy released, remaining was in the form of UV, X-rays and gamma rays. Studying the formations and death of stars is indeed one of the most interesting topics in astronomy and sure we are going to explore the lifespan of a star at our upcoming stations. At this stations, we shall be categorizing supernova and shall develop an intuition about supernova and hypernova.

Supernovas are broadly classified into two types:-

**Type I supernova:** this type of supernova occurs when one star in the binary system is a white dwarf and gets enough energy from its companion star to ignite its nuclear fusion again, leading to cataclysmic supernova outburst. Since white dwarfs are mainly composed of carbon and oxygen and not hydrogen, the remnants of the explosion lack hydrogen and thus can be identified as type I supernova. On the basis of the spectral lines of the supernova, type I supernova can be classified as type Ia supernova, type Ib supernova, and type Ic supernova. Type Ia supernova shows a strong ionized silicon absorption lines whereas type Ib and type Ic do not show these lines. Type Ib shows neutral helium lines whereas type Ic lacks these lines.




*Graph 3.1 light emitted by type II L supernovas decreases steadily whereas in type II P it stays constant before diminishing.*

**Type II supernova:** this type of supernova occurs, when nuclear fusion inside a star's core stops, resulting in sudden collapse under its own gravity. Type II supernovas show the presence of hydrogen and are further classified on the basis of their light curves into type II P and type II L. From the graph 3.1 we can see that, light emitted by type II L supernovas decreases steadily whereas in type II P it stays constant before diminishing. Other categories of type II supernova are type II N supernova and type IIb supernova. Type II N supernova show some narrow spectral lines whereas in type II L and type II P supernovas, no such lines are observed. Type IIb supernova is the combination of type II and type Ib supernovas. In earlier stages of the explosion, presences of hydrogen atoms are detected but in later stages of explosion helium spectral lines are also observed, thus the name type IIb supernova.

Stars having a mass greater than 30 solar masses are considered to undergo a super-luminous supernova having luminosity 10 times greater than a usual supernova. This superluminous supernova is also called as a hypernova. Hypernovas have further distinguished between hydrogen rich and hydrogen poor hypernova. After discovering many super-luminous supernovas, astronomers have concluded that there is the requirement to introduce more categories to classify recently discovered hypernovas.

## THE BIRTH OF A GIANT!



*Figure 4.1 Crab Nebula*

Every star begins their life journey from the cloud of dust and gas (mainly hydrogen) called as a nebula. These gas particles clump together due to gravity and give birth to a protostar. A protostar having a mass less than 0.08 $M_\odot$ fails to generate enough temperature and pressure to initiate nuclear fusion of hydrogen atoms and ends as a brown dwarf. However, International Astronomical Union says that these stars are massive enough to fuse deuterium at some points of their lives. These stars have very less luminosity and eventually dies off, cooling gradually over hundreds and millions of years.

Other protostars are able to initiate their nuclear fusion and end becoming like our sun (1 $M_\odot$). These stars fuse hydrogen atoms and form helium atoms. The energy released in this fusion illuminates the star and also counterbalances the inward force of gravity, resulting in achieving a hydrostatic equilibrium. The lifespan of such stars is almost 10 billion years and they end becoming as a white dwarf which eventually fades into a black dwarf. But before coming to an end, there are some awesome and dangerous phenomena which the star goes from. After 10 billion years, nuclear fusion stops and then gravity takes over and increase the temperature and pressure of the core to such an extent that, helium atoms starts to fuse. The energy released from such fusion drastically increases the size of the star; this is called a star's red giant phase which last for about 100 million years. After this phase,

the star's surface bursts out and the white dwarf is formed surrounded by a planetary nebula.

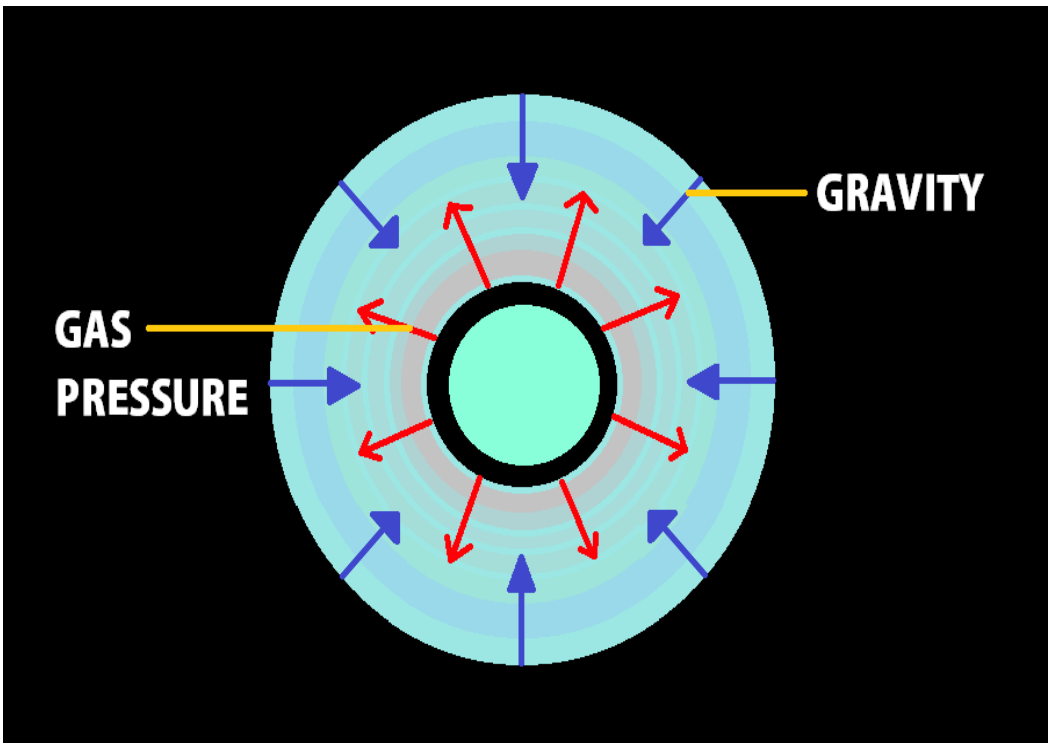



*Figure 4.2 Hydrostatic Equilibrium.*

Some stars which have masses of around 20-30 times the mass of our sun, lives shorter and more dramatic life than our sun. The lifespan of such stars is almost 10 million years and they have the capability to fuse helium atoms and can go up to the formation of iron atoms which generally occupy the star's core. Once nuclear fusion stops at iron, gravity takes over and after this rapid collapse, temperature rises so high that elements heavier than iron starts to form, including radioactive cobalt, titanium, aluminum, etc. and it later leads to a catastrophic explosion called a supernova. This massive explosion which occurs due to increase in density of the core because of crushing under its own gravity spreads out all known elements in outer space. The ultra-dense remnants of the core which are left behind after the explosion is called as a neutron star. One out of ten neutron stars ends becoming as magnetars. Magnetar is a type of a neutron star which has a huge magnetic field and emits high energy of electromagnetic radiations over its decay period of 10,000 years.

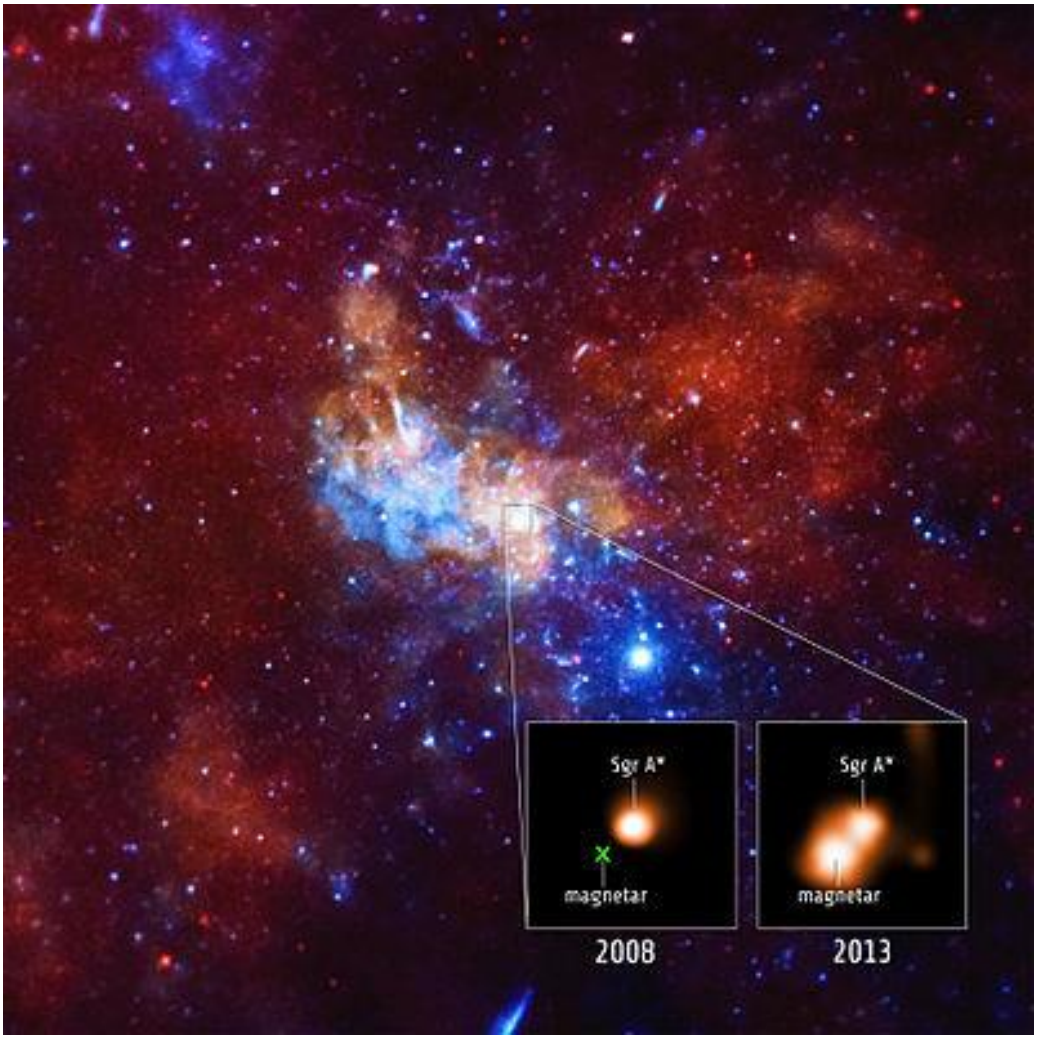



*Figure 4.3 Magnetar Near Supermassive Black Hole (source: NASA).*

**PARKER SOLAR PROBE MISSION:**

It's just amazing to see new technologies and revolution being made for space exploration as well. On 12th August 2018, NASA made a record to launch a space probe seven times closer than German-American Helios 2 spacecraft. It's the first time in NASA's history that this probe has been given a person's name, "Parker Solar Probe". Photos of Parker and a digital copy of his 1958 solar-wind paper is flying along with this probe along with a memory card which bears more than 1.1 million names. Last year NASA and ESA set a plan to send two probes closer towards the surface of the sun, one which is launched today by NASA "Parker Solar Probe" and other "Solar Orbiter" which would be launched by ESA to solve the decade-old question to understand the inner working of our nearest star. These probes have the aim to collect information and to help us in understanding the following:

1. Trace the flow of energy that heats and accelerates solar corona and solar wind.

2. Determine the structure and dynamics of plasma and magnetic fields at the sources of the solar wind.

3. Explore mechanisms that accelerate and transport energetic particles.

This mission has been planned for seven years and in these seven years, NASA's $1.5Billion Parker Solar Probe will perform 24 close flybys of the sun, getting within 6.16 million km (3.83 million miles) of the solar surface at its closest approach. The Solar Probe would encounter its first tight pass on the upcoming November when sun's gravity will accelerate the Solar Probe to one among the top speeds of around 690,000 km/h (430,000 mph). During the closest approach, the Solar Probe has to withstand 500 times more solar radiation that we experience on earth and a temperature of about 1,370 degree Celsius (2,500 degrees F). The mission would help us to understand the stellar behavior and its causes on space weather. The Parker Solar Probe is equipped with 2.3 meters (7.5 foot-wide), 11.4 centimeters (4.5 inch-thick) shield made of advance carbon composite material, which would keep spacecraft's most of the scientific instruments at around room temperature. Between these super-heated carbon-carbon composite material, there is a layer of carbon foam which together makes the Thermal Protection System (TPS) of the NASA's Parker Solar Probe. This TPS which is attached at the front of the Solar Probe can withstand a temperature of about 1,3750 degree Celsius (2,500 degrees F). The probe will be powered by twin Solar Arrays which uses pressurized water as its coolant. These instruments, along with others will measure the sun's electric and magnetic fields and waves, observe super-energetic particles, and photograph the corona and inner regions of the heliosphere.

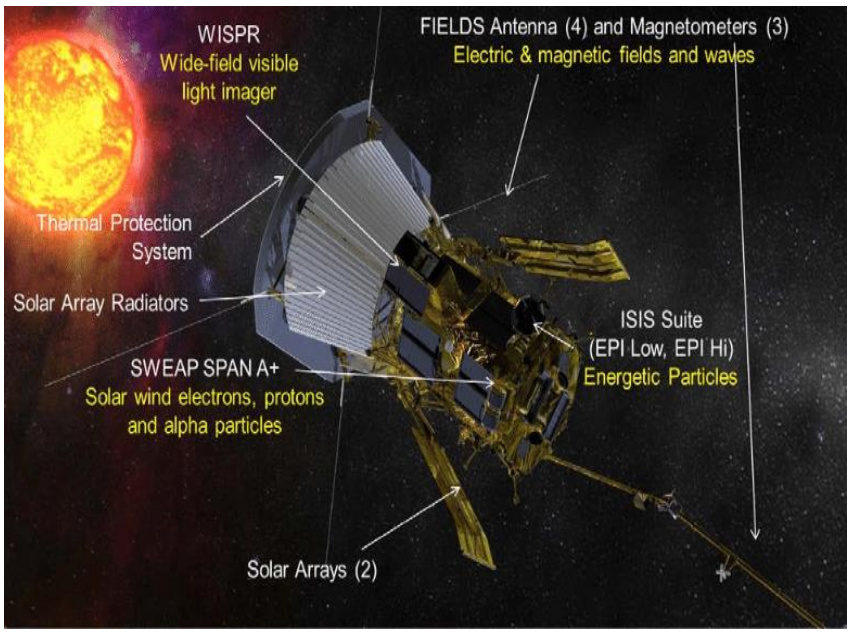



*Figure 4.4 Parker Solar Probe (source: Research-Gate).*

Our sun is filled with many mysteries; the sun emits charged particles called solar winds. These particles are highly dangerous for us humans. However, we are protected by these radiations via Earth's magnetic field. The solar wind comes zooming at a velocity of about 4.5 million to 2.9 million km/h (900,000 to 1.8 million mph) by the time it reaches the Earth's orbit. Theoretically, these solar winds should be motionless at the solar surface, but then what accelerates them at such a humongous speed is unknown. Another mystery is the Corona (last layer of the sun's atmosphere). In principle, the temperature must decrease as

we go away from the sun's core, however, what we find is that the temperature at Corona is 300 times hotter than the Photosphere (the lowest layer of the sun's atmosphere)! It's, therefore, the sun's gravity can't hold the charged particles at Corona leading them to escape in the form of solar winds. However, theories like these have not got any solid pieces of evidence. Therefore, NASA's Parker Solar Probe is going to play a very important role in solving many such stellar mysteries, understanding the behavior of solar weather, and to keep our planet safe from major upcoming solar disasters.

## CHASING THE DARK STAR

Black holes are generally made when blue giants and supergiants with solar masses greater than twenty undergo supernova. These stars instead of forming a neutron star, they collapse further due to their masses being more than 4 $M_{\odot}$ which is greater than a usual size of a neutron star. The collapse of the stars reaches the gravitational singularity which gives birth to a black hole. Black holes have a smaller diameter than its original star. A black hole of a mass of the earth can be of the diameter of just as your palm! Supermassive black holes have a very huge diameter and they are found at the center of almost every galaxy. Even our Milky Way galaxy has a supermassive black hole (Sagittarius A*) which has a mass of four million solar mass. Other than a supermassive black hole our galaxy also has ten million to a billion black holes.

It's a common misconception that black holes suck everything. The gravitational influence of a black hole is the same as that of its original star. So if a planet somehow saves itself from the destruction of supernova, it will be orbiting the black hole in the same manner as it was orbiting the then star. Another misconception is that black holes are circular in shape as far as its outer appearance is concerned. But in fact, black holes are spherical in structure and is predicted that it is connected to a white hole. Any object which gets in contact with black hole's event horizon gets sucked in; it's the point of no return. Now if you think of this scenario very carefully, you will notice that if the object is sucked in that means information and energy is lost and entropy of the universe decreases which contradicts our physical laws. The solution to this problem might be the Hawking radiation.



*Figure 15.1 Black Hole*

It's a prediction done by Mr. Hawking that black holes might be emitting photons and other particles which should result in evaporation of black hole. The thermal energy radiated by a black hole is inversely proportional to its mass. What it means is that since black holes have huge mass, it's difficult to detect these radiations. A black hole of 1 $M_{\odot}$ will emit thermal energy of temperature 62 nanokelvins which is far less than the 2.7 K temperature of Cosmic Microwave Background radiation.

You will be amazed to know that in principle, high-energy collisions can make a black hole. One such machine is the Large Hadron Collider (LHC) where micro black holes are created due to high-energy collisions. But practically no such black holes are detected in LHC since these micro black holes evaporate within $10^{-25}$ seconds. It's also being predicted that gravitons (hypothetical particle of gravity) produced in the collision process might be escaping through these micro black holes and causing difficulty for the detectors to detect it. Traveling through a black hole is not a funny business, but time travel is possible and you will be amazed that we do have the theoretical blueprint of the time machine (Shall be exploring at later stations)!

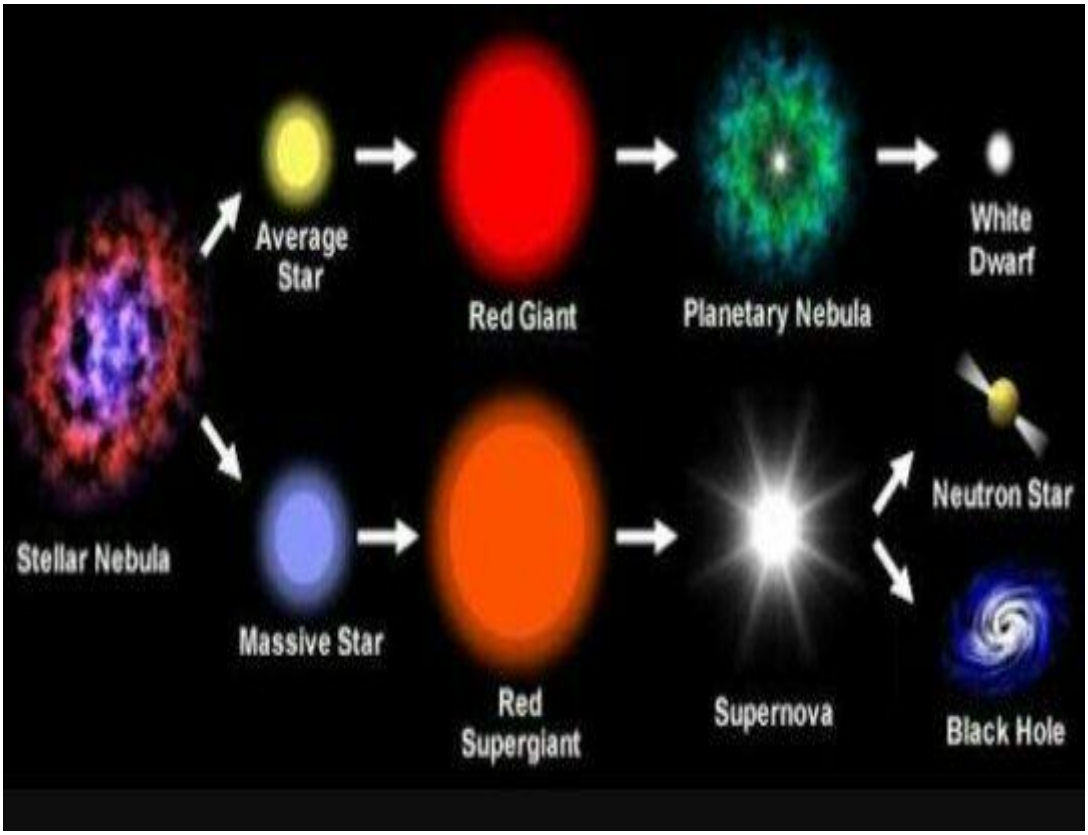



*Figure 5.2 Summary of Stellar Evolution.*

## HOW WAS A BLACK HOLE PREDICTED MATHEMATICALLY?

In 1798, Pierre Laplace calculated the escape velocity of a massive body. He kept changing the mass and radii until he found that the escape velocity of that particular body is the speed of light. Since no particle with mass can ever go beyond the speed of light that massive body with small radii which has an escape velocity greater than light will appear dark. Such bodies or dead stars were called ‘dark stars'. In 1915, Albert Einstein published his theory of General Relativity. A few months later Schwarzschild gave a solution to Einstein's field equations. What he did is that he investigated the consequence of concentrating the entire mass of an exactly spherical body at a mathematical point. He thus discovered Schwarzschild metric (the metric has been derived at the next station):

$$ds^2 = [1 \div (1 - r_s/r)]\, dr^2 - (1 - r_s/r)\, c^2dt^2 + r^2(d\Theta^2 + \sin^2\Theta d\phi^2)$$

This metric gave us the space interval around a spherically dense object. Here, $r_s$ and r represent Schwarzschild radius (radius of that eternally spherical object) and distance of an external body (eg. Particle with mass) from the center of the sphere. The term $[1 \div (1 - r_s/r)]$ and $(1 - r_s/r)$ tells the extreme warping of space-time. This dense, uncharged, non-rotating, eternal-spherical dark object was termed as a black hole by John Wheeler in mid-1960s. Other mathematicians like Roy Kerr, Hans Reissner, and Gunnar Nordstrom contributed their efforts on other kinds of black holes.

# TYPES OF BLACKHOLES

1. **Schwarzschild**: Non- rotating
   Uncharged
   Perfectly spherical
   Eternal

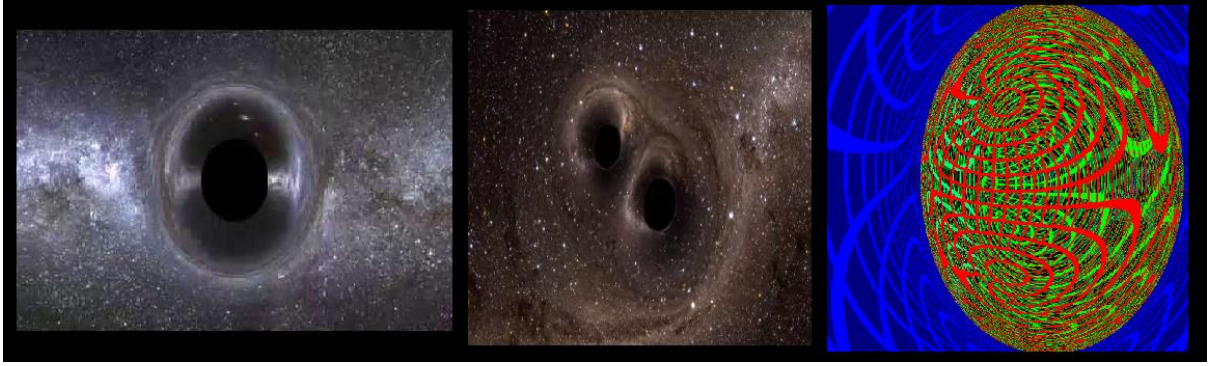
2. **Reissner - Nordstrom**: Non- rotating

3. **Kerr** : Rotating
   Flattened at the poles

4. **Kerr-Nordstrom** : Rotating

5. **Plank-Mass** : Mass of 0.00000001 kilograms
   Size is 100 billion billion times smaller than a proton

6. **Primordial** : Mass greater than 10 trillion kilograms



## What are the different types of black holes?

1 Schwarzschild: These are non-rotating, uncharged, perfectly spherical, eternal black holes. They are defined only by their total mass.

2 Reissner-Nordstrom: These black holes are defined by their mass and charge, but are non-rotating.

3 Kerr: These black holes are rotating and thus are flattened at the poles. Therefore, they are defined by mass and angular momentum.

4 Kerr-Nordstrom: These possess mass, charge and are rotating.

5 Plank-Mass: These have a mass of 0.00000001 kilograms and a size that is 100 billion billion times smaller than a proton.

6 Primordial: These can have a mass greater than 10 trillion kilograms and were formed soon after the big bang and can still exist today.

## How can we make a black hole?

Take a planet or a star (eternal) and squeeze it equal to or below the Schwarzschild radius ($r_s$), and there you have made an eternal black hole. The relation between Schwarzschild radius ($r_s$) and mass (M) of original star or planet can be given as:

$$r_s = (2GM) \div c^2$$

Therefore, a black hole of a mass of the earth can be of the diameter of just as your palm!

## DERIVING THE SCHWARZSCHILD METRIC

The general theory of relativity published by Albert Einstein in 1915 gave a different understanding about gravity. It described gravity as the 'virtual' bending of spacetime which otherwise was seen as the force between two massive objects in the Newtonian gravity. Einstein's great insight was that matter distorts and curves the spacetime metric in three-dimensional space and one dimension of time. This relationship of the distortion in the spacetime metric is described by famous Einstein's field equation:

$$R_{ab} - (½)Rg_{ab} + \Lambda g_{ab} = (8\pi G T_{ab})/c^4$$

On the left-hand side of the equation we have terms involving the Ricci tensor $R_{ab}$ and its trace, the Ricci scalar R. These tensors are functions of the metric which measure the amount of curvature present at each point of our spacetime. The $g_{ab}$ here is the spacetime metric. On the right-hand side we have G, Newton's gravitational constant; c, the speed of light; and $T_{ab}$ which is called the energy-momentum tensor. This describes the amount, type and distribution of energy and matter in space. A few months later after publishing General Relativity paper, Schwarzschild gave a solution to Einstein's field equations which we call the Schwarzschild metric. Before starting with the derivation we would assume a few things:

1. A spherical symmetric spacetime is one that is invariant under rotations and taking mirror image.
2. A static spacetime is one in which all metric components are independent of time.
3. A vacuum solution is one which satisfies $T_{ab} = 0$, which implies $R_{ab} = 0$ from Einstein's field equation.
4. Metric signature used here are (+, +, +, -).

The first step is to make a general metric and then to find the dependency of each component. The second step is to find Christoffel symbols and Ricci tensor. The third step is to define A and B functions and then substituting all factors in the general metric.

STEP1: DEFINING THE GENERAL METRIC:

Since all metric components are independent of time coordinate, therefore, we can make a metric of following:

$$\Delta s^2 = g_{11}\Delta r^2 + g_{22}\Delta\Theta^2 + g_{33}\,\Delta\phi^2 + g_{44}\,\Delta t^2 \qquad \ldots(i)$$

On each hypersurface with constant t, constant Θ, constant ϕ, $g_{11}$ should only depend on r. Therefore, by spherical symmetry $g_{11} = A(r)$. Similarly, on each hypersurface with constant t, constant Θ, constant ϕ, $g_{44}$ should only depend on r. Therefore, by spherical symmetry $g_{44}$ =

B(r). On each hypersurface with constant t and constant r, a 2-dimensional sphere metric is required, which can be represented as:

$$\Delta\Omega^2 = r^2(\Delta\Theta^2 + \sin^2\Theta\Delta\phi^2)$$

Now if we change the Θ and ϕ components on this hypersurface, even then, the metric would remain unchanged due to spherical symmetry. If we compare the given hypersurface with the components in eq(i), we get,

$$g_{22}\Delta\Theta^2 + g_{33}\,\Delta\phi^2 = r^2(\Delta\Theta^2 + \sin^2\Theta\Delta\phi^2)$$

$$g_{22}[\Delta\Theta^2 + (g_{33}/g_{22})\Delta\phi^2] = r^2(\Delta\Theta^2 + \sin^2\Theta\Delta\phi^2)$$

Therefore, from comparison we get $g_{22} = r^2$ and $g_{33} = r^2\sin^2\Theta$. Therefor eq(i) can be re-written as:

$$\Delta s^2 = A(r)\Delta r^2 + r^2\Delta\Theta^2 + r^2\sin^2\Theta\Delta\phi^2 + B(r)\Delta t^2 \qquad ...(ii)$$

with A and B as yet undetermined functions of r. Note that if A or B is equal to zero at some point, the metric would be singular at that point.

STEP2: FINDING THE CHRISTOFFEL SYMBOLS AND RICCI TENSOR:

Christoffel symbols can be calculated by using the formula:

$$\Gamma^a_{bc} = (½)\, g^{ad}[g_{bd,c} + g_{cd,b} - g_{bc,d}]$$

(I think a video would be the best source to understand the calculation and extraction of the required equations from the Christoffel equations. Kindly visit: https://www.youtube.com/watch?v=Axhz7NAk4BM&t=12s).

The Ricci tensor can be calculated by using formula:

$$R_{ab} = \Gamma^d_{ad,b} - \Gamma^d_{ab,d} + \Gamma^d_{ae}\,\Gamma^e_{db} - \Gamma^d_{ed}\,\Gamma^e_{ab}$$

Since we have assumed a vacuum solution, therefore, $R_{ab} = 0$. Therefore,

$$\Gamma^d_{ad,b} - \Gamma^d_{ab,d} + \Gamma^d_{ae}\,\Gamma^e_{db} - \Gamma^d_{ed}\,\Gamma^e_{ab} = 0$$

For solving different values of rows and column in metrix (watch the vedio) we conclude that only few $\Gamma^a_{bc}$ terms are non-zero. After substituting respective $\Gamma^a_{bc}$ terms in $R_{ab}$ we get four equations:

$$4A'B^2 - 2rB''AB + rA'B'B + rB'^2A = 0 \qquad ....(1)$$

$$rA'B + 2A^2B - 2AB - rB'A = 0 \qquad ....(2)$$

$$-2rB''AB + rA'B'B + rB'^2A - 4B'AB = 0 \qquad ....(3)$$

$$\sin^2\Theta(rA'B + 2A^2B - 2AB - rB'A) = 0 \qquad ....(4)$$

Where A' or B' are first derivative and A" or B" are second derivative. Subtracting first and third equations we get: A'B + AB' = 0; which in fact is the derivative solution for A(r).B(r) = K. Substituting K in second equation gives:

$$rA' = A(1 - A)$$

which has a general solution of $A(r) = [1 + (1/Sr)]^{-1}$. Therefore $B(r) = K = [1 + (1/Sr)]$.

STEP3: SUBSTITUTING ALL FACTORS IN METRIC:

Now since we have defined the functions A and B we can substitute them in eq(ii) and we get our Schwarzschild interval as:

$$\Delta s^2 = [1 \div (1 - r_s/r)]\, \Delta r^2 - (1 - r_s/r)\, \Delta t^2 + r^2(\Delta\Theta^2 + \sin^2\Theta\Delta\phi^2)$$

Note: here value of c = 1; if c > 1 then $\Delta s^2 = [1 \div (1 - r_s/r)]\, \Delta r^2 - (1 - r_s/r)\, c^2\Delta t^2 + r^2(\Delta\Theta^2 + \sin^2\Theta\Delta\phi^2)$. As r reaches to infinity the metric approaches that of the Minkowski metric and the spacetime manifold resembles that of Minkowski space (Will be discussing at later stations).

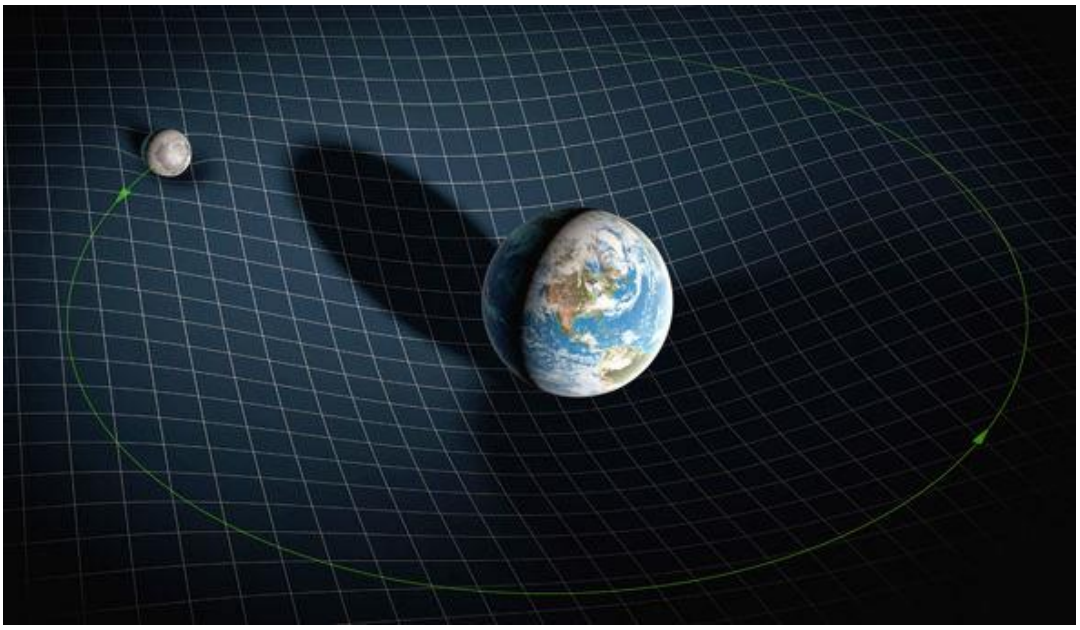
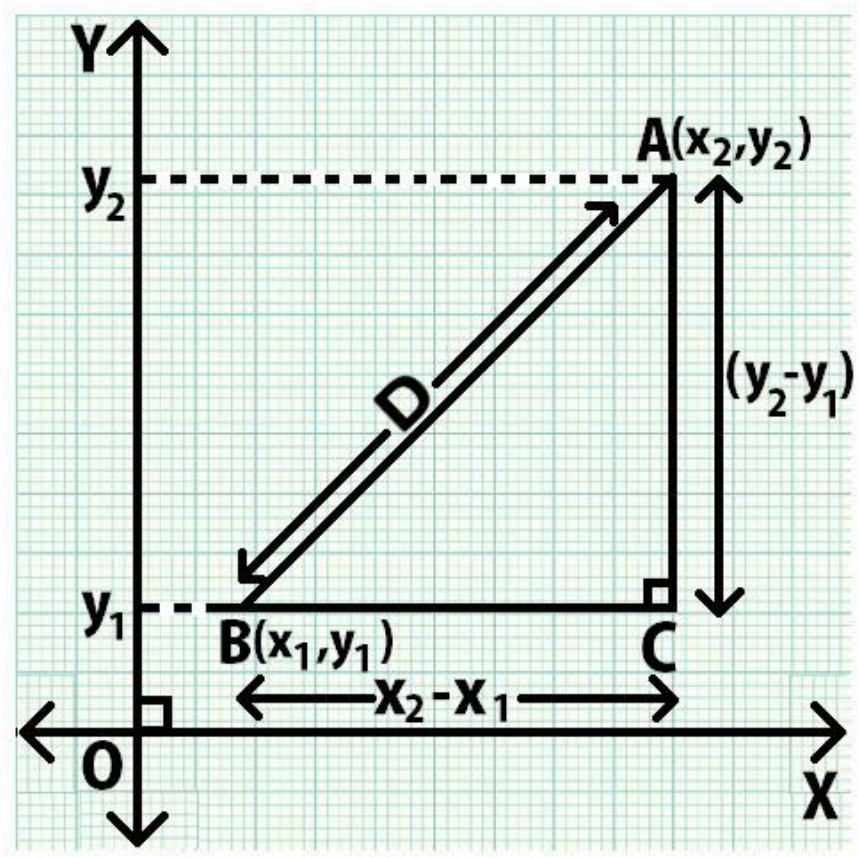


*Figure 7.1 Visualising Spacetime Curvature (source: Humanizer-News).*

## MINKOWSKI SPACETIME INTERVAL THROUGH BASIC ALGEBRA

## PART1



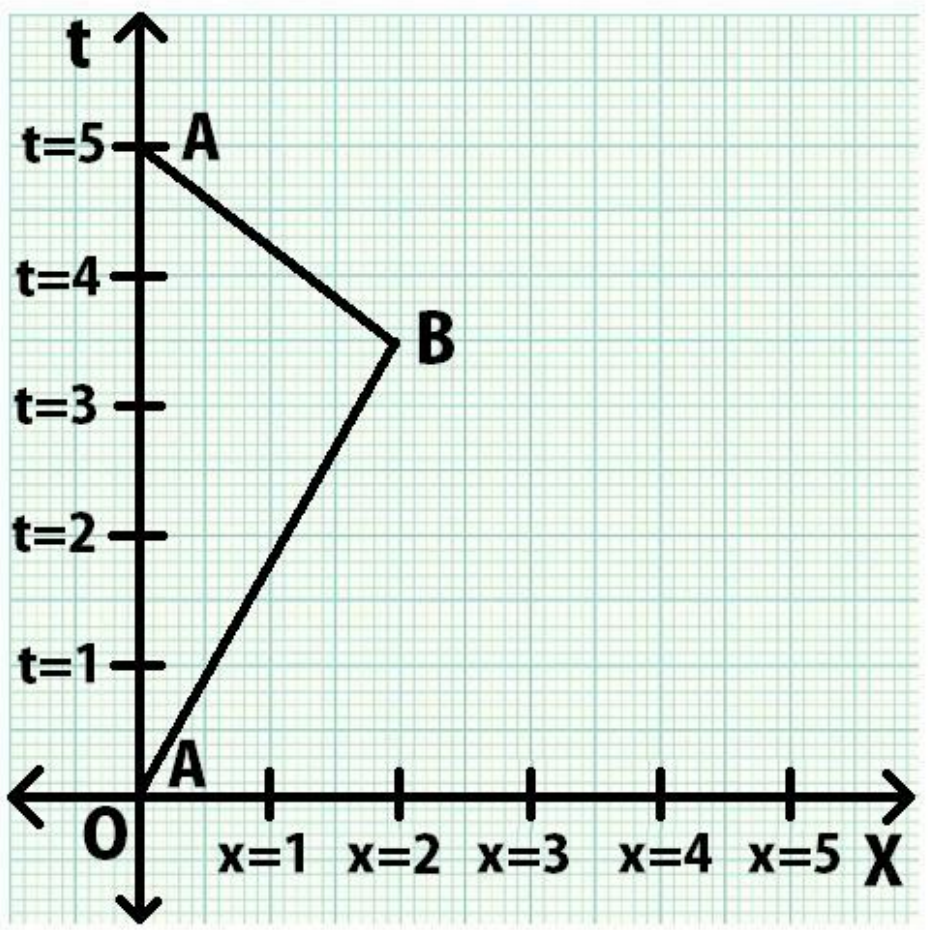

*Graph 8.1*

To understand relativity you just need an understanding of basic algebra. However, things become truly nasty when we start thinking about the events in different frames of reference. One of the best solution to visualize the transformation between the reference frames between any observers is Minkowski's spacetime diagram. Consider the graph 8.1, to calculate the distance between the two points we can simply apply distance formula:

$$D^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

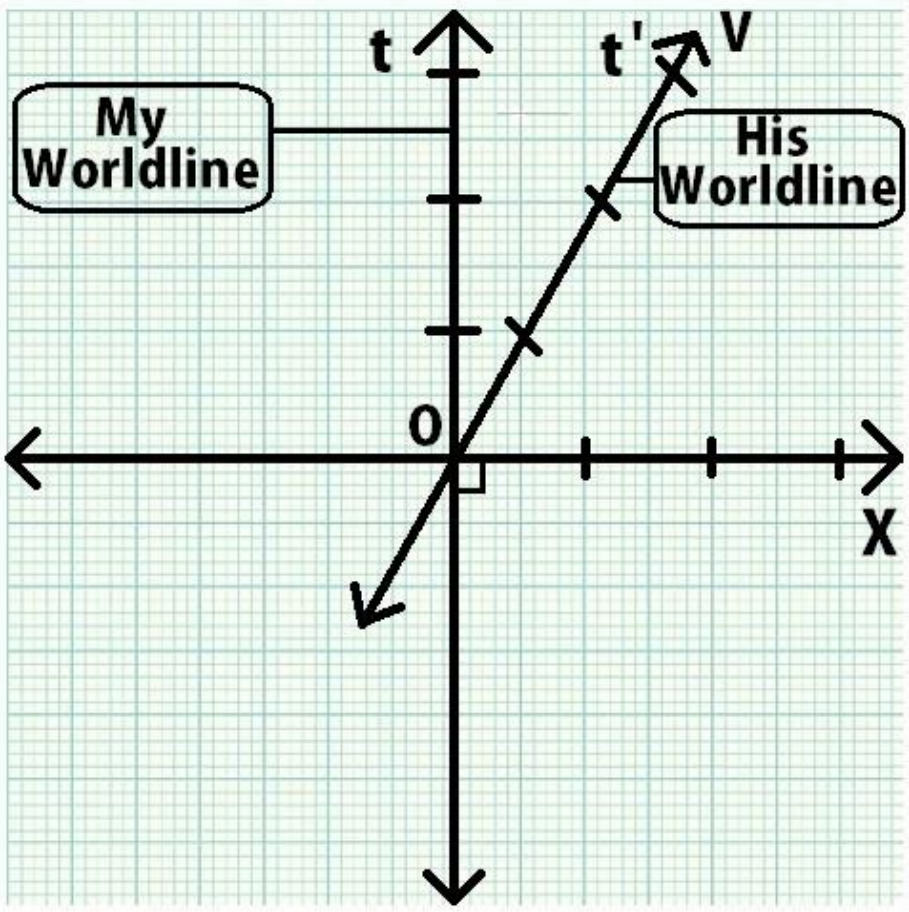
Which basically comes from Pythagoras theorem. Now we simply replace our y-axis with time t and we get a graph similar to graph 8.2. In graph 8.2 we have plotted two events; one I going from point A to point B and two I coming to my original position i.e. point A. Do note that this is a linear i.e. one-dimensional motion but since we have added time as our second coordinate, we are plotting my events in two-dimensional spacetime. Let's consider graph3 where the events happening are from my perspective and not from your/ third person's perspective. For the simplicity of imagination let's say I see a person riding on a horse. From my perspective I am stationary and the rider is moving with velocity 'v'. But from rider's perspective/ reference frame (constant velocity/ no acceleration), he is stationary and I am moving. Therefore if I want to plot my spacetime diagram it would be like graph 8.3.1 and if the rider is plotting the graph, then, his spacetime diagram would be like graph 8.3.2. In both the graphs, we observe that time and distance covered by him or me is the same as that of me or him. The graph basically defines the Newtonian understanding of space and time as two different entities and time and space to be absolute. However, in relativity

world we know that time dilates and length contracts. Therefore the graph 8.3.1 and 8.3.2 are wrong.

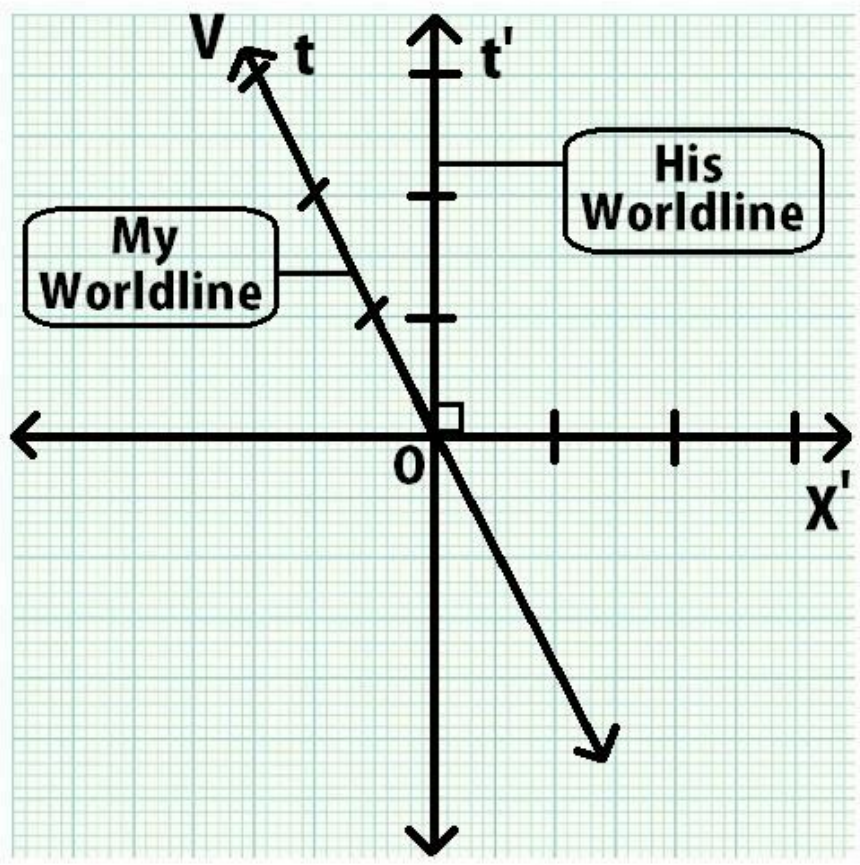



*Graph 8.2*

1. *I going from point A to point B*
2. *I coming to my original position.*




*Graph 8.3.1*




*Graph 8.3.1*

In relativity space and time are two components of a single spacetime, where space and time are not absolute, it varies for different observers. 'Transformations' are what required to change from one reference frame to another reference frame. Such transformations were developed by Galileo Galilei and then upgraded by Lorentz. The problem with Galilean transformation was that space and time were independent of the velocity, whereas the Lorentz transformation showed the dependency of space and time on velocity by inserting the Lorentz factor. So when we change the reference frame from me to the rider the correct spacetime diagram for graph 8.3.2 will be like graph 8.4.2 and the ticks (mark) between the time and space intervals (U') for the rider can be calculated by:

$$U' = U\,[\sqrt{(1+\beta^2)} \div \sqrt{(1-\beta^2)}]$$

Where,

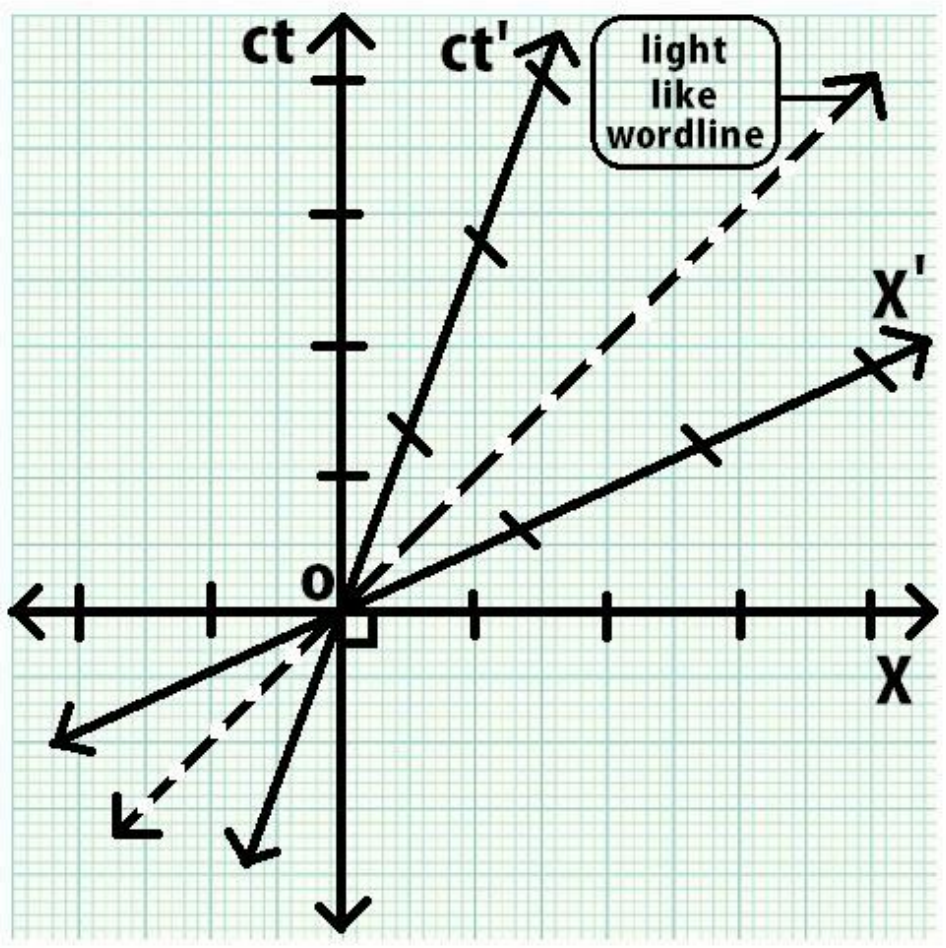
U = unit length of x and ct axes

U' = unit length of x' and ct' axes

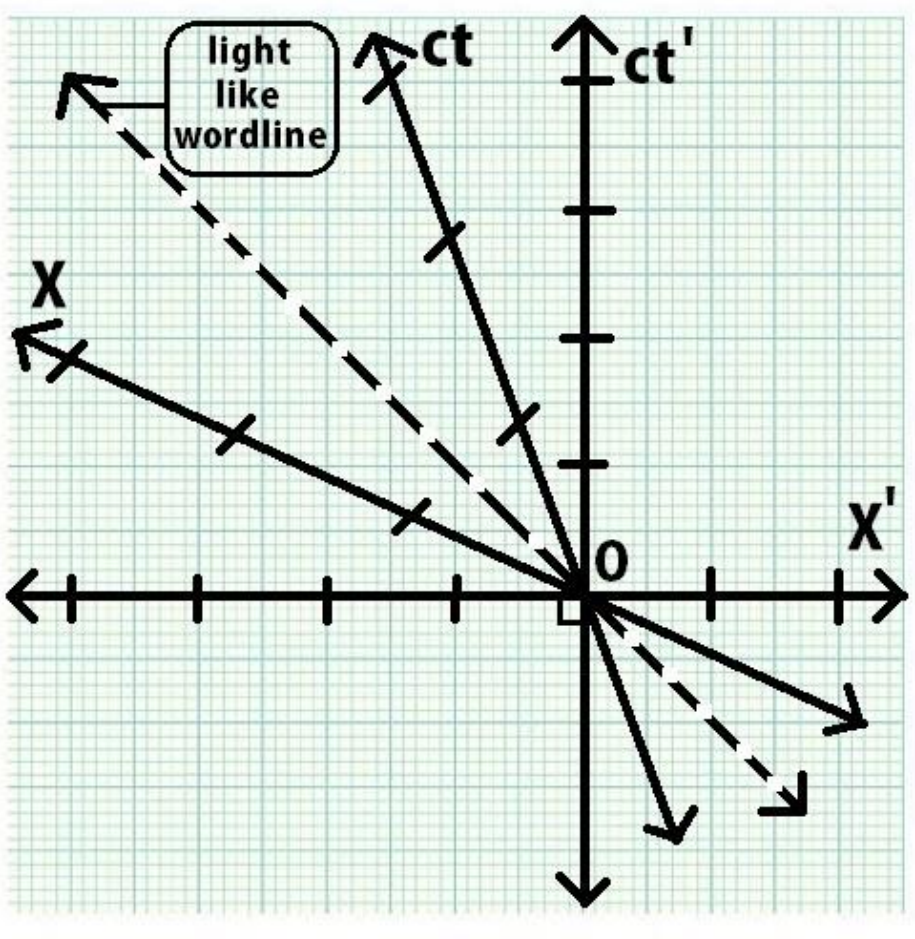
$\beta = v/c = \tan(\alpha)$ [will be discussed shortly]

Note that instead of time-axis we denoted ct-axis, this is because space and time are not absolute and we can measure ct in terms of meters (similar to space units) which will give a

dimensionless slope (β). Therefore the correct spacetime diagram for a given event is given in graph 8.4.1 and 8.4.2.




*Graph 8.4.1*




*Graph 8.4.2*

Now one may ask that what would be the slope for a given velocity, then, if you have paid attention, the slope (β) can be calculated by v/c, therefore at v = c, β = 1, therefore, the light waves will always travel a $45^0$ angle. The x'-axis is considered to be at same angle α from x-axis which is the same as that of ct'-axis with respect to ct-axis. Another point to mention is that no matter what your reference frame be, the speed of light is constant for all inertial observers, which do satisfies the second postulate of special relativity.

## INTRODUCTION TO SPECIAL RELATIVITY:

Relativity is all about relating the situation with one's perspective. Let's start with an example: consider two men (man1 and man2). Man1 is standing stationary on the side of the road and sees man2 passing on the road in his car. If we think with man1's perspective, then he would say that he is stationary and man2 is moving relative to him (man1). But when we think from man2's perspective, man2 would say that he is stationary and man1 and the entire surrounding is moving relative to him (man2). So that's what relativity is all about, we get two different answers and the amazing fact is that all these answers are correct!

The two postulates given by Einstein for special relativity are as follows:

1] Laws of physics remain the same for all inertial observers.

2] Vacuum speed of light is the same for all observers.

On the basis of these two postulates Einstein derived two formulae:

$$X' = \gamma (X - Vt)$$

$$t' = \gamma (t - VX/c^2)$$

Where,

$\gamma$ = Lorentz factor = $1 \div \sqrt{(1 - (v^2/c^2))}$.

V = relative velocity.

X = x-coordinate of man1.

X' = x-coordinate of man2.

t = time measured for man1.

t' = time measured for man2.

And the equations are originally called as Lorentz transformations. The first and second equation tells about the length contracted and time dilated for man1 from man2's perspective. From man2's perspective, man1 will undergo the following changes:

$$X = \gamma (X' + Vt)$$

$$t = \gamma (t + VX'/c^2)$$

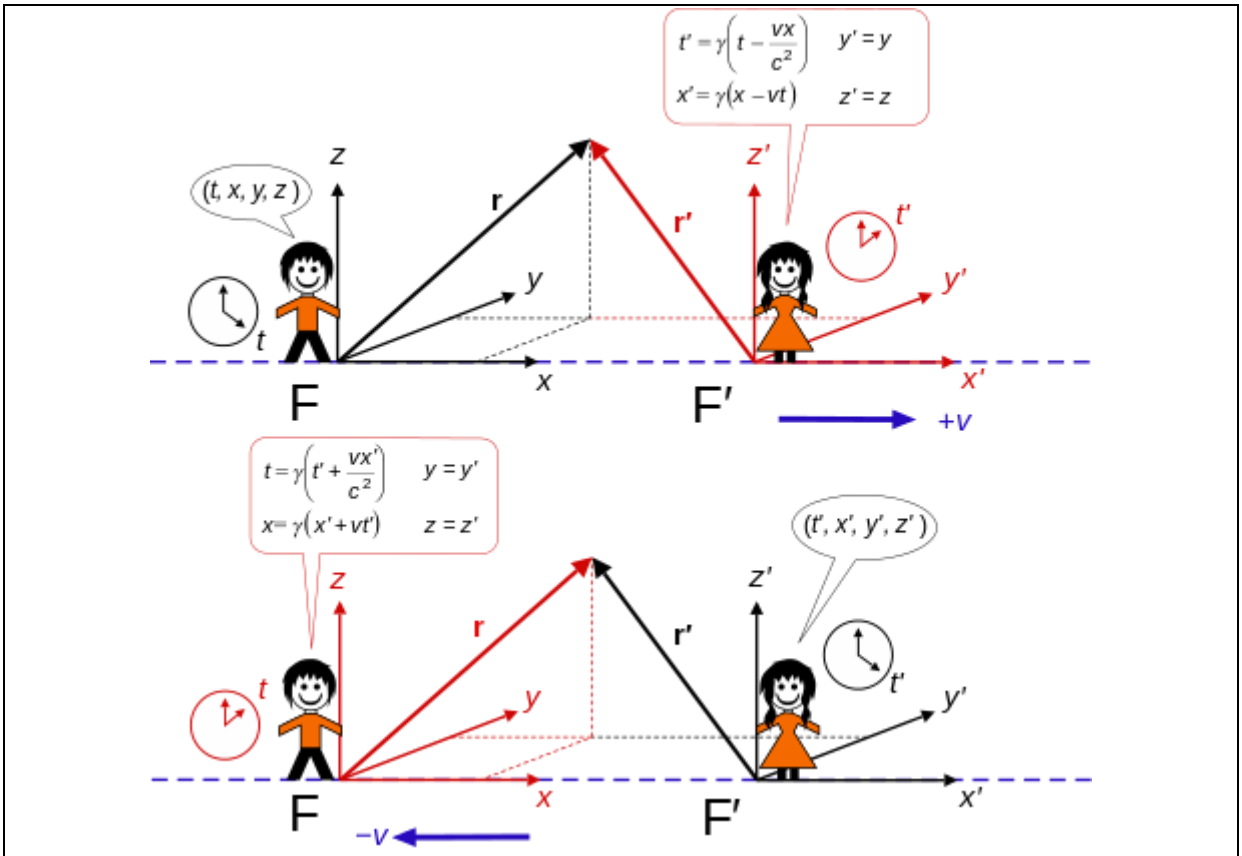



*Figure 8.1 The spacetime coordinates of an event, as measured by each observer in their inertial reference frame (in standard configuration) are shown in the speech bubbles.*
***Top:*** *frame F′ moves at velocity v along the x-axis of frame F.*
***Bottom:*** *frame F moves at velocity −v along the x′-axis of frame F′. (Source: Wikipedia)*

## MINKOWSKI SPACETIME INTERVAL THROUGH BASIC ALGEBRA

## PART2

At our previous station, we saw how light travels at a $45^0$ angle and any particle will always have a velocity less than the speed of light. Therefore the events for an ordinary matter with real mass will occur within the region as shown in fig 9.1 and there we have our light-cone. One thing to note is that the cone is in two dimensions representing one-dimensional motion. For four-dimensional universe like us, it's difficult to build such a cone, therefore we usually work with a two-dimensional cone. The figure clearly tells us the O to be the present for that particle and A to be the future event of the particle. The cone below O represents the past light-cone and cone above O represents the future light-cone. The dotted line in fig 9.2 represents the motion of the particle throughout that event and by definition is called the worldline of that particle.

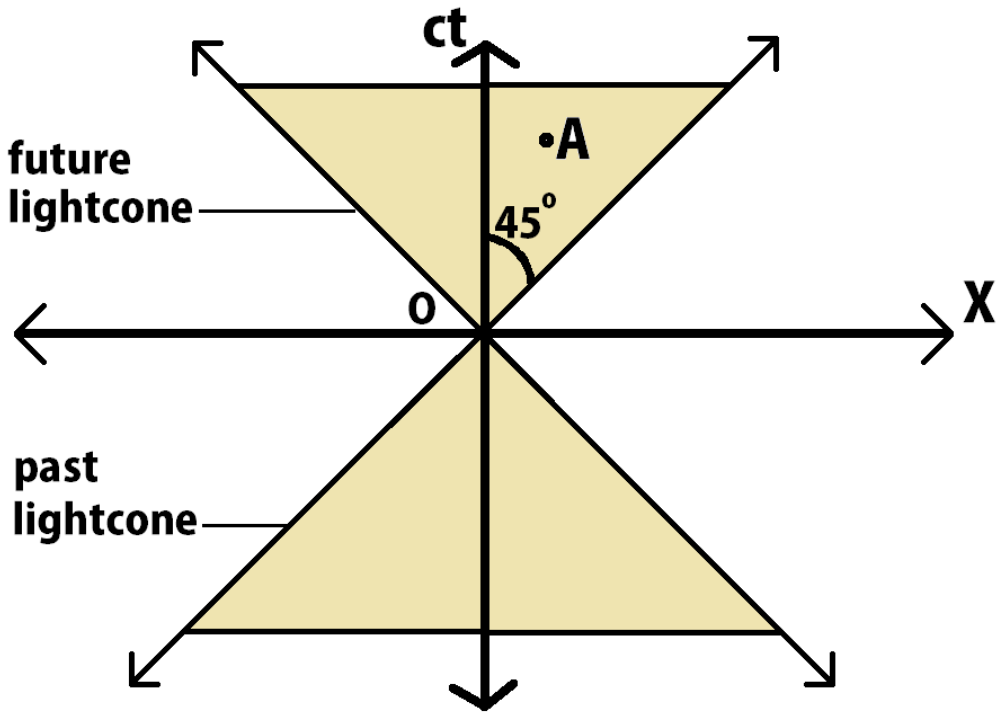



*Figure 9.2*

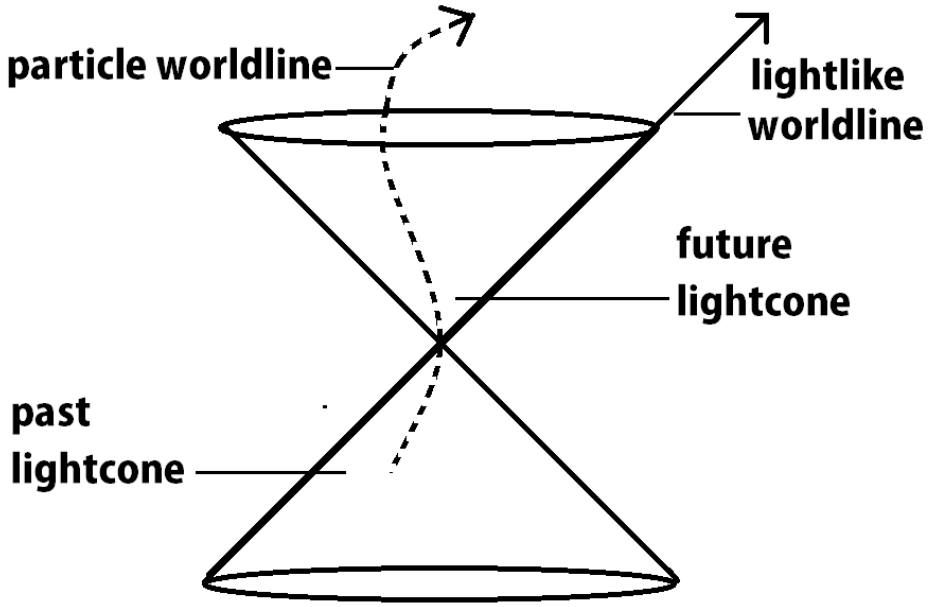



*Figure 9.3*

In relativity, any observers would not agree about the time required to complete an event, neither do they agree about the distance covered by them. In analogy, from my reference frame, I will be stationary and the car guy is moving and it's his watch which is ticking slower than me. On other hand, in car guy's reference frame, it's me who is moving and he will observe my clock to be ticking slower than his. Though they don't agree about

time or events, neither of them are wrong (As we saw earlier)! Any two observers may not agree about the time interval between the events or space between things at any given moment or about the chronological order of the events. Someone's sequence of event in past maybe someone's sequence of event in the future, therefore, past, present, and future are not a universal division of events, instead, it varies for different observers. Such a statement opens many philosophical questions, that, can we change our future? Is reality subjective? However, in fig 9.1 we saw that the event ‘caused' in past light-cone ‘effects' the future light-cone. That means an effect cannot occur from a cause that is not in past light-cone of that event and a cause cannot have an effect outside its future light-cone. Therefore, there has to be something which has a universal agreement and which also imply about the nature of reality; this is called causality which is the relationship between causes and effects. Every inertial observer will disagree about the present, past, future, time, distance, etc. but they do agree about causality or causal events. Such a causal event to which all inertial observers would agree is the spacetime interval or the spacetime separation between two events. The spacetime interval can be given as: $\Delta s^2 = \Delta x^2 - c^2\Delta t^2$ (which is somewhat like distance formula we saw in part1) the negative sign denotes the forward evolution.

Therefore, causality is ‘reality' of our universe and thus, the reality is not a three-dimensional space which evolves in time, instead, it's the four-dimensional non-Euclidean (hyperbolic geometry or basically not flat) mathematical space with no evolution and no time but ‘spacetime'. Its points correspond to events in 4D spacetime and anything which correspond to geometric relations between that 4D spacetime is real eg. Causal relations which are nothing but our spacetime interval.

## PENROSE DIAGRAM

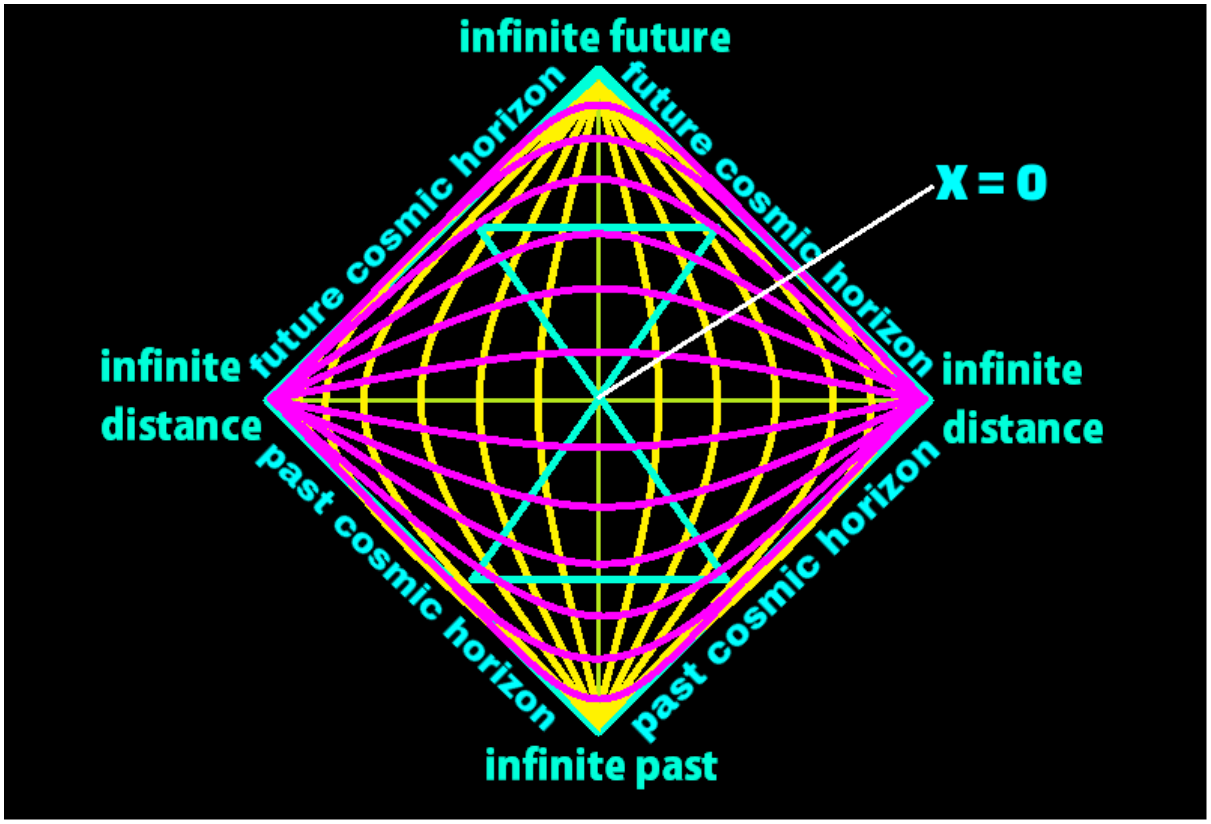



*Figure 10.1 Penrose Diagram*

Representing an event near a black hole in the Minkowski spacetime diagram is not a convenient way of representation, since, light in such diagram will not travel at $45^0$ angle when we place a black hole inside it. Therefore we need something more updated version of the spacetime diagram. Such a version is provided by Penrose diagram (fig 10.1). In such diagrams, the light will always travel at $45^0$ angle. Do note that as Minkowski spacetime is flat and two dimensional, in the same manner, Penrose diagram is also flat and two dimensional. A standard Penrose diagram as seen in fig 10.1 has infinite space and infinite time joining at the corners. In layman terms, we can show an infinite amount of spacetime in the finite size of the Penrose diagram. The vertical lines represent the line of constant position and the horizontal lines represent the fixed moment of time. If the center point of the hyperbolic grid is 'Now' at the origin x = 0, note how space and time intervals become smaller and smaller as you move to the corners of this diagram. These corners represent 'asymptotic infinity' plotted as a single point. If you move left or right you encounter the two points that represent the negative (-x) and positive (+x) space-like infinities. If you move vertically along the time axis, you encounter the past (-t), time-like infinity and the future (+t) time-like infinity. For spherically symmetric spacetimes, every point in the diagram corresponds to a 2-dimensional sphere. An infinite static Minkowski universe, coordinates (x,t) is related to Penrose coordinates (u,v) by:

$$\tan(u \pm v) = x \pm t$$

The corners of the Penrose diamond, which represent the space-like and time-like conformal infinities, are $\pi/2$ from the origin.

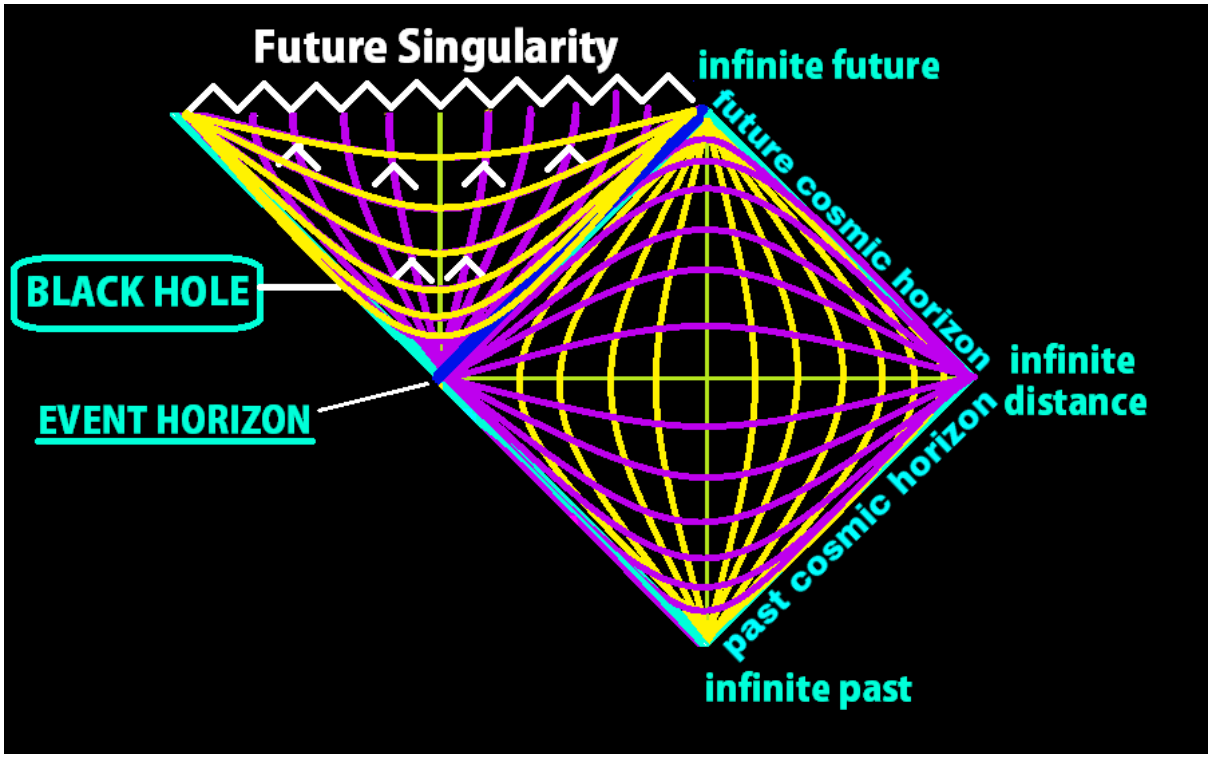



*Figure 10.2 Representing Black Hole in Penrose Diagram.*

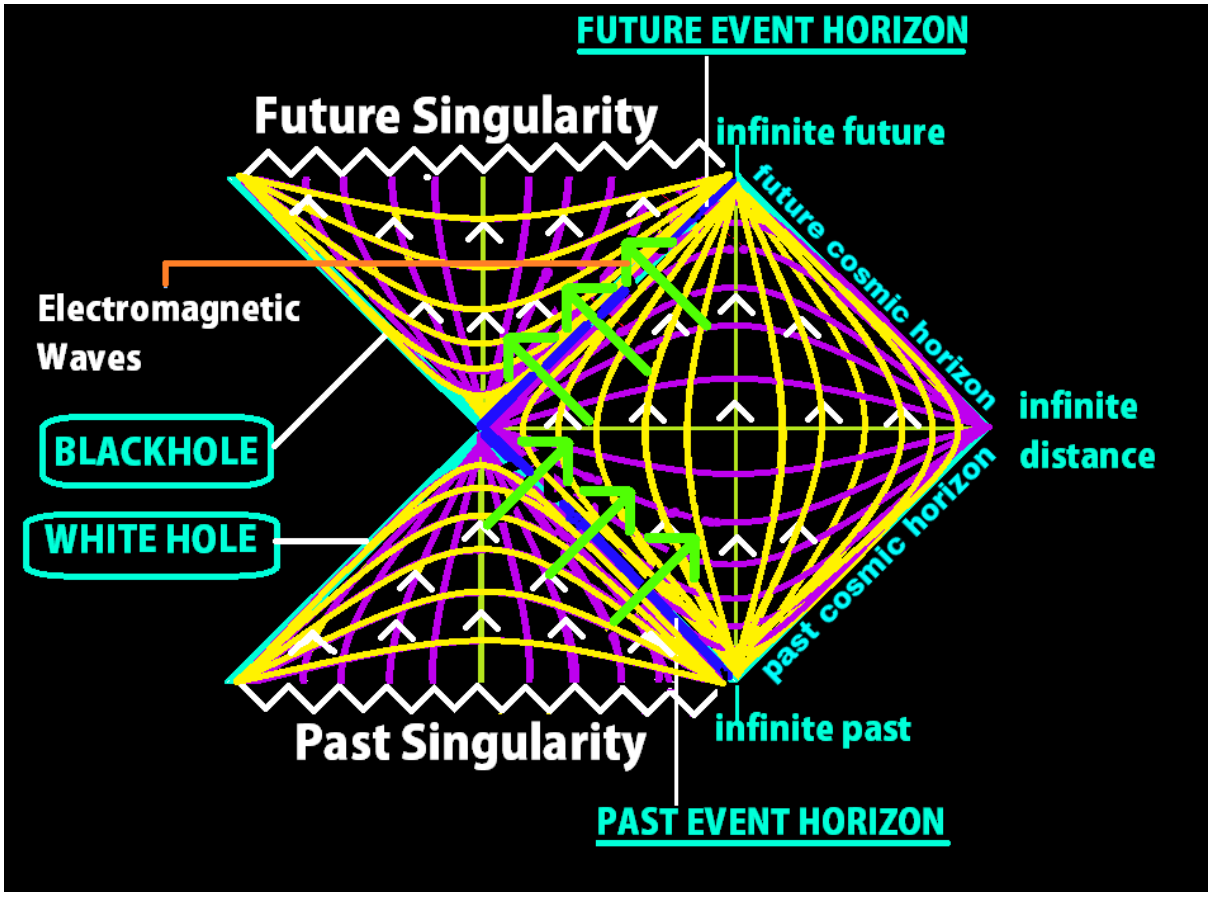



*Figure 10.3 Representing White Hole in Penrose Diagram.*

Let's consider a non-rotating, uncharged and eternal Schwarzschild black hole. The Penrose diagram with such black hole will look like fig 10.2. Do note that the line represents the event horizon of a black hole and the hyperbolic grid represents our universe. The light-cone can cross the event horizon and can enter the black hole, however it can never come back to our universe since the future light-cone is heading towards the singularity. Therefore, we can never see light coming out from the inside of the black hole. Inside a black hole, one space and time switch their roles i.e. space behaves as time and time behaves as space. We soon will be proving this mathematically at our future stations. In fig 10.2 you can see that vertical lines inside the event horizon represent time and horizontal lines inside the event horizon represents space which is totally opposite of what we saw in our universe. Thus, the singularity inside the Schwarzschild black hole is designated as the future singularity.

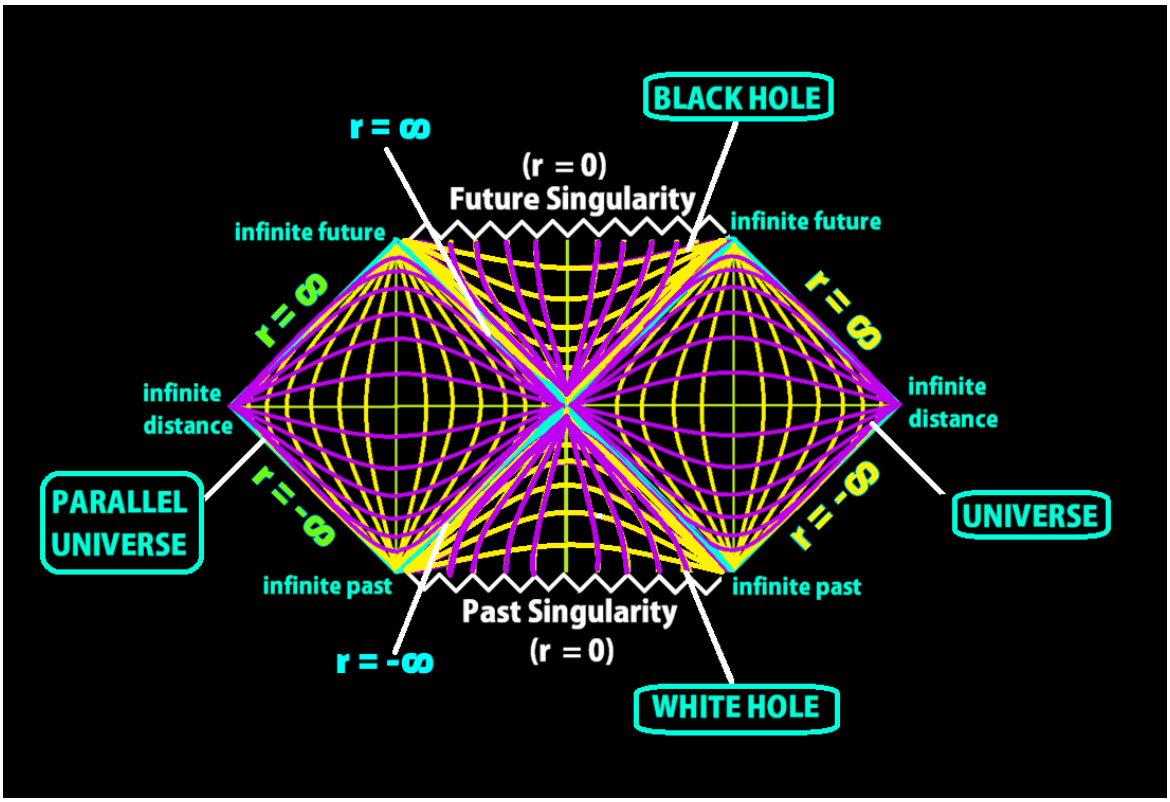



*Figure 10.4 Parallel Universe*

Now let's extend our Penrose diagram as shown in fig 10.3. If you think about that extended region you will observe that light can never enter that region, but can come out of it. From light cone perspective, the future light-cone points towards us and the past light-cone points towards the past singularity. Surprisingly, we have just build a white hole! White holes are somewhat opposite of black holes. The singularity of the white hole lies in the past instead of being in future, like that of the black hole. Not only this but if we take this to even further, the mathematical symmetry would uncover a mysterious parallel universe (fig 10.4)! Now that's where we can really say that finally, we know where our parallel universe is exactly. But if you try to use the Penrose diagram and light-cones, you will observe that no matter what you do, it's not feasible to enter our parallel universe, until and unless you travel beyond the speed of light, because that would expand the light-cone with an angle greater than $45^0$ and in the end we may reach our parallel world!

# RING SINGULARITY

At this station, we shall be introducing with Kerr black holes and shall be specifically talking about ring singularity. A rotating uncharged black hole is considered as Kerr Blackhole. These black holes are defined by their mass (M) and angular momentum (J). The Kerr metric, given in Boyer–Lindquist coordinate system is:

$$ds^2 = (\Sigma \div \delta)dr^2 - [1 - ((r_s r) \div \Sigma)]c^2dt^2 + \Sigma\, d\Theta^2 + [r^2 + a^2 + ((r_s r a^2 \sin^2\Theta) \div \Sigma)]\sin^2\Theta d\phi^2 - [(2r_s r a \sin^2\Theta) \div \Sigma]cdtd\phi$$

Where, r, Θ, ϕ are standard spherical coordinate system and;

$a = J/Mc$

$\Sigma = r^2 + a^2 \cos^2\Theta$

$\delta = r^2 - r_s r + a^2$

We shall be talking about more properties in detail at our upcoming stations. Presently we shall see one of the amazing facts seen inside a rotational black hole, which otherwise is not seen in Schwarzschild black holes. Singularity, instead of being a single point is actually like a ring.

In the Penrose diagram a Kerr black hole has four different regions of spacetime and a ring singularity. The fig 10.1 explicitly represents four different regions of Kerr black holes.

1. The spacetime region of our universe and parallel universes.
2. The spacetime region in purple.
3. The spacetime region in red.
4. The spacetime region of antigravity universe (yellow).

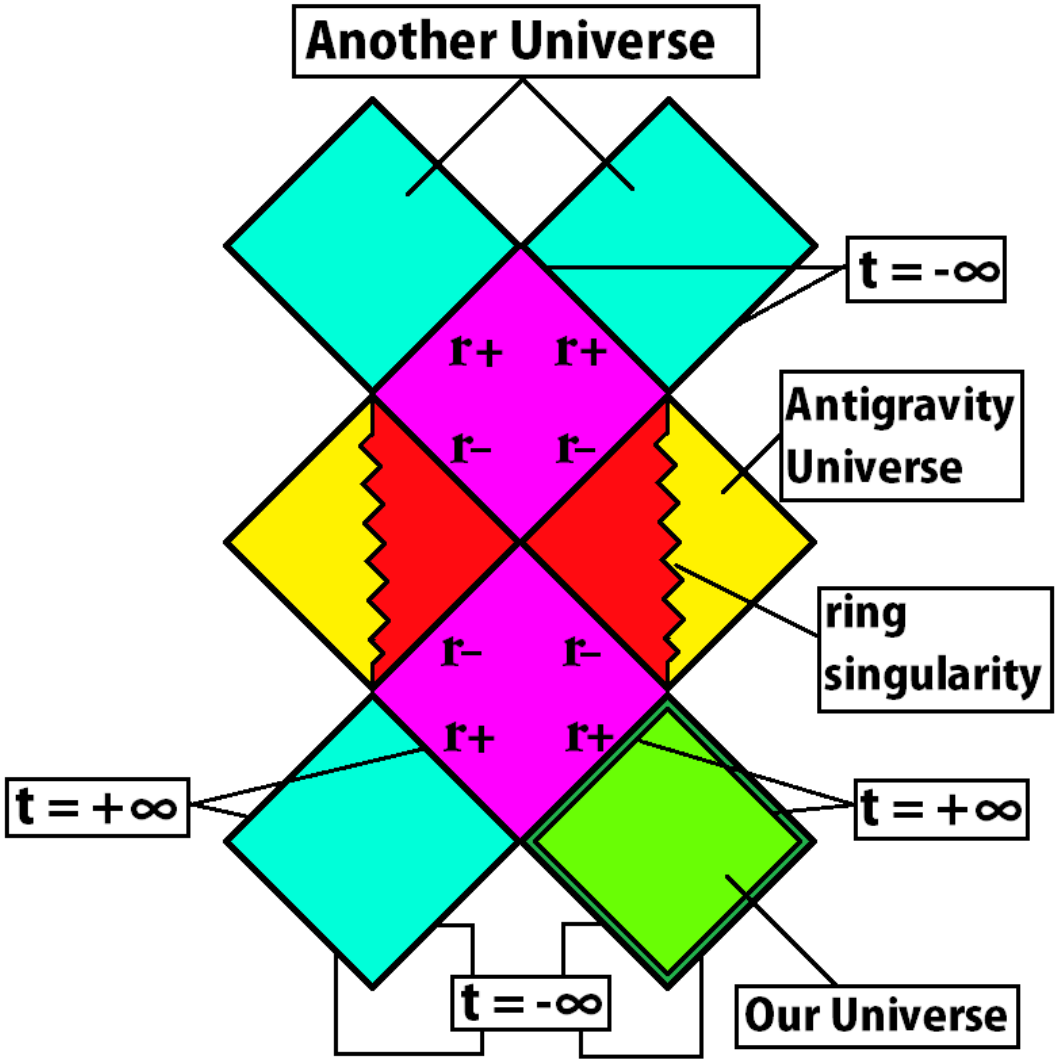



*Figure 11.1 Representing Kerr Black Hole in Penrose Diagram.*

You can clearly see that the diagram explicitly explains the existence of wormhole and an anti-gravity universe (where gravity is repulsive!). As for all Penrose diagrams, light rays travel on 45-degree lines and light cones point up (future) or down (past) and have an opening angle of 90 degrees. Unlike a Schwarzschild black hole, a traveler can completely avoid the ring singularity if he enters the rotating black hole along its rotation axis. In front of him would be the circular Ring of Death, and he can even head directly for the center of this ring but will exit the black hole in a bizarre ‘anti-gravity' universe very different from his own. In this universe, the mathematics of General Relativity predicts that gravity will be repulsive, not attractive! The Kerr black hole thus also allows the possibility of the existence of wormhole inside a rotating uncharged black hole!

## MATHEMATICAL UNDERSTANDING OF BLACK HOLES

## PART1

One of the amazing fact of the black hole is that, if a star like our sun (hypothetical case!) is converted to black hole, then, the planets would still be rotating along their respective orbits, until and unless they do not emit gravitational radiation, causing their orbit to decay and crossing their last stable orbit. Therefore, black holes do not suck everything. They only suck in two conditions:

1. If a body orbits close to the event horizon in an elliptical orbit, it emits gravitational radiation, then, its orbit will eventually decay in millions of years.
2. If a disk of gas is formed around a black hole, then, through friction matter will slowly slide into the black hole over time.

The last stable orbit (also called as the static limit) ($r_i$) of such planets/ satellites/ objects which after crossing they will eventually fall into the black hole can be calculated by:

1. In case of a non-rotating black hole:
   a. For massive particle:
      $r_i = (6GM) \div c^2$
      $r_i = 3.r_s$
   b. For photon:
      $r_i = (3GM) \div c^2$
2. In case of rotating black holes:
   $r_i = [(GM) \div c^2 ].[3 + z_2 \pm \sqrt{}(3 - z_1)(3 + z_1 + 2z_2)]$

The proper acceleration that a rocket or any materialistic object would need to hover at a distance R for a black hole of mass M can be given as:

$$a = [GM] \div [R^2\sqrt{}(1 - 2GM/Rc^2)]$$

The mathematics of General relativity shows us that time not only changes with relative velocity but also with the gravitational field. More the gravitational field more slowly the clock ticks and less the gravitational field relatively faster the clock ticks. The formula for time dilation for a non-rotating spherical black hole in General Relativity is given by:

$$t' = t\sqrt{}(1 - 2GM/rc^2) = t\sqrt{}(1 - r_s/r)$$

For an object free-falling in circular orbit which it may if the orbital radius is greater than 1.5 $r_s$ then time dilation can be calculated by:

$$t' = t\sqrt{}[1 - (1.5r_s)/r]$$

Where,

t' = time for an outside observer

t = time for an object

M = mass of the black hole

$r_s$ = Schwarzschild radius

r = radial distance from the observer

c = speed of light

So for an outside observer, any object approaching the event horizon of the black hole will measure the time moving slowly relative to his own clock. When the object reaches the event horizon, the outside observer measures the object's clock to be stopped ticking, technically he will see the object entering the black hole at infinity. However, from the object's point of view, the time does not stop and thus he will enter the black hole. But in reality, no one knows what exactly the object would experience/ perceive things from his point of view. Therefore, it's a reality that is observer-dependent.

Similarly, as time, the wavelength of light emitted by any object orbiting around the black hole (eg. dust or gas) is also shifted towards the red part of the spectrum, also called as red-shift. So if a gas particle at a distance R from the center of the black hole is emitting a wavelength of length $\lambda_0$, then, by the time it reaches an observer it would be redshifted to $\lambda$. For a weak gravitational field like our planets, dwarf planets, etc. the Gravitational Redshift is given by:

$$Z = GM \div Rc^2$$

Where Z is the Gravitational redshift. However, for stronger gravitational fields like the surface of the neutron star or near the event horizon of a black hole, Z is given as:

$$Z = [1 \div \sqrt{(1 - (2GM)/(Rc^2))}] - 1$$

One of the interesting fact about the black hole is that the entities space and time behaves very differently inside the black hole. Inside the black hole, space behaves as time and time behaves as space! We will see this soon what exactly this statement means. Causality tells us that cause and effect, governs the relative universe. The classical formulation of spacetime interval which governs the flow of cause and effect was given by Minkowski:

$$\Delta s^2 = \Delta x^2 - c^2\Delta t^2$$

Here, $\Delta x$ is the difference in space/ position coordinate which two observers will always have, $\Delta t$ is the difference in time coordinate which these observers will always record. The observers will never agree with relative distance covered in space and the time intervals for the given event. However, they both will definitely record the same spacetime interval. The negative sign in the equation represents forward evolution. Therefore, the entity $\Delta x$ is behaving like space coordinate and $\Delta t$ is behaving like time coordinate, which it should obviously do.

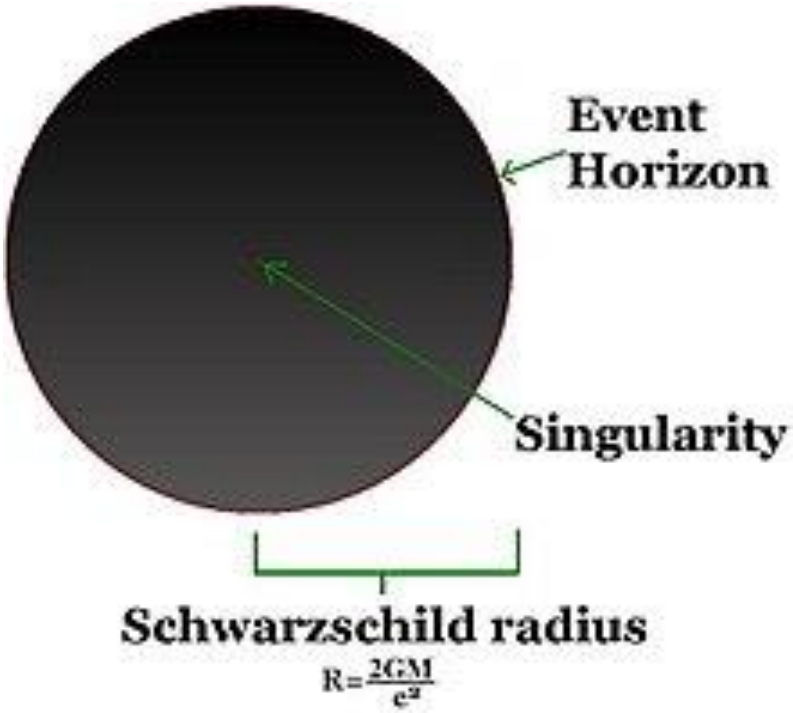



*Figure 42.1 Schwarzschild Black Hole (source: Wikipedia)*

Let's mathematically see how space and time behaves differently inside a black hole. Consider the Schwarzschild interval again (we are not considering the rotational motion here):

$$\Delta s^2 = [1 \div (1 - r_s/r)]\, \Delta r^2 - (1 - r_s/r)\, c^2 \Delta t^2$$

At a far distance from a black hole, r becomes infinite and we get back to our Minkowski's spacetime interval equation. Inside the event horizon $r < r_s$ and the equation becomes:

$$\Delta s^2 = - [1 \div (1 - r_s/r)]\, \Delta r^2 + (1 - r_s/r)\, c^2 \Delta t^2$$

This switching of the sign in the spacetime interval represents that the events occurring inside the event horizon are not causal events. This reversing the causality shows flipping in the behavior of space and time. To see how to imagine such an abstract idea, check out how we have mapped these structures on the Penrose diagram.

## MATHEMATICAL UNDERSTANDING OF BLACK HOLES

## PART2

So we still are on our journey to a black hole and we are yet to reach the singularity where physicist believe quantum mechanics may remove these singularities. At our previous station, we concluded with how space and time switch roles inside the event horizon of black hole, at this stationswe would be continuing with some more exciting features of non-rotating and rotating black holes mathematically. In the end, I will tell you how energy can be extracted from a black hole!

Consider the Schwarzschild interval:

$$\Delta s^2 = [1 \div (1 - r_s/r)]\, \Delta r^2 - (1 - r_s/r)\, c^2 \Delta t^2 + r^2(\Delta\Theta^2 + \sin^2\Theta \Delta\phi^2)$$

When $r = r_s$ we say that the particle has reached the event horizon of the black hole. In Schwarzschild interval we get singularities at $r = 0$ and at $r = r_s$, however, the singularity at $r = r_s$ is just an illusion created by Schwarzschild interval. When we consider the same condition in other coordinate systems (for example Lemaitre coordinates, Eddington–Finkelstein coordinates, Kruskal–Szekeres coordinates, Novikov coordinates, or Gullstrand–Painlevé coordinates) the metric becomes regular and can be extended to values of r smaller than $r_s$. Another way to think is that at $r = r_s$ the space-like coordinate and time-like coordinate disappears from the equation but the particle do experience time and eventually crosses the event horizon. To see whether singularity at $r = 0$ is true or not, consider the Kretschmann invariant which is independent of the choice of coordinates:

$$K = (48G^2M^2) \div (c^4r^6)$$

Here at $r = 0$, K becomes infinite and we get a true singularity, but at $r = r_s$, K has a finite value. Therefore, $r = r_s$ is called a coordinate singularity and $r = 0$ is called the gravitational singularity. One of the most amazing facts of singularity is that they are a generic feature of the theory of relativity and not just a special exotic special case.

We have talked a lot about Schwarzschild non-rotating black holes, however, in our universe, we see a majority of the celestial bodies being rotational, so as applies to black holes as well and surprisingly the math predicts mindboggling stuff about them. The uncharged, non-rotating black holes are also called Kerr black holes. These black holes are defined by their mass (M) and angular momentum (J). The Kerr metric given in Boyer–Lindquist coordinate system is:

$$ds^2 = (\Sigma \div \delta)dr^2 - [1 - ((r_s r) \div \Sigma)]c^2dt^2 + \Sigma\, d\Theta^2 + [r^2 + a^2 + ((r_s r a^2 \sin^2\Theta) \div \Sigma)]\sin^2\Theta d\phi^2 - [(2r_s r a \sin^2\Theta) \div \Sigma]cdtd\phi$$

Where, r, Θ, ϕ are standard spherical coordinate system and;

$a = J/Mc$

$\Sigma = r^2 + a^2 \cos^2\Theta$

$\delta = r^2 - r_s r + a^2$

The charge (Q) and angular momentum (J) for any black hole are constrained to its mass (M) by the following relation (note- we are taking $G = c = 1$ for simplicity of the math):

$$Q^2 + (J/M)^2 \leq M^2$$

Any black hole having charge will possess the same charge as that of its original star. However, its angular momentum may vary to satisfy the given condition. A black hole with $Q = 0$ will *give:

$$J/M^2 \leq 1$$

This is called the fraction of maximality. During the collapse of uncharged and rotating blue star, it has to satisfy this condition to form a black hole. The theory of General Relativity also predicts that if this condition is not satisfied i.e. the fraction becomes greater that one, then a naked singularity without the event horizon is born. However, what we have observed so far is that the star's excess angular momentum is lost in the form of gravitational waves and the condition is satisfied to form a rotating and uncharged black hole (in our case (Kerr black hole)).(*if $G \neq c \neq 1$, then $cJ/GM \leq 1$).

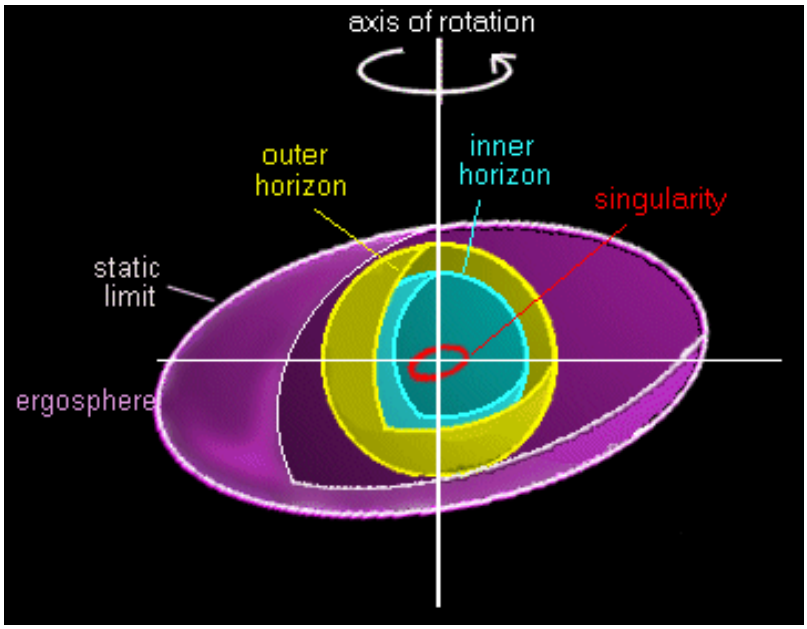



*Figure 13.1 Kerr Black Hole (source: NASA)*

When a body rotates, it also 'drags' the fabric of spacetime, such phenomenon is called frame-dragging. An extreme version of frame-dragging is observed within the ergosphere of the Kerr's black hole. Ergosphere is a pumpkin shaped region at which the particle cannot stay still/ stable (discussed in the later session). Within the ergosphere, the rotational black hole has two event horizons which occur/depend upon the angular velocity and latitude Θ; the inner event horizon occurs at:

$$r_{inner} = [r_s \pm \sqrt{(r^2_s - 4a^2)}] \div 2$$

The outer event horizon can be found at:

$$r_{outer} = [r_s \pm \sqrt{(r^2_s - 4a^2\cos^2\Theta)}] \div 2$$

Where a = J/Mc. These two equations are actually the outcome of a quadratic equation of $1/g_{rr} = 0$ and $g_{tt} = 0$ respectively which also predicted respective coordinate singularities for a rotational black hole. The real singularity of a rotational black hole is different than Schwarzschild black holes. Instead of being a singular point, it is a singular ring; ring singularity (here for the proof)! Ergosphere which lies outside the outer event horizon is a sphere where particle rotates faster and gains more energy. Because they are still outside the event horizon, they may escape the black hole. The net process is that the rotating black hole emits energetic particles at the cost of its own total energy. The possibility of extracting

spin energy from a rotating black hole was first proposed by the mathematician Roger Penrose in 1969 and is thus called the Penrose process (discussed in the later session). Rotating black holes in astrophysics are a potential source of large amounts of energy and are used to explain energetic phenomena, such as gamma-ray bursts. If a particle initially falls with or without angular momentum from infinity to a black hole, it gains angular momentum during the infall. So even if the particle is having a revolution opposite to that of the black hole, the particle will eventually be forced to rotate in the direction of the revolution of the black hole. The angular momentum observed by an external observer can be given as:

$$\Omega = (r_s ra) \div ((r^2 + a^2c^{-2})^2 - a^2c{-}2\Delta\sin^2\theta)$$

A rotating black hole has the same static limit at its event horizon (except at the poles) but there is an additional surface outside the event horizon named the "ergosurface" given by:

$$(r - M)^2 = M^2 - J^2\cos^2\Theta$$

It's here where the particle is forced to co-rotate irrespective of its original rotation/ non-rotation with the black hole.

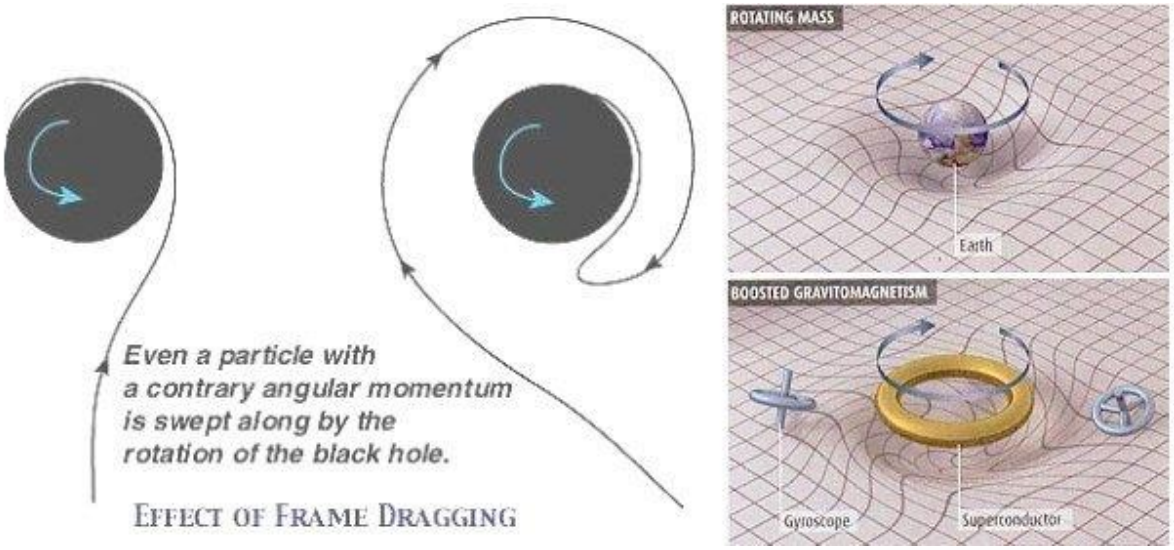



*Figure 13.5 Effect of Frame Dragging (source: Relativity-Review)*

So now I will tell you how to extract energy from a black hole via the Penrose process. The wiki explains (I think this is a good understandable analogy) that in the process, a lump of matter enters into the ergosphere of the black hole, and once it enters the ergosphere, it is forcibly split into two parts. For example, the matter might be made of two parts that separate by firing an explosive or rocket which pushes its halves apart. The momentum of the two pieces of matter, when they separate can be arranged so that one piece escapes from the black hole (it "escapes to infinity"), whilst the other falls past the event horizon into the black hole. With the careful arrangement, the escaping piece of matter can be made to have greater mass-energy than the original piece of matter, and the infalling piece has negative mass-energy. Although momentum is conserved the effect is that more energy can be extracted than was originally provided, the difference being

provided by the black hole itself. In summary, the process results in a slight decrease in the angular momentum of the black hole, which corresponds to a transference of energy to the matter. The momentum lost is converted to energy extracted. However, the maximum theoretical efficiency would be around 20% as per the mathematical calculations by Chandrasekhar in 1983.

## QUANTUM MECHANICAL DESCRIPTION OF BLACK HOLES

So far we theorized and analyzed black holes at a macroscopic scale, but now we shall be entering into the microscopic scale to analyze the quantum behavior of black holes. Extreme density and high temperature are the key ingredients to make a black hole. Such conditions are available inside a star or at some initial stages after the big bang. Scientists theorized that in the early stages after the big bang, there's a possibility of forming primordial black holes with the mass ranging from $10^{-5}$ g (Planck's mass) to hundreds of thousands of solar masses. As per this theory, black holes with plank masses would have evaporated by now, since small black holes evaporate faster than massive black holes. Therefore, we can never detect their presence, however, we might be able to detect the primordial black holes which had hundreds of thousands of solar masses during their birth.

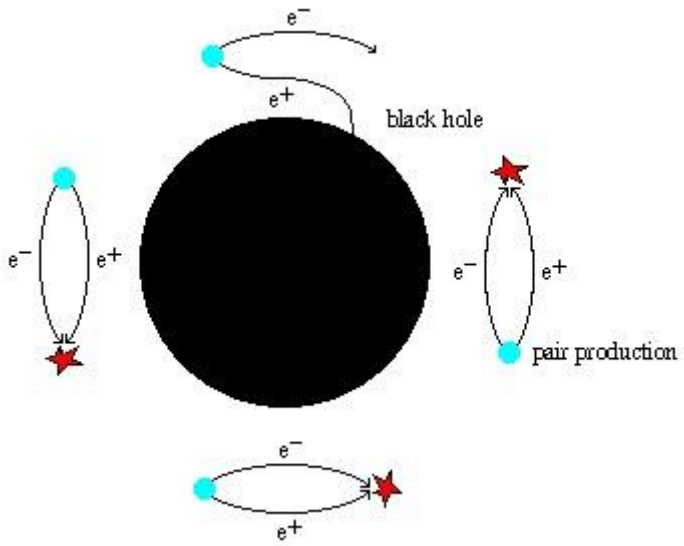
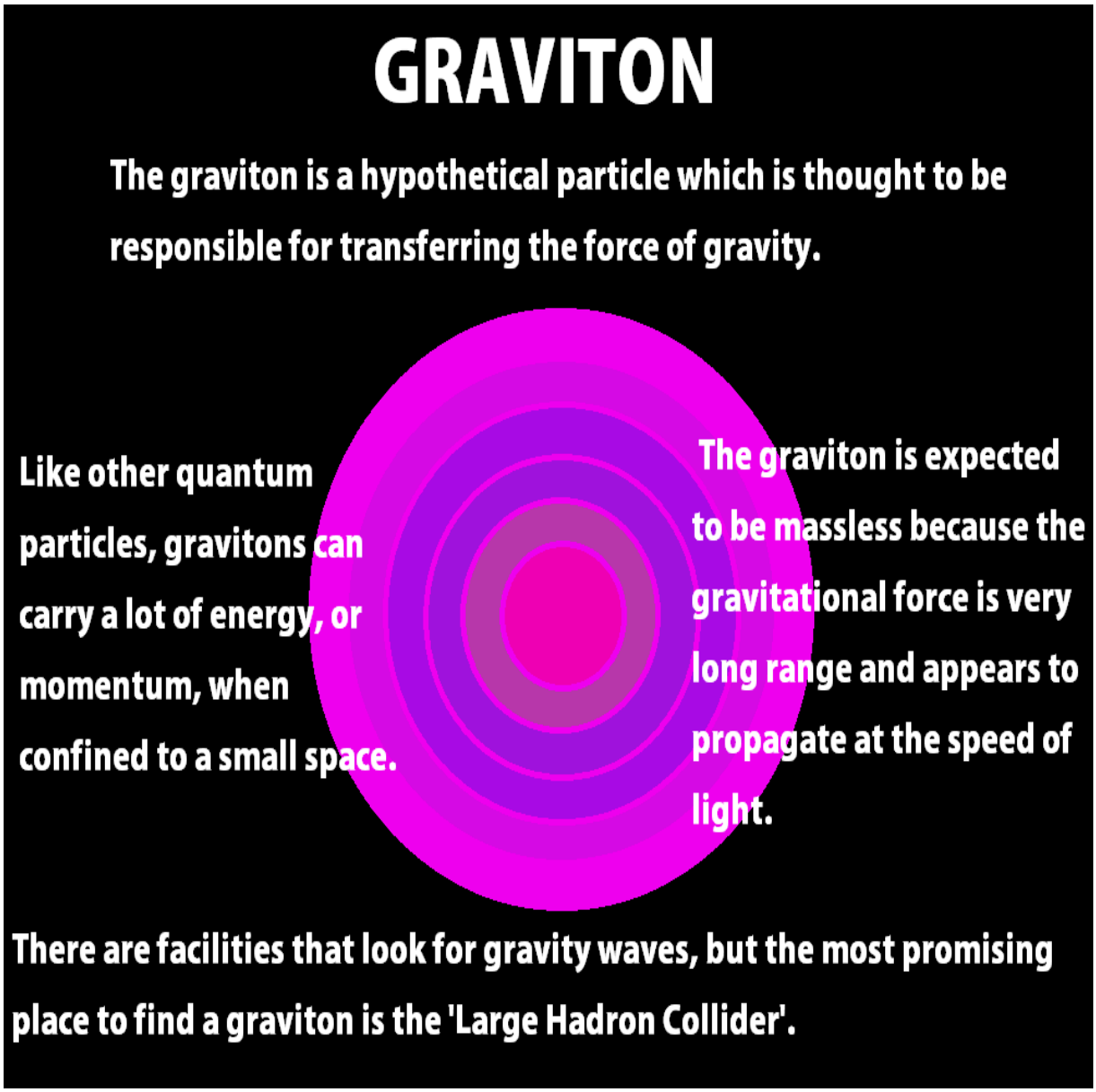



*Figure 14.1 Hawking's Radiation*

When objects fall inside the black hole, the information (arrangement of particles in that object) gets lost forever. But then it leads to many paradoxes which we will be discussing soon. We know that the total entropy of the system always increases, but what we see here (objects falling inside the black hole) is that information is lost and therefore the entropy of the system has been decreased. In 1974, Hawking came out with an idea that black holes aren't black, instead, they are constantly emitting thermal radiation whose temperature is inversely proportional to the mass of a black hole. He proposed that near the event horizon there are virtual particles (particles which we can never be detected due to uncertainty principle) which are continuously coming in and out of existence from the

vacuum of space. These virtual particles are in fact pair of matter (having positive mass) and antimatter (having negative mass) where antimatter gets inside the black hole due to extreme bending of spacetime. Therefore, it leads in adding a negative mass to black hole, causing it to decrease its mass. Due to lack of the virtual-anti partner of the matter, the virtual appearance of matter turns out to be real and it's ejected in the form of thermal radiations which we refer as Hawking's radiation. If you have paid attention so far at this station, you might notice that we are describing quantum mechanical behaviour at the vicinity of the black hole but not inside the black hole. This is because describing quantum mechanics inside the black hole or of the black hole is not possible with our present well established theories. The only theory which can describe the quantum mechanics of a black hole is the theory of quantum gravity. Many physicists are still working on this theory, however, there is still time for the theory to deepen its roots.



## ARE BLACK HOLES REAL?

There's no doubt they are and scientists have also been successful in taking the image of its event horizon! However, many people among the crowd still don't believe it. If light can never escape a black hole, then, how can we ‘see' them? The answer is by observing/ detecting its effects on astronomical bodies. The Event Horizon Telescope (EHT) is a collection of many telescopes across the globe to capture the images of the event horizon. EHT thus will test the theory of General Relativity at its extreme capabilities. Detecting a direct image of a black hole would need a telescope as big as our planet since we need to capture every single data from the light (invisible, mainly infrared) emitted ‘across' (from the heating of the gas) a black hole. The heated gas mainly emits infrared (where gases glow the most) and eventually reaches earth. This light appears circular near the black hole, but as it reaches the earth it becomes parallel. Therefore, capturing this data will require a telescope as big as our planet, otherwise, we would be capturing incomplete data since only light (signal) (as seen in the figure 15.1) will enter our telescope while other (signals) will be lost forever. To solve this problem scientists across the globe has set many satellites across the globe (including Antarctica) and are capturing a majority of the signals and synchronizing all the data from all the telescopes to get a sharp image of the black hole's event horizon. More synchronized the satellites, sharper the image would be.

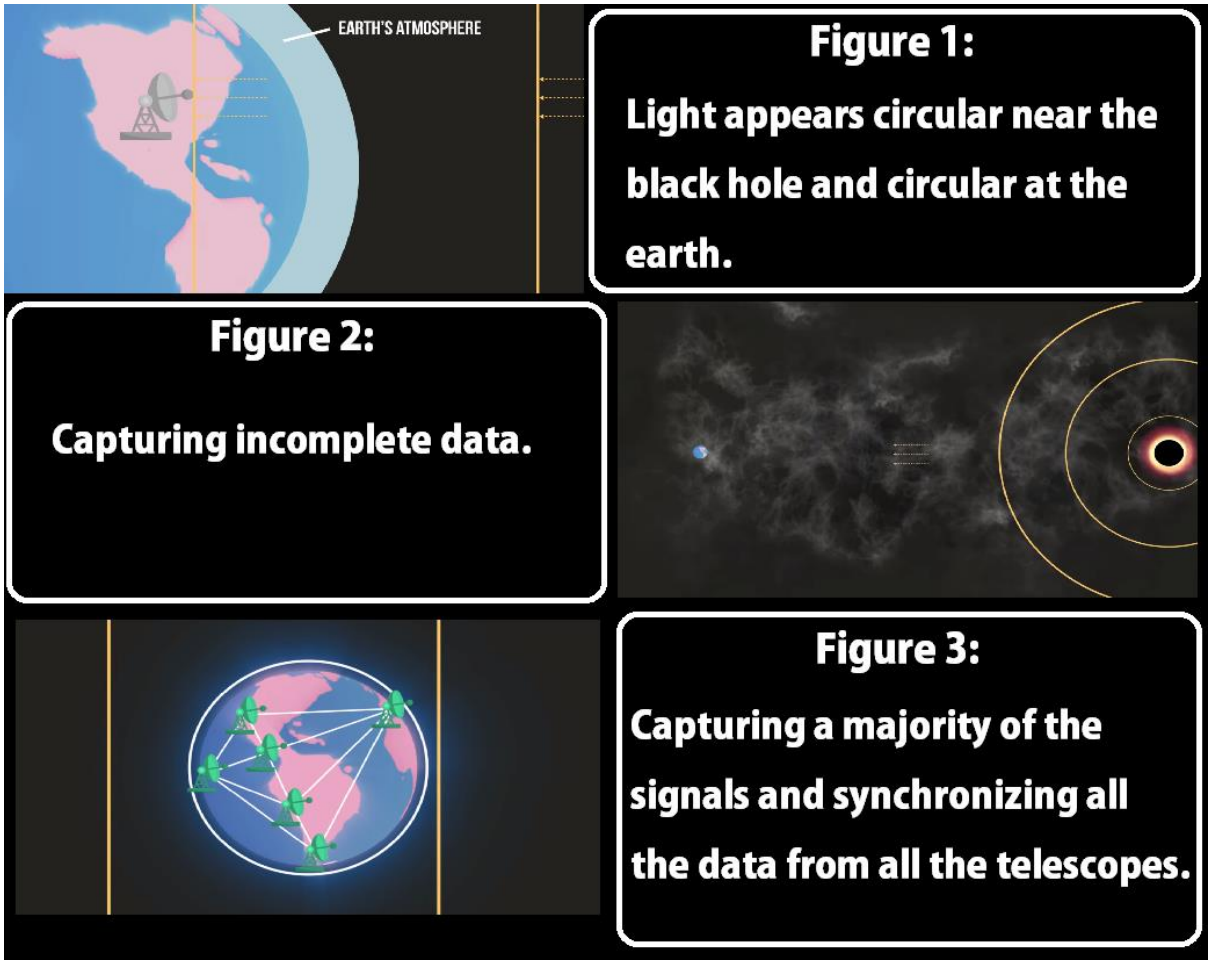



*Figure 15.1*

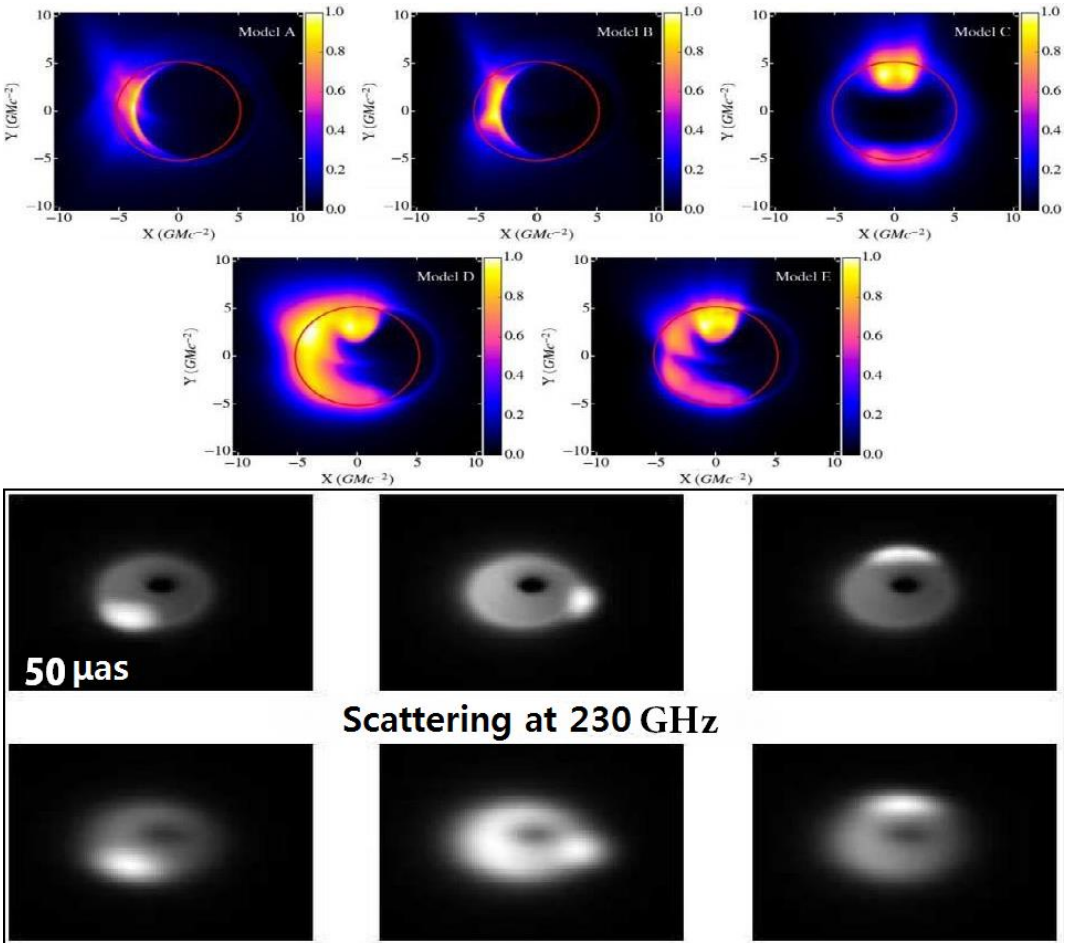



*Figure 16.2 After Synchronising the Satellite Signals.*

Another way by which the existence of black holes can be predicted is by observing the motion and behavior of celestial bodies. Every star has its binary partner and scientists has found many stars which are coupled with their unknown, ‘dark', x-ray emitting binary partner whose mass was calculated by measuring the intensity of the light and results were compared with the theoretical data which showed a possible presence of extremely dense and massive object (Recollect the "The Birth of a Giant!” station where we talked about the masses at which neutron stars and black holes are formed). Therefore, we saw an indirect evidence of black hole via X-ray binaries. Another great evidence among many is the detection of the gravitational waves whose source was figured out to be two black holes! The data from both LIGO facilities, match exactly with each other and with the theoretical data as shown in the image.

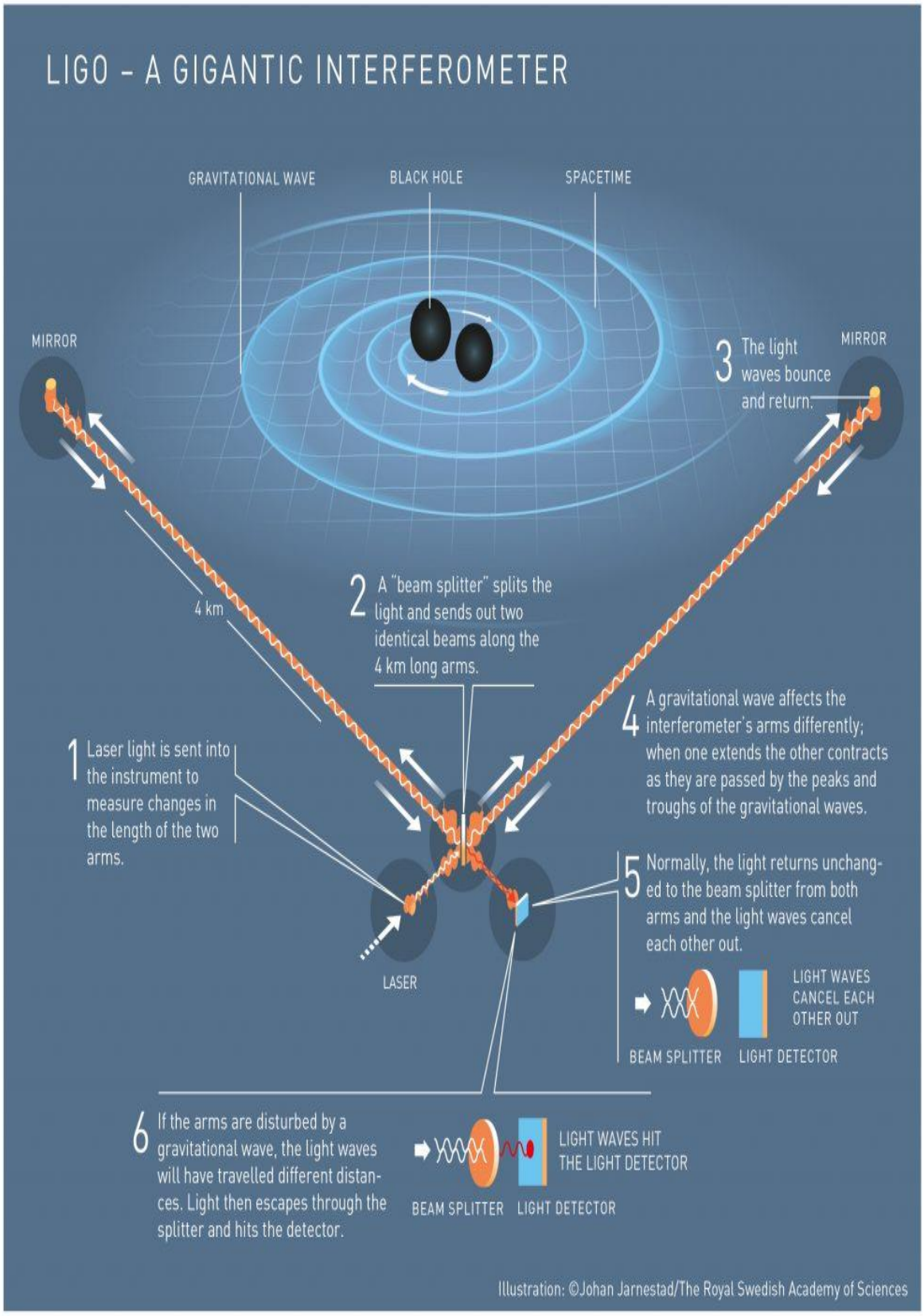



*Figure 15.7 LIGO (Source: PBS.org)*

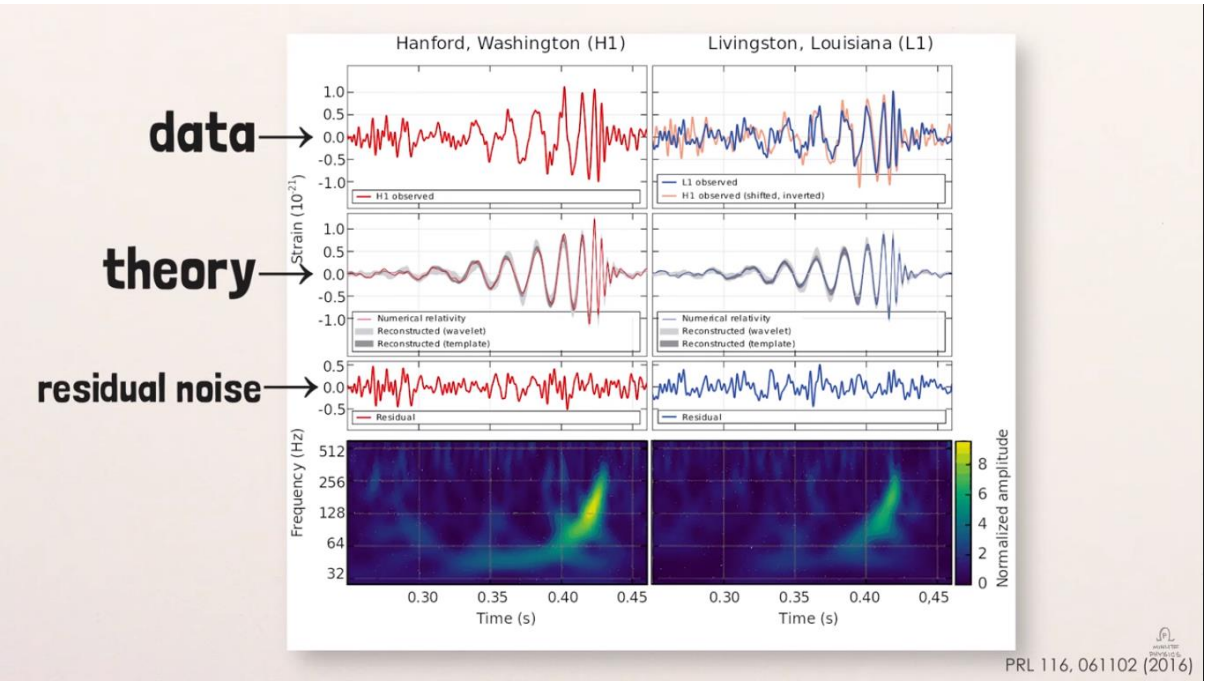



*Figure 15.8 Detection of Gravitational Waves (Source: Minute-Physics)*

## MATHEMATICAL DESCRIPTION OF WORMHOLES

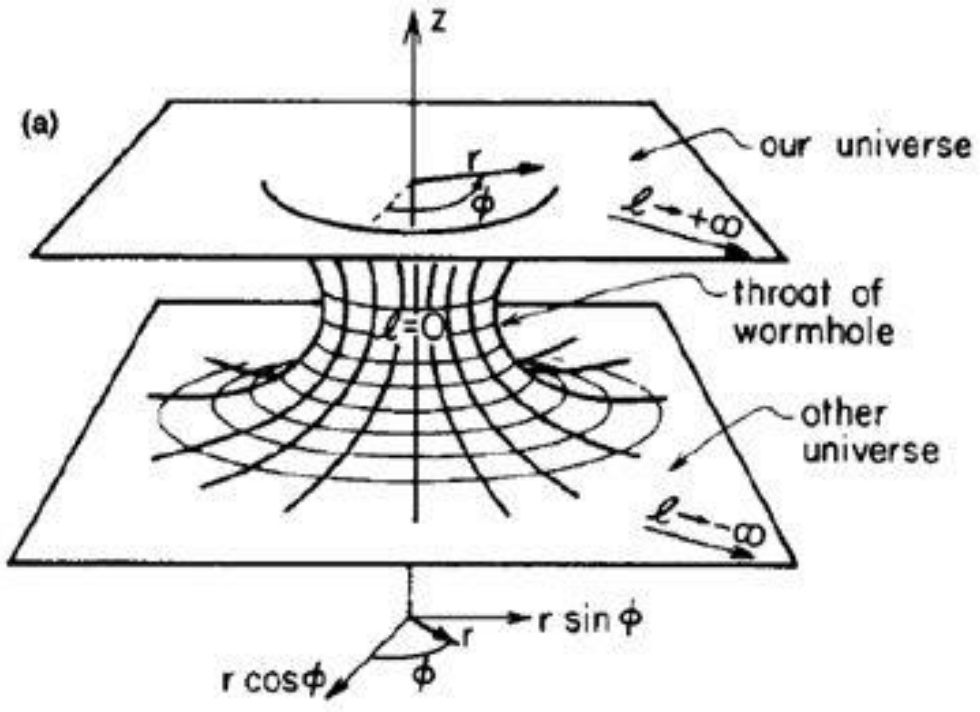



*Figure 16.1 Morris-Thorne Metric Wormhole (Source: i.pinimg.com )*

Wormholes were first studied mathematically in relativity as early as 1921 by the German mathematician Herman Weyl. However, it wasn't until Kip Thorne began studying them in the 1980s that they started to be taken seriously by relativists. Solving Einstein's field equation is not an easy task if we inserted the matter in the equation and then solved for the metric, instead we can straight away insert the metric and derive the conditions for

matter and that's what Kip Thorne did in his 1988 paper, ‘wormholes in spacetime and their use for interstellar travel.' This metric for traversable wormhole is known as the Morris-Thorne metric:

$$ds^2 = dl^2 – c^2dt^2 + (b^2 + l^2)(d\Theta^2 + \sin^2\Theta d\phi^2)$$

Where s denotes the spacetime interval, c denotes the speed of light, t denotes the time-like coordinate, r is the space-like coordinate (Note: the r is replaced by l and $\sqrt{(b^2 + l^2)}$ and is similar to Schwarzschild metric at $r_s = 0$ or when r is at infinity). The negative sign in front of $c^2dt^2$ denotes the forward evolution and Θ and ϕ are spherical polar parameters just like in Schwarzschild metrics. The b denotes the radius of the tunnel from distance l (l has a range of –infinity to +infinity). One of the amazing things to note is that the equation has no gravitational or ‘real' singularity. For ordinary matter, the energy density is always a positive entity defined as:

$$p + \rho \geq 1$$

Where p is pressure added to ρ which denotes energy density (energy per unit volume). Mathematically, to keep b≥1 the energy density of the matter should be negative instead of being positive which it is otherwise. Such matter will thus have a negative mass and therefore called as exotic matter! It does violate the ‘null energy condition' but physicists and mathematicians still working to figure out such properties in dark energy and dark matter which contribute 96% energy density of our universe.

## WORMHOLES AND TIME TRAVEL

Wormholes also referred as Einstein-Rosen Bridge is a hypothetical tunnel which connects two points in spacetime. It thus deduces the path to reach the distant locations in space-time. In spite of the prediction by General Relativity and so far unobserved experimental evidence, scientists have the recipe to create and travel through a wormhole.

At a quantum level, space-time is highly unstable. Virtual particles come in and out of existence making the so called ‘empty space’ not empty. This energy state of spacetime is called as quantum foam or spacetime foam. And as per the theoretical blueprint, if we stabilize this quantum foam long enough, we would be able to create a wormhole. But the energy required to stabilize the spacetime foam is very high, even the world's most powerful particle accelerator is insufficient to make the required energy. But by blasting lasers with plasma we can create an enormous amount of energy which will stabilize the spacetime foam and would eventually give birth to a wormhole.

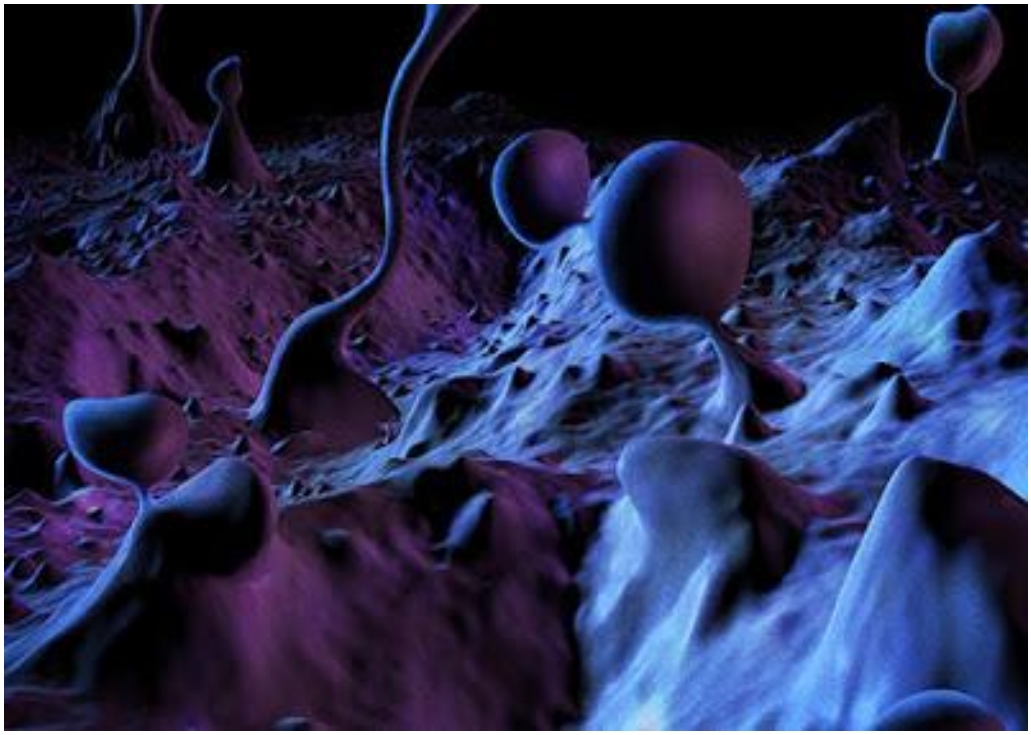


*Figure 17.1 Quantum Foam (Source: Space-Art)*

After making the wormhole, the next step would be to enlarge it so that large objects can pass through. And even this can be made possible via something called as negative energy. Negative energy gives a kind of anti-gravitational effects, basically, negative energy explains the nature of certain fields (as explored at the previous station). It thus tells us that after the big bang, there must be an equal amount of negative energy liberated along with the positive energy from which our universe is made. So stabilizing the quantum foam and then giving it some negative energy would make our natural time machine possible.

In 2015 scientists at the Autonomous University of Barcelona created a magnetic wormhole. This wormhole is not a space-time wormhole but instead a magnetic wormhole, which creates monopoles which do not exist in nature. It sounds very weird as far as I am concerned, but I would like to share this link with you. It's said that this magnetic wormhole has excellent application in MRI machines and also in making invisibility cloak since it's made from metamaterials. To understand metamaterials and invisibility cloak in more detail, check this link: https://phys.org/news/2016-02-invisibilityengineering-metamaterials.html.

## OPEN QUESTIONS

1. **ENTROPY AND THERMODYNAMICS**:

In 1971, Hawking showed under general conditions that the total area of the event horizons of any collection of classical black holes can never decrease, even if they collide and merge. This result, now known as the second law of black hole mechanics, is remarkably similar to the second law of thermodynamics, which states that the total entropy of a

system can never decrease. As with classical objects at absolute zero temperature, it was assumed that black holes had zero entropy. If this were the case, the second law of thermodynamics would be violated by entropy-laden matter entering a black hole, resulting in a decrease of the total entropy of the universe. Therefore, Bekenstein proposed that a black hole should have an entropy and that it should be proportional to its horizon area.

The link with the laws of thermodynamics was further strengthened by Hawking's discovery that quantum field theory predicts that a black hole radiates blackbody radiation at a constant temperature. This seemingly causes a violation of the second law of black hole mechanics, since the radiation will carry away energy from the black hole causing it to shrink. The radiation, however also carries away entropy, and it can be proven under general assumptions that the sum of the entropy of the matter surrounding a black hole and one-quarter of the area of the horizon as measured in Planck units is in fact always increasing. This allows the formulation of the first law of black hole mechanics as an analog of the first law of thermodynamics, with the mass acting as energy, the surface gravity as temperature and the area as entropy. The formula for the Bekenstein–Hawking entropy (S) of a black hole, which depends on the area of the black hole (*A*), the constants speed of light (c), the Boltzmann constant (k), Newton's constant (G), and the reduced Planck constant (h) is given as follows:

$$S = (¼)\ (c^3kA)/Gh$$

One puzzling feature is that the entropy of a black hole scales with its area rather than with its volume since entropy is normally an extensive quantity that scales linearly with the volume of the system. This odd property led Gerard't Hooft and Leonard Susskind to propose the holographic principle (discussed at the next station), which suggests that anything that happens in a volume of spacetime can be described by data on the boundary of that volume.

Although general relativity can be used to perform a semi-classical calculation of black hole entropy, this situation is theoretically unsatisfying. In statistical mechanics, entropy is understood as counting the number of microscopic configurations of a system that have the same macroscopic qualities (such as mass, charge, pressure, etc.). Without a satisfactory theory of quantum gravity, one cannot perform such a computation for black holes. Some progress has been made in various approaches to quantum gravity. In 1995, Andrew Strominger and Cumrun Vafa showed that counting the microstates of a specific supersymmetric black hole in string theory reproduced the Bekenstein–Hawking entropy. Since then, similar results have been reported for different black holes both in string theory and in other approaches to quantum gravity like loop quantum gravity.

## 2. INFORMATION LOSS PARADOX:

Because a black hole has only a few internal parameters, most of the information about the matter that went into forming the black hole is lost. Regardless of the type of matter which goes into a black hole, it appears that only information concerning the total mass, charge, and angular momentum are conserved. As long as black holes were thought to persist forever this information loss is not that problematic, as the information can be thought of as existing inside the black hole, inaccessible from the outside, but represented on the event horizon in accordance with the holographic principle. However, black holes

slowly evaporate by emitting Hawking radiation. This radiation does not appear to carry any additional information about the matter that formed the black hole, meaning that this information appears to be gone forever.

The question of whether the information is truly lost in black holes has divided the theoretical physics community. In quantum mechanics, loss of information corresponds to the violation of vital property called unitarity, which has to do with the conservation of probability. It has been argued that loss of unitarity would also imply a violation of conservation of energy. Over recent years evidence has been building that indeed information and unitarity are preserved in a full quantum gravitational treatment of the problem.

### 3. THE FIREWALL PARADOX:

According to the quantum field theory in curved spacetime, a single emission of Hawking radiation involves two mutually entangled particles. The outgoing particle escapes and is emitted as a quantum of Hawking radiation; the infalling particle is swallowed by the black hole. Assume a black hole formed a finite time in the past and will fully evaporate away in some finite time in the future. Then, it will only emit a finite amount of information encoded within its Hawking radiation. Assume that at time t, more than half of the information had already been emitted. According to widely accepted research by physicists like Don Page and Leonard Susskind, an outgoing particle emitted at time t must be entangled with all the Hawking radiation the black hole has previously emitted. This creates a paradox: a principle called "monogamy of entanglement" requires that, like any quantum system, the outgoing particle cannot be fully entangled with two independent systems at the same time; yet here the outgoing particle appears to be entangled with both the infalling particle and, independently, with past Hawking radiation.

In order to resolve the paradox, physicists may eventually be forced to give up one of three time-tested theories: Einstein's equivalence principle, unitarity, or existing quantum field theory. One possible solution, which violates the equivalence principle, is that a "firewall" destroys incoming particles at the event horizon. A 2016 analysis of LIGO data shows tentative signs of echoes caused by a fuzzy event horizon; such echoes may be possible in firewall or fuzzball theories but should not occur in classical general relativity. Over the next two years, additional LIGO data should establish whether the echoes were just random noise, or whether they are instead evidence of a violation of classical general relativity.

## HOLOGRAPHIC PRINCIPLE

Back in the early 1970's, Stephen Hawking wrote an astonishing equation which included quantum mechanics, relativity and information. Hawking's rather simple equation bought us closer to understanding the relationship between quantum physics and black holes. When an object gets closer to a black hole, the tidal forces crush the object. The information (arrangement of particles in some sense) of this objects shatters away completely. But the question which physicist raised was that, is the information lost? Or is it still there? Where exactly the information goes?

In Quantum mechanics, information can never be lost, it remains permanently in our universe. However, the phenomena of shattering of information at black hole gave rise to information paradox (explored at the previous station). Hawking's description to this phenomena was that the energy remains but the information disappears and that's where scientists started debating among themselves, eventually, an assumption was made, that information is stored in every volume of space. But any patch of space can become a black hole, nature's densest file cabinet, which stores information in bits of an area. This was first proposed by Gerard Hooft, it was given a precise string-theory interpretation by Leonard Susskind who combined his ideas with previous ones of Hooft and Charles Thorn. As pointed out by Raphael Bousso, Thorn observed in 1978 that string theory admits a lower-dimensional description in which gravity emerges from it in what would now be called a holographic way. Therefore, it was named as Holographic Principle and what it described is that all the things which are 'falling' inside a black hole are somehow captured in a preserved image of the horizon itself. So in layman terms, the object's information (arrangement of particles) is still there on the event horizon and if we somehow extracted that information we could eventually remake that object! Some string physicists also thing, since the information is stored at event horizon that means all the information might be on the event horizon. Therefore, we and everything in 'our' universe probably is a simulation on the surface of a black hole!

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